Knowledge in Lineland

(Extended Abstract)

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ABSTRACT

In this paper we investigate a concrete epistemic situation: there are agents (humans, robots, cameras,...) and propositions (lamps on or off, obstacles dangerous or not,...) located in Lineland. We express properties with the standard epistemic logic language like "Agent A knows that agent B knows that lamp L is on". We give some words about model-checking, satisfiability problem and common knowledge.

Categories and Subject Descriptors

I.2.4 [**Theory**]: Epistemic modal logic. Knowledge representation.

General Terms

Theory

Keywords

Knowledge representation. Spatial reasoning. Epistemic modal logic.

1. INTRODUCTION

In this article, we introduce a spatially grounded epistemic logic based on the simple case: a line. Our approach is different from [2] and even [4]. Here a model is directly a drawing like Figure 1 or 2 and not a Kripke model. This is motivated essentially because constructing the Kripke model by hand for a problem (e.g. Muddy Children etc.) gives the impression that we solve the problem by formalize it. With our approach, a problem is directly represented by its drawing (Figure 1).

☼	<	>	☼
a's forehead_dirty	a	b	b's forehead_dirty

Figure 1: Muddy-children

This logic provides a pedagogic graphical model-checker for students[1] based on the same idea that [1]. On the other hands, it may have some application in spatial reasoning in robotics or video games like in [3].

[1]You can find a model-checker implemented in Java/Scheme at `http://www.irit.fr/~Francois.Schwarzentruber/agentsandlamps/`, .

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<	☼	>	■	>	☼
1	p_1	2	p_2	3	p_3

Figure 2: Example of a world

2. SYNTAX

Our logic is based on the language of $S5_n$ [2]:

Definition 1 (language).
Let ATM, AGT be two countable sets of respectively atomic propositions and agents. The language $\mathcal{L}_{AGT}$ is defined by the following rule:

$$\varphi ::= \top \mid p \mid \varphi \wedge \varphi \mid \neg\varphi \mid K_a\psi$$

where $p \in ATM$ and $a \in AGT$.

As usual, $\varphi \vee \psi =^{def} \neg(\neg\varphi \wedge \neg\psi)$. $\hat{K}_a\psi =^{def} \neg K_a \neg\psi$.

The formula p is read as "the lamp p is on" and $K_a\psi$ means "agent a knows that p is true".

3. SEMANTICS

The semantics is not defined with a class of models but geometrically. A *world* is a situation where all agents have a *location* (*position* and *direction* where they look) in the line and all *lamps* (atomic propositions) have a *position* and a *state* (on or off). Formally:

Definition 2 (world).
A *world* w is a tuple $\langle \leq, d, \pi \rangle$ where

- $\leq$ is a total order over $AGT \cup ATM$;
- $d : AGT \to \{\texttt{left}, \texttt{right}\}$;
- $\pi : ATM \to \{\bot, \top\}$.

The set of all worlds is noted W. The order $\leq$ enumerates agents and lamps from left to right. $d(a)$ denotes the direction where the agent a looks. π is a valuation.

Example 1. *The Figure 2 gives us an example of a world* $\langle \leq, d_{AGT}, \pi \rangle$. *We have:*

- $1 \leq p_1 \leq 2 \leq p_2 \leq 3 \leq p_3$;
- $d(1) = \texttt{right}; d(2) = \texttt{left}; d(3) = \texttt{left}$;
- $\pi(p_1) = \top; \pi(p_2) = \bot; \pi(p_3) = \top$;

Now we are going to define the epistemic relation over worlds. wR_au means that agent a can not distinguish w from u, i.e. agent a sees the same things in w and u. Formally:

Definition 3 (epistemic relation). Let $a \in AGT$. We define the *epistemic relation* R_a on the set worlds W by wR_av iff:

- If $d(a) = \texttt{right}$,
  1. for all $x \in AGT \cup ATM$, ($a \leq_w x$ iff $a \leq_v x$);
  2. for all $x, y \in AGT \cup ATM$ such that $a \leq_w x$ and $a \leq_w y$, we have ($x \leq_w y$ iff $x \leq_v y$);
  3. for all $x \in AGT$, $a \leq_w x$ implies $d_w(x) = d_v(x)$;
  4. for all $x \in ATM$, $a \leq_w x$ implies $\pi_w(x) = \pi_v(x)$.
- Similarly, if $d(a) = \texttt{left}$ replace $\leq_w$ by $\geq_w$.

Briefly, suppose that wR_au and that $d(a) = \texttt{right}$. In this case, $a \leq_w x$ means x is on the left of a. As $d(a) = \texttt{right}$ means that a is looking to the left, $a \leq_w x$ means that a sees x. The condition 1. means that agent a sees the same lamps and agents in both w and v. The condition 2. means that if two objects x or y are seen by a in w (and also in v because it equivalent from 1.) then they are in the same order both in w and v.The condition 3. means that an agent x seen by a has the same direction both in w and v. The condition 4. means that a lamp seen by agent a has the same state both in w and v. If an object x is not seen by a in w, then 1. gives it is also not seen in v but there is no more constraints over the position, direction or state of the object. Until now, we have finally defined a model $\mathcal{M} = \langle W, (R_a)_{a \in AGT}, \nu \rangle$ where ν maps each world $w \in W$ to π_w. From now, the truth conditions is standard:

Definition 4 (truth conditions). Let $w \in W$. We define $w \models \varphi$ by induction:

- $w \models p$ iff $\pi(p) = \top$;
- Truth conditions for boolean connectives are standard;
- $w \models K_a\psi$ iff for all w', wR_aw' implies $w' \models \psi$.

We say that a formula φ is valid iff $\forall w \in W, w \models \varphi$.

3.1 Some validities

Since R_a is an equivalence relation on W, then the axioms T, 4 and 5 of classical epistemic logic are valid. But there are more validities in $L^{☼1D}$ than in $S5_n$.

The semantics of K_ap in $L^{☼1D}$ corresponds to the fact that the agent a sees the light p and the light p is on. Informally, $K_1(p \vee q)$ means that agent 1 has a proof that $p \vee q$. In other words, either he sees p on, or he sees q on. Hence, either K_1p or K_1q. More generally:

Proposition 1. *Let $\varphi, \psi \in \mathcal{L}_{AGT}$ such that agents and lamps appearing in φ and ψ are disjoint.*
$\models_{L^{☼1D}} K_1(\varphi \vee \psi) \rightarrow K_1\varphi \vee K_1\psi$.

Interestingly, we have $\models_{L^{☼1D}} K_1K_2p \wedge K_2K_1p \rightarrow (K_1K_2)^+p$ where "$(K_1K_2)^+$" denotes any finite sequence of K_1 and K_2. That is to say common knowledge comes only from $K_1K_2p \wedge K_2K_1p$ like in Figure 3.

More surprising is the fact that common knowledge is not guaranteed by $K_1K_2\varphi \wedge K_2K_1\varphi$ for all φ. Consider the world w of Figure 4. We have $w \models K_1K_2\neg K_2p \wedge K_2K_1\neg K_2p$. But, we have $w \not\models K_1K_2K_1\neg K_2p$.

In fact, $L^{☼1D}$ lacks the property of uniform substitution.

$<$ ☼ $>$
1 p 2

Figure 3: Common-knowledge of p

$<$ $<$ ■
2 1 p

Figure 4: $w \models K_1K_2\neg K_2p \wedge K_2K_1\neg K_2p \wedge \neg K_1K_2K_1\neg K_2p$

4. TWO DECISION PROBLEMS

4.1 Definitions

Definition 5 (model-checking of $L^{☼1D}$). We call *model-checking of* $L^{☼1D}$ the following problem:

- Input: a formula $\varphi \in \mathcal{L}_{AGT}$, a world w (where only atoms and agents occurring in φ are taken in account);
- Output: Yes iff we have $w \models_{L^{☼1D}} \varphi$. No, otherwise.

In the Definition 5, we do not care about propositions or agents not in the formula φ. In particular, the data structure for the order $\leq$ is a *finite* list representing a permutation over agents' and propositions' occurring in φ .

Definition 6 ($L^{☼1D}$-satisfiability problem). We call $L^{☼1D}$-satisfiability problem the following problem:

- Input: a formula $\varphi \in \mathcal{L}_{AGT}$;
- Output: Yes iff there exists a w s.th. $w \models_{L^{☼1D}} \varphi$.

Proposition 2. *The model-checking of $L^{☼1D}$ and satisfiability problem are in PSPACE.*

Moreover, if AGT is infinite, we can reduce those two problems to Quantified propositional logic satisfiability problem and then the two problems are indeed PSPACE-complete.

5. CONCLUSION

We have presented a spatially grounded epistemic logic. One advantage is that a model is very close to the reality it represents. Furthermore, model-checking and satisfiability remains in PSPACE as for $S5_n$. From now, there are many perspectives: study in more details complexities when AGT is finite, find an axiomatization. And above all study Flatland...

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6. REFERENCES

[1] Jon Barwise and John Etchemendy. *Tarski's World: Version 4.0 for Macintosh (Center for the Study of Language and Information - Lecture Notes)*. Center for the Study of Language and Information/SRI, 1993.

[2] Joseph Y. Halpern and Yoram Moses. A guide to completeness and complexity for modal logics of knowledge and belief. *Artificial Intelligence*, 54(3):319–379, 1992.

[3] Ethan Kennerly, Andreas Witzel, and Jonathan A. Zvesper. Thief belief (extended abstract). Presented at Logic and the Simulation of Interaction and Reasoning 2 (LSIR2) Workshop at IJCAI-09, July 2009.

[4] Rohit Parikh, Lawrence S. Moss, and Chris Steinsvold. Topology and epistemic logic. In *Handbook of Spatial Logics*, pages 299–341. 2007.