Complexity of Multiagent BDI Logics with Restricted Modal Context

(Extended Abstract)

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ABSTRACT

In this paper we present and discuss a novel language restriction for modal logics for multiagent systems that can reduce the complexity of the satisfiability problem from EXPTIME-hard to NPTIME-complete. In the discussion we focus on a particular BDI logic, called TeamLog, which is a logic for modelling cooperating groups of agents and which possesses some of the characteristics typical to other BDI logics. All the technical results can be found in the dissertation [5].

Categories and Subject Descriptors

I.2.4 [**ARTIFICIAL INTELLIGENCE**]: Knowledge Representation Formalisms and Methods—*Modal Logic*

General Terms

Theory

Keywords

Multiagent Theories, BDI, Teamwork, Modal Logic, Satisfiability

1. INTRODUCTION

One of the most influential models of agency is the *beliefs-desires-intentions (BDI) model* [2] and logical formalisms based on the BDI model [3, 10] are among the most important in the field of multiagent systems. One of the characteristics of these multimodal formalisms is adopting, along with standard modal systems K_n, KD_n or $\mathrm{KD45}_n$, *mixed axioms* that interrelate modalities representing different aspects of agent description. Examples of such axioms are *realism axioms* [3, 10] and *introspection axioms* [4].

It is well known that the extension of these formalisms with fixpoint modalities representing group aspects of multiagent systems [9, 11, 1, 4] lead to EXPTIME-hardness of the satisfiability problem, even if modal depth of formulas is bounded by 2 [8, 7].

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To deal with this problem we propose a new kind of language restriction called *modal context restriction*. In [6] we applied this restriction to standard systems of multimodal logics enriched with fix point modalities and showed that it leads to PSPACE-completeness and, when combined with modal depth restriction, to NPTIME-completeness of the satisfiability problem. In this paper we present modal context restrictions for BDI logics, choosing, as a 'working' formalism, TeamLog [4], a well known and important formalism that focuses on teamwork.

2. THE FORMALISM

TeamLog is a logical framework proposed to formalize individual and group aspects of BDI systems [4]. It is a multimodal formalism with the set of modal operators based on a non-empty and finite set of agents, $\mathcal{A}$: $\Omega^{\mathrm{T}} = \Omega^{\mathrm{B}^+} \cup \Omega^{\mathrm{G}} \cup \Omega^{\mathrm{I}^+}$, where $\Omega^{\mathrm{B}^+} = \Omega^{\mathrm{B}} \cup \{[\mathrm{B}]^+_G : G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}\}$, $\Omega^{\mathrm{I}^+} = \Omega^{\mathrm{I}} \cup \{[\mathrm{I}]^+_G : G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}\}$, $\Omega^{\mathrm{B}} = \{[\mathrm{B}]_j : j \in \mathcal{A}\}$, $\Omega^{\mathrm{G}} = \{[\mathrm{G}]_j : j \in \mathcal{A}\}$ and $\Omega^{\mathrm{I}} = \{[\mathrm{I}]_j : j \in \mathcal{A}\}$.[1] Operators $[\mathrm{B}]_j$, $[\mathrm{G}]_j$ and $[\mathrm{I}]_j$ stand for beliefs, goals and intentions of agent j, respectively, while $[\mathrm{B}]^+_G$ and $[\mathrm{I}]^+_G$ are fixpoint modalities standing for common beliefs and mutual intentions of group G, respectively. The propositional multimodal language $\mathcal{L}^{\mathrm{T}}$ of TeamLog and its semantics are defined in the usual way (see [4] for details).

An important aspect of the formalism are mixed axioms, interrelating different attitudes of individual agents. The fact that for each agent j intentions are a subset of goals, is reflected in the **goals-intentions compatibility** axiom $[\mathrm{I}]_j\varphi \to [\mathrm{G}]_j\varphi$. The fact that each agent j is fully aware of his goals and intentions is reflected in **positive** and **negative introspection** axioms: $[O]_j\varphi \to [\mathrm{B}]_j[O]_j\varphi$ and $\neg[O]_j\varphi \to [\mathrm{B}]_j\neg[O]_j\varphi$, where $O \in \{\mathrm{G}, \mathrm{I}\}$.

As was shown in [7], the TeamLog satisfiability problem is EXPTIME-complete.

3. MODAL CONTEXT RESTRICTION

We start by defining the notion of modal context restriction for general language of multimodal logic. First we need a notion of modal context of a formula within a formula. Let $\mathcal{L}$ be a multimodal language defined over some set of unary modal operators Ω.

[1] For the sake of conciseness we will use a more compact notation for operators of TeamLog, replacing that standard ones from [4].

Definition 1. Let $\{\varphi, \xi\} \subseteq \mathcal{L}$. The *modal context of formula ξ within formula φ* is a set of finite sequences over Ω, $\mathrm{cont}(\xi, \varphi) \subseteq \Omega^*$, defined inductively as follows:

- $\mathrm{cont}(\xi, \varphi) = \varnothing$, if $\xi \notin \mathrm{Sub}(\varphi)$,
- $\mathrm{cont}(\varphi, \varphi) = \{\varepsilon\}$,
- $\mathrm{cont}(\xi, \neg\psi) = \mathrm{cont}(\xi, \psi)$, if $\xi \neq \neg\psi$,
- $\mathrm{cont}(\xi, \psi_1 \wedge \psi_2) = \mathrm{cont}(\xi, \psi_1) \cup \mathrm{cont}(\xi, \psi_2)$, if $\xi \neq \psi_1 \wedge \psi_2$,
- $\mathrm{cont}(\xi, \Box\psi) = \Box \cdot \mathrm{cont}(\xi, \psi_j)$, if $\xi \neq \Box\psi$ and $\Box \in \Omega$,

where $\mathrm{Sub}(\varphi)$ denotes the set of all subformulas of φ and $\Box \cdot S = \{\Box \cdot s : s \in S\}$, for $\Box \in \Omega$ and $S \subseteq \Omega^*$.

Definition 2. A *modal context restriction* is a set of finite sequences over Ω, $R \subseteq \Omega^*$, constraining possible modal contexts of subformulas within formulas. We say that a formula $\varphi \in \mathcal{L}$ *satisfies a modal context restriction* $R \subseteq \Omega^*$ iff for all $\xi \in \mathrm{Sub}(\varphi)$ it holds that $\mathrm{cont}(\xi, \varphi) \subseteq R$.

In this paper we propose two modal context restrictions of the language of TeamLog that lead to PSPACE completeness of the satisfiability problem. The restrictions are presented below.

Definition 3. Let

$$\mathbf{R_1} = \Omega^* \setminus \left(\Omega^* \cdot \left[\bigcup_{G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}} (S_{\mathrm{I}}(G) \cup S_{\mathrm{IB}}(G)) \cup \bigcup_{G \in \mathrm{P}(\mathcal{A}), |G| \geq 2} S_{\mathrm{B}}(G)\right] \cdot \Omega^*\right),$$

where

$$S_{\mathrm{IB}}(G) = \bigcup_{j \in G} [\mathrm{I}]_G^+ \cdot ([\mathrm{B}]_j)^* \cdot T_{\mathrm{B}}(\{j\}) \cdot T_{\mathrm{I}}(\{j\}), \text{ and}$$
$$S_O(G) = [O]_G^+ \cdot T_O(G),$$
$$T_O(G) = \{[O]_j : j \in G\} \cup \{[O]_H^+ : H \in \mathrm{P}(\mathcal{A}), H \cap G \neq \varnothing\},$$

for $O \in \{\mathrm{B}, \mathrm{I}\}$. The set of formulas in $\mathcal{L}^{\mathrm{T}}$ satisfying restriction $\mathbf{R}_1$ will be denoted by $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$.

Definition 4. Let

$$\mathbf{R_2} = \Omega^* \setminus \left(\Omega^* \cdot \left[\bigcup_{G \in \mathrm{P}(\mathcal{A}) \setminus \{\varnothing\}} (S_{\mathrm{I}}(G) \cup S_{\mathrm{IB}}(G)) \cup \bigcup_{G \in \mathrm{P}(\mathcal{A}), |G| \geq 2} \tilde{S}_{\mathrm{B}}(G)\right] \cdot \Omega^*\right),$$

where

$$\tilde{S}_{\mathrm{B}}(G) = [\mathrm{B}]_G^+ \cdot \left(\{[\mathrm{G}]_j : j \in G\} \cup \bigcup_{O \in \{\mathrm{B}, \mathrm{I}\}} T_O(G)\right)$$

and S_{IB}, S_{I} and T_O, for $O \in \{\mathrm{B}, \mathrm{I}\}$, are defined like in the case of restriction $\mathbf{R}_1$. The set of formulas in $\mathcal{L}^{\mathrm{T}}$ satisfying restriction $\mathbf{R}_2$ will be denoted by $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$.

Restriction $\mathbf{R}_1$ forbids any operator $[O]_j$ or $[O]_H^+$, with $O \in \{\mathrm{B}, \mathrm{I}\}$ in the context of $[O]_G^+$, if $j \in G$ or $H \cap G \neq \varnothing$. Additionally the restriction forbids subsequences contained in S_{IB}. Forbidding subsequences from S_{IB} is related to axioms of positive and negative introspection of intentions. Restriction $\mathbf{R}_2$ is a refinement of restriction $\mathbf{R}_1$ which forbids any operator $[O]_j$ or $[O]_H^+$, with $O \in \{\mathrm{B}, \mathrm{G}, \mathrm{I}\}$ in the context of $[\mathrm{B}]_G^+$, if $j \in G$ or $H \cap G \neq \varnothing$. Thus any formula $\varphi \in \mathcal{L}^{\mathrm{T}}$ satisfying restriction $\mathbf{R}_2$, satisfies restriction $\mathbf{R}_1$ as well, that is $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}} \subseteq \mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$. Notice that if $|\mathcal{A}| = 1$, then $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}} = \mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$.

We have the following results regarding the complexity of the TeamLog satisfiability problems for formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ and $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$.

Theorem 1. *The* TeamLog *satisfiability problem for formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$ is PSPACE-complete. Moreover, it is NPTIME-complete if model depth of formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_2}}$ is bounded by a constant.*

Theorem 2. *The* TeamLog *satisfiability problem for formulas from $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ is PSPACE-complete, even if modal depth of formula is bounded by a constant ≥ 2.*

It is worth noting that the second result was obtained despite the fact that formulas of $\mathcal{L}^{\mathrm{T}}_{\mathbf{R_1}}$ can enforce exponential path in the model.

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