Knowledge and Control

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ABSTRACT

Logics of propositional control, such as van der Hoek and Wooldridge’s CL-PC [14], were introduced in order to represent and reason about scenarios in which each agent within a system is able to exercise unique control over some set of system variables. Our aim in the present paper is to extend the study of logics of propositional control to settings in which these agents have incomplete information about the society they occupy. We consider two possible sources of incomplete information. First, we consider the possibility that an agent is only able to “read” a subset of the overall system variables, and so in any given system state, will have partial information about the state of the system. Second, we consider the possibility that an agent has incomplete information about which agent controls which variables. For both cases, we introduce a logic combining epistemic modalities with the operators of CL-PC, investigate its axiomatization, and discuss its properties.

Categories and Subject Descriptors

I.2.11 [Distributed Artificial Intelligence]: Multiagent Systems; I.2.4 [Knowledge representation formalisms and methods]

General Terms

Theory

Keywords

epistemic logic, propositional control, partial observability

1. INTRODUCTION

The Coalition Logic of Propositional Control (CL-PC) was introduced by van der Hoek and Wooldridge as a formalism for reasoning about how agents and coalitions can exercise control in multiagent environments [14]. The logic models situations in which each agent has control over some set of propositions; that is, each agent is associated with some set of variables that the agent is able to “see”. Partial observability interacts with control in several important ways. First, and most obviously, an agent may be uncertain about the value of the variables in the system. We call this type of uncertainty partial observability, and it is very naturally modelled by assigning to every agent a set of variables that the agent is able to “see”. Partial observability interacts with control in several important ways. For example, if I control the variable $q$ and my goal is to achieve the formula $p \iff \neg q$, then if I can observe the value of $p$, I can readily choose a value for $q$ that will result in my goal being achieved: I simply choose the opposite to the value of $p$. However, if I cannot see the value of $p$, then I am in trouble. Second, and perhaps more unusually, there may be uncertainty about which agent controls which variables. Here, too, we might conceivably have a situation in which an agent is able to bring about some state of affairs, but does not know that they are able to bring it about, because it is not...
aware that it controls the appropriate variables.

The aim of the present paper is to develop extensions to CL-PC that are able to capture these types of uncertainty. The remainder of the paper is structured as follows. After presenting some definitions that will be used throughout the remainder of the paper, in Section 2, we present the epistemic extension to CL-PC for the case that agents have complete knowledge about how the control of variables is actually distributed over the agents, but they may lack information about what is factually true. Subsequently, in Section 3, we then look at formalising the case where agents have full knowledge about factual truth, partial knowledge about who controls what, and are completely ignorant about other’s information regarding control. We also sketch an even more general setting where both factual truth and control may be uncertain. We conclude in Section 4.

We begin with some definitions, which are used throughout the remainder of the paper. First, let $\mathbb{B} = \{true, false\}$ be the set of Boolean truth values. We assume that the domains we model contain a (finite, non-empty) set $N = \{1, \ldots, n\}$ of agents ($|N| = n$, $n > 0$). The environment is also assumed to contain a (fixed, finite) set $\mathcal{A} = \{p, q, \ldots\}$ of Boolean variables. Each agent $i$ in $N$ will be assumed to control some subset $\mathcal{A}_i$ of atoms $\mathcal{A}$, with the intended interpretation that if $p \in \mathcal{A}_i$, then $i$ has the unique ability to assign a value (true or false) to $p$. We require that the sets $\mathcal{A}_i$ form a partition of $\mathcal{A}$, i.e., $\mathcal{A}_i \cap \mathcal{A}_j = \emptyset$ for $i \neq j$, and $\mathcal{A}_1 \cup \cdots \cup \mathcal{A}_n = \mathcal{A}$. Thus every variable is controlled by some agent; and no variable is controlled by more than one agent. A coalition is simply a set of agents, i.e., a subset of $N$. We typically use $C, C', \ldots$ as variables standing for coalitions. Where $C \subseteq N$, we denote by $\mathcal{A}_C$ the set of variables under the collective control of the agents in $C$: $\mathcal{A}_C = \bigcup_{i \in C} \mathcal{A}_i$. A valuation is a total function $\theta : \mathcal{A} \rightarrow \mathbb{B}$, which assigns a truth value to every Boolean variable. Let $\Theta$ denote the set of all valuations. Where $C$ is a coalition, a $C$-valuation is a function $\theta_C : \mathcal{A}_C \rightarrow \mathbb{B}$; thus a $C$-valuation is a valuation to variables under the control of the agents in $C$. Given a set $X$ of atoms and two valuations $\theta_1$ and $\theta_2$, we write $\theta_1 \equiv_X \theta_2$ to mean that $\theta_1$ and $\theta_2$ agree on the value of all variables in $X$, i.e., $\theta_1(p) = \theta_2(p)$ for all $p \in X$.

2. PARTIAL OBSERVABILITY

In this section, we develop an Epistemic Coalition Logic of Propositional Control with Partial Observability – ECL-PC(PO) for short.

This logic is essentially CL-PC extended with epistemic modalities $K_i$, one for each agent $i \in N$. These epistemic modalities have a conventional (SS) possible worlds semantics. The interpretation we give to epistemic accessibility relations is as follows. We assume each agent $i \in N$ is able to see a subset $V_i \subseteq \mathcal{A}$ of the overall set of Boolean variables; that is, it is able to correctly perceive the value of these variables. A valuation $\theta'$ is then $i$-accessible from valuation $\theta$ if $\theta$ and $\theta'$ agree on the valuation of variables visible to $i$, i.e., $\theta \equiv_{V_i} \theta'$. Formally, the language of ECL-PC(PO) is defined by the following BNF grammar:

$$\phi ::= p \mid \neg \phi \mid \phi \lor \psi \mid \diamond \phi \mid K_i \phi$$

where $p \in \mathcal{A}$, and $i \in N$. As in CL-PC [14], a formula $\diamond \phi$ means that $i$ can assign values to the variables under its control in such a way that, assuming no other variables are changed, $\phi$ becomes true. As in epistemic logic [6], a formula $K_i \phi$ means that the agent $i$ knows $\phi$.

The remaining operators of classical logic ("$\land$, and", "$\lor$, or" – implies, "$\rightarrow$, if") are assumed to be defined as abbreviations in terms of $\neg, \lor$ as usual. We define the box dual operator of $\diamond$, as: $\Box_i \phi \equiv \neg \diamond_i \neg \phi$. We also assume the existential dual $M_i$ ("maybe") of the $K_i$ operator is defined as: $M_i \phi \equiv \neg K_i \neg \phi$. For coalitions, we define (this definition is justified in [14]):

$$\Box_i \phi_i \equiv \Box_1 \cdots \Box_i \phi_i.$$

Coming to the semantics, a frame for CL-PC is simply a structure $\langle N, \mathcal{A}_1, \ldots, \mathcal{A}_n \rangle$, where $N$ is the set of agents in the system, and each $\mathcal{A}_i$ is the set of variables under the control of agent $i$; a model for CL-PC combines such a frame with a valuation $\theta \in \Theta$, which gives an initial value for every Boolean variable [14]. Frames for ECL-PC(PO) extend CL-PC frames with a set of variables $V_i \subseteq \mathcal{A}$ for each agent $i \in N$. Formally, an ECL-PC(PO) frame, $F$, is a $(2n + 1)$-tuple

$$F = \langle N, \mathcal{A}_1, \ldots, \mathcal{A}_n, V_1, \ldots, V_n \rangle,$$

- $N = \{1, 2, \ldots, n\}$ is a (finite, nonempty) set of agents.
- The sets $\mathcal{A}_i$ form a partition of $\mathcal{A}$.
- $V_i \subseteq \mathcal{A}$ is the set of variables whose values are visible to $i$.

It will often make sense to assume $V_i \supseteq \mathcal{A}_i$, i.e., each agent can see the value of the variables it controls; however, we will not impose this as a requirement. We leave aside the question for now of what settings there are in which this assumption does not hold.

The truth value of an ECL-PC(PO) formula is inductively defined wrt. a frame $F$ and a valuation $\theta$ by the following rules ($\models$ stands for a ‘direct semantics’, [14]):

$$F, \theta \models \phi \quad \text{iff} \quad \theta(p) = true \quad (p \in \mathcal{A})$$

- $F, \theta \models \neg \phi \quad $iff $ F, \theta \not\models \phi$
- $F, \theta \models \phi \lor \psi$ $\quad$iff $ F, \theta \models \phi \lor F, \theta \models \psi$
- $F, \theta \models \diamond \phi \quad$iff $ \exists \theta' \in \Theta : \theta' \equiv_{V_i} \theta \land M_i \theta' \models \phi$
- $F, \theta \models K_i \phi \quad$iff $ \forall \theta' \in \Theta : \theta' \equiv_{V_i} \theta \Rightarrow M_i \theta' \models \phi$

We denote the fact that $\phi$ is true in all models by $\models \phi$. We let $\Lambda_1 = \{\phi \mid \models \phi\}$ be the logic of all the formulas valid in all ECL-PC(PO) models.

**EXAMPLE 1.** Suppose we have a frame $F$ with two agents, $N = \{1, 2\}$ and two Boolean variables, $\mathcal{A} = \{p, q\}$, with $\mathcal{A}_1 = \{p\}$ and $\mathcal{A}_2 = \{q\}$, and $V_2 = \{p, q\}$. Thus agent 1 can only see the value of the variable it controls, while agent 2 can see the values of both variables. Let $\theta_1(p) = \theta(q) = \text{true}$. Now, we have:

- $F, \theta \models \diamond_1 (p \iff \neg q)$
- Agent 1 can set his variable $p$ in such a way that $p$ and $q$ have different values.
- $F, \theta \models \neg K_1 q \land \neg K_1 \neg q \land K_1 (K_2 q \lor K_2 \neg q)$
- Agent 1 does not know the value of variable $q$, but he does know that 2 knows the value of $q$.
- $F, \theta \models K_1 \diamond_1 (p \iff \neg q) \land \neg \diamond_1 K_1 (p \iff \neg q)$
- Agent 1 knows that he can make $p$ and $q$ take on different values (because he controls $p$, and hence can make it different to $q$ in any given state). However, agent 1 cannot choose values for the variables he controls in such a way that he knows that $p$ and $q$ take on different values.
- $F, \theta \models K_2 \diamond_1 ((K_2 p \lor K_2 \neg p) \land (K_2 q \lor K_2 \neg q))$
- Agent 2 knows that whatever truth values 1 chooses for her variables, 2 will know the value of $p$ and of $q$.
- $F, \theta \models K_2 ((p \land q) \land \diamond_1 (\neg p \land \neg \diamond_2 (\neg p \land q)))$
- Agent 2 knows that $(p \land q)$ and that 1 can bring about that $\neg p$ which 2 can further narrow down to $(\neg p \land \neg q)$. 

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The elements of $\mathcal{A}$, $\text{CTRL}$ and $\text{VIEW}$ are called basic propositions. For any set $\Phi$ of basic propositions, call $L(\Phi) = \{x, \neg x \mid x \in \Phi\}$ the set of literals over $\Phi$. For a basic proposition $x$, let $\ell(x) \in \{x, \neg x\}$. So e.g., $\ell(p) \rightarrow \Box_i \ell(p)$ stands both for $p \rightarrow \Box_i p$ and for $\neg p \rightarrow \Box_i \neg p$. A propositional description $\pi$ is a conjunction over $L(\mathcal{A})$ where each $p \in \mathcal{A}$ occurs exactly once. Let $\Gamma$ be the set of propositional descriptions. A control description $\gamma$ is a conjunction over $\mathcal{C}$ such that for every $p \in \mathcal{A}$, there is exactly one $i \in N$ such that $\text{ctrls}(i, p)$ occurs in $\gamma$. Let $\Sigma$ be the set of visibility descriptions. A full description is a conjunction $\pi \land \gamma \land \varsigma$, where $\pi, \gamma$ and $\varsigma$ are as explained above.

Given a propositional description $\pi \in \Pi$, we shall note $\pi^1$ the conjunction of literals in $\pi$ that are under the control of agent $i$ and $\pi^2$ the conjunction of literals that are not under its control. Of course $\pi \equiv \pi^1 \land \pi^2$. In the same vein, we shall note $\pi^3$ the conjunction of literals in $\pi$ that are seen by agent $i$ and $\pi^4$ the conjunction of literals in $\pi$ that are not seen by it. Again $\pi \equiv \pi^3 \land \pi^4$.

As its name suggests, a full description $(\pi \land \gamma \land \varsigma)$ fully characterises a situation: it specifies which atoms are true and which are false (this is $\pi$), it specifies which agents control which variables (through $\gamma$) and it specifies exactly which propositional variables each agent can see (through $\varsigma$). So semantically, it is immediately clear that any formula will be a disjunction of such full descriptions (namely, descriptions of those situations where $\varphi$ is true), but our task is now to show that this is derivable in the logic.

The next Lemma states a few theorems derivable within our axiomatic system, all of which are instrumental in the proofs of Theorem 2 and of Theorem 3.

**Lemma 2.** Let $\pi, \gamma$ and $\varsigma$ be propositional, control and visibility descriptions, respectively (and so are their 'primed' version). For $P \subseteq \mathcal{A}$, let $\pi_1(L(P))$ be a conjunction over $L(P)$ and let $\pi_2(L(\mathcal{A} \setminus P))$ be a conjunction over $L(\mathcal{A} \setminus P)$.

Then, the following are derivable in $\Lambda_2$:

1. $\neg \text{ctrls}(i, p) \rightarrow (\ell(p) \rightarrow \Box_i \ell(p))$
2. $\text{sees}(i, p) \rightarrow (\ell(p) \rightarrow K_i \ell(p))$
3. $(\ell(\text{ctrls}(i, p)) \rightarrow \Box_N \ell(\text{ctrls}(i, p))$
4. $(\ell(\text{sees}(i, p)) \rightarrow \Box_N \ell(\text{sees}(i, p))$
5. $\bigwedge_{p \in P} \text{ctrls}(i, p) \land \bigwedge_{p \not\in P} \neg \text{ctrls}(i, p) \rightarrow (\pi_1(L(P)) \land (\pi_2(L(\mathcal{A} \setminus P)) \rightarrow \Box_i \pi_2(L(\mathcal{A} \setminus P))

6. $\Diamond_i (\pi^1 \land \pi^2) \rightarrow \pi^3$
7. $\bigwedge_{p \in P} \text{sees}(i, p) \land \bigwedge_{p \not\in P} \neg \text{sees}(i, p) \rightarrow M_i \pi_2(L(\mathcal{A} \setminus P)) \land (\pi_1(L(P)) \rightarrow K_i \pi_1(L(P))$
8. $M_i (\pi^3 \land \pi^4) \rightarrow \pi^3$
9. $\Box_N \varphi \rightarrow \Box_i \Box_N \varphi$
10. $\Box_N \varphi \rightarrow K_i \Box_N \varphi$
11. $(\pi \land \gamma \land \varsigma) \rightarrow (\pi \land \Box_N \gamma \land \Box_N \varsigma)$

**Theorem 2 (Normal Form).** Every formula $\varphi$ is provably equivalent to a disjunction of full descriptions, i.e., for every $\varphi$ there exists a $k$ and $\pi_1, \gamma_1, \varsigma_1$ ($1 \leq i \leq k$) such that

$$\vdash \varphi \leftrightarrow \bigvee_{j \leq k} (\pi_j \land \Box_N \gamma_j \land \Box_N \varsigma_j)$$

Figure 1: Axiomatics of $\Lambda_1$. The meta-variable $i$ ranges over $N$, $C_i$ and $C_2$ over $2^N$, $\varphi$ represents an arbitrary formula of $\text{ECL-PC}(\text{PO})$, $p$ ranges over $\mathcal{A}$, $\varphi_X$ is the conjunction of literals true in any valuation of $X \subseteq \mathcal{A}$.
Proof. By Lemma 2.11, it follows from
\[ \vdash \varphi \iff \bigvee_{j \leq k} (\pi_j \land \gamma_j \land \varsigma_j) \]
which we prove now by induction on the structure of \( \varphi \).

We will make use of the fact that the sets of propositional (\( \Pi \)), control (\( \Gamma \)) and visibility (\( \Sigma \)) descriptions are finite. Roughly speaking, a triple \( (\pi, \gamma, \varsigma) \) represents a state. The idea behind the normal form is that a formula can be represented by a subset \( X \subseteq \Pi \times \Gamma \times \Sigma \), which translates in the language as a (typically large) disjunction of formulas of the form \( \pi \land \gamma \land \varsigma \).

One base case is for \( \varphi \) being a proposition in \( \Phi \).

\[ \vdash p \iff \bigvee_{\pi_i \in \Pi} \bigvee_{\gamma_i \in \Gamma} \bigvee_{\varsigma_i \in \Sigma} (\pi_i \land \gamma_i \land \varsigma_i) \]

The statement \( \pi_i \vdash p \) means that \( p \) appears as a positive literal in \( \pi_i \). The two other base cases \( \varphi = \text{ctrls}(i, p) \) and \( \varphi = \text{sees}(i, p) \) are analogous.

Now we suppose for induction that \( \varphi \) can be transformed into an equivalent formula \( \bigvee_{j \leq k} (\pi_j \land \gamma_j \land \varsigma_j) \).

Case \( \psi = \neg \varphi \): "\( \psi \) is represented by the complement of the states representing \( \varphi \)."

\[ \vdash \psi \iff \bigvee_{j \leq k} \bigvee_{(\pi, \gamma, \varsigma) \not\in \Phi} (\pi \land \gamma \land \varsigma) \]

Case \( \psi = \lor \varphi_1 \lor \varphi_2 \): since the normal form itself is a disjunction, this case is straightforward.

Case \( \psi = \land \varphi_1 \lor \varphi \): similar to \( \psi \) is \( \land \varphi \).

Case \( \psi = \lor \varphi_1 \lor \varphi_2 \): by induction hypothesis

\[ \vdash \psi \iff \bigvee_{j \leq k} M_i (\pi_j \land \gamma_j \land \varsigma_j) \]

By modal logic

\[ \vdash \psi \iff \bigvee_{j \leq k} M_i (\pi_j \land \gamma_j \land \varsigma_j) \]

By Lemma 2.10 and Lemma 2.11

\[ \vdash \psi \iff \bigvee_{j \leq k} (M_i \pi_j \land K_i \Box_N \gamma_j \land K_i \Box_N \varsigma_j) \]

By S5(\( \mathcal{K} \))

\[ \vdash \psi \iff \bigvee_{j \leq k} (M_i \pi_j \land K_i \Box_N \gamma_j \land K_i \Box_N \varsigma_j) \]

Applying our notation and Lemma 2.11 and Lemma 2.10

\[ \vdash \psi \iff \bigvee_{j \leq k} (M_i (\pi_j \land \gamma_j \land \varsigma_j)) \]

By Lemma 2.8

\[ \vdash \psi \iff \bigvee_{j \leq k} (\pi_j \land \gamma_j \land \varsigma_j) \]

Finally,

\[ \vdash \psi \iff \bigvee_{j \leq k} \bigvee_{\delta_j \in \Pi(\pi_j)} ((\pi_j \land \gamma_j \land \varsigma_j)) \]

where \( \Pi(\pi_j) \) is the set of propositional descriptions restricted to the set of atoms occurring in \( \pi_j \), that is, that are not seen by \( i \).

We require some subsidiary definitions. We begin by defining an alternative, possible worlds semantics for ECL-PC(PO). Given a frame \( F \), a Kripke model for ECL-PC(PO) is a structure

\[ \mathcal{K} = (W, R_1^0, \ldots, R_n^0, R_1^K, \ldots, R_n^K, \pi) \]

where \( W = \emptyset \) is a set of worlds, which correspond to possible worlds \( \approx \), \( R^0 \subseteq W \times W \), and \( R^K \subseteq W \times W \), where these latter relations are defined as:

\[ R^0(w, w') \text{ iff } w \equiv_{\approx \setminus \approx_1} w' \text{, and } R^K(w, w') \text{ iff } w \equiv_{\approx} w' \]

Finally, \( \pi : W \rightarrow 2^k \) gives the set of Boolean variables true at each world. The key clauses for \( \models^\mathcal{K} (\text{‘Kripke semantics}) \) are then as follows:

\[ K, w \models^\mathcal{K} p \text{ iff } p \in \pi(w) \text{ (} p \in \approx \text{)} \]

\[ K, w \models^\mathcal{K} \land \varphi \text{ iff } \exists w' \in W \text{ s.t. } R^0(w, w') \text{ and } K, w' \models^\mathcal{K} \varphi \]

\[ K, w \models^\mathcal{K} \lor \varphi \text{ iff } \forall w' \in W \text{ s.t. } R^K(w, w') \text{ and } K, w' \models^\mathcal{K} \varphi \]

Lemma 3. Let \( F, \theta \) be an ECL-PC(PO) frame and associated valuation, let \( \mathcal{K} \), \( K \) be the corresponding Kripke model and world, and let \( \varphi \) be an arbitrary ECL-PC(PO) formula. Then:

\[ F, \theta \models^\mathcal{K} \varphi \text{ iff } K, w \models^\mathcal{K} \varphi \]

We assume the standard definitions of maximally consistent sets and their existence via Lindenbaum’s lemma (see, e.g., [4, p.196]). We proceed to construct a canonical model:

\[ \hat{K} = (\hat{W}, \hat{R}_1^0, \ldots, \hat{R}_n^0, \hat{R}_1^K, \ldots, \hat{R}_n^K, \hat{\pi}) \]

where:

- \( \hat{W} \) is the set of all \( \approx_1 \) maximally consistent sets;
- \( \hat{R}_i^0(w, w') \text{ iff } \phi \in w' \text{ implies } \lor \phi \in w; \)
- \( \hat{R}_i^K(w, w') \text{ iff } \phi \in w' \text{ implies } M_i \phi \in w \text{; and} \)
- \( \hat{\pi}(w) = \approx \cap w. \)

The following is a standard result for canonical models:

Lemma 4 (Truth Lemma.). Let \( \hat{K} = (\hat{W}, \hat{R}_1^0, \ldots, \hat{R}_n^0, \hat{R}_1^K, \ldots, \hat{R}_n^K, \hat{\pi}) \) be a canonical model, \( w \in \hat{W} \) be a world in \( \hat{K} \), and \( \varphi \) be an arbitrary ECL-PC(PO) formula. Then:

\[ \hat{K}, w \models^\hat{\mathcal{K}} \varphi \text{ iff } \varphi \in w. \]

The truth lemma above gives rise to completeness wrt. a set of models, but it is not the kind of models we have associated with ECL-PC(PO). In the intended models, the modalities \( K_i \) and \( \land \) are defined with respect to valuations that are ‘similar’ with respect to the appropriate sets of atoms, while in the canonical model, those modal operators are defined as necessity operators with respect to a relation between maximal consistent sets that is defined in terms of membership of formulas in these sets. We now have to show that, in the canonical model, these two ways of looking at the modalities coincide. For this, our normal form Theorem 2 will be crucial.

But first we restrict ourselves to a generated submodel of \( \hat{K} \). To be more precise, for the canonical model \( \hat{K} \) just obtained, and \( w \in \hat{W} \), let \( \hat{K}_w \) be the model generated by \( w \) in the following sense. Let \( \hat{R}_i^0 \) be \( \hat{R}_i^0 \cup \cdots \cup \hat{R}_{i-1}^0 \). Then, define \( W_0 = \{ v \mid \hat{R}_i^0(w, v) \} \), and all relations \( \hat{R}_i^0 \) and \( \hat{R}_i^0 \) and valuation \( \hat{\pi}_0 \) are the old relations and valuation restricted to the new set \( W_0 \). The following is a known result about generated submodels:

\[ \forall \varphi \forall v \in W_0 \hat{K}_w, v \models^\hat{\mathcal{K}} \varphi \text{ iff } \hat{K}_w, v \models^\hat{\mathcal{K}} \varphi \]

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THEOREM 3 (\(\hat{K}_w\) SIMULATES AN ECL-PC(PO) FRAME.). Let \(\hat{K}\) be as defined above, and take \(w \in \hat{W}\). Consider the model \(\hat{K}_w\). Define, for every \(i \in N\), the sets \(A_i = \{p | \text{ctrls}(i, p) \in w\}\), and \(V_i = \{p | \text{sees}(i, p) \in w\}\). Then, in \(\hat{K}_w\), the accessibility relations satisfy the following properties:

1. \(\hat{R}_{\hat{K}_w}(v, v') \iff \pi_a(v) \equiv_{A_i} \pi_a(v')\).

2. \(\hat{R}_{\hat{K}_w}(v, v') \iff \pi_a(v) \equiv v, \pi_a(v')\).

PROOF. Consider the first item. Suppose \(\hat{R}_{\hat{K}_w}(v, v')\), which means that for any \(\phi, \varphi \in v' \Rightarrow \square_{\varphi} \phi \in v\). Take any \(p \in A \setminus A_i\). We show that \(p \in v \iff p \in v'\). Suppose \(p \in v\). By definition of \(A_i\), we have \(\text{ctrls}(i, p) \notin w\), and, since \(w\) is a maximal consistent set, \(\neg \text{ctrls}(i, p) \in w\). By Lemma 2, item 4 (take \(t(\text{ctrls}(i, p) = \text{ctrls}(i, p))\) we have \(\square_{\varphi} \neg \text{ctrls}(i, p) \in w\), and, since \(v\) is \(\hat{R}_{\hat{K}_w}\)-reachable from \(w\), we have \(\neg \text{ctrls}(i, p) \in v\). This gives \((\neg \text{ctrls}(i, p) \wedge p) \in v\), which, by Lemma 2, item 1 gives \(\Box_{\varphi} p \in v\). Now for contradiction, if \(p \notin v'\), we would have \(\neg p \in v',\) and by definition, \(\Box_{\varphi} \neg p \in v\), which contradicts \(\Box_{\varphi} p \in v\). The reasoning for \(p \notin v\) goes similar.

For the converse, suppose \(\pi_a(v) \equiv_{A_i} \pi_a(v')\), i.e., \(v \cap (A \setminus A_i) = v' \cap (A \setminus A_i)\). Take an arbitrary \(\varphi \in v\), we have to show that \(\Box_{\varphi} \varphi \in v\). By Theorem 2, we know that \(\varphi\) is equivalent to a disjunction as specified in (1), and since \(v\) is a maximal consistent set, there must be (uniquely) a propositional description \(\pi\), a control description \(\gamma\) and a visibility description \(\varsigma\) such that \((\pi \wedge \Box_{\gamma} \Box_{\varsigma}) \in v\). Since \(v\) and \(v'\) are both reachable from the same generating world \(w\), we have \((\Box_{\gamma} \wedge \Box_{\varsigma}) \in v\) and hence, by \((\text{comp})\) and \((T(1))\)

\[
(\Box_{\gamma} \wedge \Box_{\varsigma}) \in v
\]

Let us decompose \(\pi\) into \(\pi_1 \wedge \pi_2\), where \(\pi_1\) uses all the atoms \(p\) from \(A \setminus A_i\), and \(\pi_2\) uses all the atoms from \(V_i\). By Lemma 2, item 7, we have

\[
M_i \pi_1 \in v
\]

Moreover \(\pi \in v'\) means trivially that \(\pi_2 \in v'\). Moreover by assumption \(v \cap (V_i) = v' \cap (V_i)\). Hence, \(\pi_2 \in v\). By Lemma 2, item 7, we then have that

\[
K_i \pi_2 \in v
\]

Collecting equations (5), (6) and (7), and using the modal validity \(\vdash (\Box_{\gamma} \wedge \Box_{\varsigma}) \rightarrow (\Box_{\gamma} \wedge \Box_{\varsigma})\), we obtain \(M_i (\pi_1 \wedge \pi_2) \wedge \gamma \wedge \varsigma) \in v\). By Lemma 2.11, we conclude \(M_i (\pi_1 \wedge \pi_2) \wedge \Box_{\gamma} \wedge \Box_{\varsigma} \in v\) which means that \(M_i \varphi \in v\).


\[
(\Box_{\gamma} \wedge \Box_{\varsigma}) \in v
\]

THEOREM 4 (COMPLETENESS OF \(A_\lambda\)). \(A_\lambda\) is sound and complete with respect to the class of ECL-PC(PO) frames.

PROOF. Soundness is observed in Lemma 1. For completeness, take a \(A_\lambda\)-consistent formula \(\varphi\). Consider a maximal consistent set \(w\) with \(\varphi \in w\). We know that \(\hat{K}_w, w \models \varphi\). Take the generated model \(\hat{K}_w\). We know that again \(\hat{K}_w, w \models \varphi\), and moreover, by Theorem 3, \(\hat{K}_w\) simulates an ECL-PC(PO) frame.

3. UNCERTAINTY ABOUT OWNERSHIP

The next type of uncertainty we consider relates to which agents control which variables. We refer to the logic we develop to capture such situations as the ECL-PC(UO), where “UO” stands for “uncertainty of ownership”. The syntax of ECL-PC(UO) is identical to that of ECL-PC(PO), and so we will not present the syntax again here. In the semantics however, we substitute for every agent the set of propositions that it can see the value of, with a set of propositions which it sees the ownership of.

Given a set of agents \(N\), atomic variables \(A\), and control partition \(A_1, \ldots, A_n\), a controls observation for agent \(i\) is as set \(\Omega_i \subseteq A\). The interpretation of \(\Omega_i\) is that \(p \in \Omega_i\) means that agent \(i\) knows who has control over the variable \(p\), that is, the agent \(j \in N\) such that \(p \in A_j\). Given this, we define a frame \(F\) for ECL-PC(UO) as:

\[
F = \langle N, A_1, \ldots, A_n, \Omega_1, \ldots, \Omega_n \rangle
\]

where:

- \(N\) and \(A_i \subseteq A\) are as before, and
- \(\Omega_i\) is the controls observation for agent \(i\).

We now define a relation on frames, which will be used to give a semantics to our epistemic modalities. Let

\[
F' = \langle N, A_1, \ldots, A_n, \Omega_1, \ldots, \Omega_n \rangle
\]

be two frames that contain the same agents and the same base set of propositional variables. Then we write \(F \simeq F'\) to mean that (1) \(\Omega_1 = \Omega_1'\) and (2) for all \(p \in \Omega_i\) and for all \(j \in N\) we have \(A_i \cap \Omega_j = A_j' \cap \Omega_i\). Thus, roughly, \(F \simeq F'\) means that \(F'\) and \(F\) agree on the variables that \(i\) can see the ownership of, and moreover, for each of those variables, the control is assigned to the same agents in both frames.

Formally, the key steps in the semantics are defined as follows:

\[
F, \theta \vdash p \iff \theta(p) = true \quad (p \in A)
\]

\[
F, \theta \vdash \Box_{\phi} \varphi \iff \exists \theta' \in \Theta : \theta' \equiv_{A_\lambda} \theta \text{ s.t. } M, \theta' \vdash \varphi
\]

\[
F, \theta \vdash K_i \varphi \iff \forall F' : F' \simeq_i F \implies F', \theta \vdash \varphi
\]
EXAMPLE 2. Suppose we have a frame \( F \) in which \( N = \{1, 2\} \), \( A_1 = \{p\} \), \( A_2 = \{q\} \), \( \Omega_1 = \emptyset \), \( \Omega_2 = \{p, q\} \). In this case, agent 1 has no information about which agent controls which variable: As far as this agent is concerned, any partition of controlled variables to agents is possible. Let \( \theta(p) = \theta(q) = true \). We have:

- \( F, \emptyset \models K_1(p \land q) \land K_2(p \land q) \)

Unlike ECL-PC(PO), agents have no uncertainty about the actual value of variables. Thus both agents know that both variables are true in the valuation \( \emptyset \).

- \( F, \emptyset \models \diamond_1(\neg p \land q) \land \neg K_1(\neg p \land q) \)

In fact, agent 1 can bring about \( \neg p \land q \): he controls the variable \( p \) and can choose \( \neg p \land q \). However, because he is uncertain about whether he controls \( p \), he does not know that he has the ability to choose \( \neg p \land q \).

- \( F, \emptyset \models K_1(p \land q) \land K_2(\neg p \land q) \)

Agent 2 can choose a value for \( q \) so as to bring about \( p \land \neg q \) (assuming agent 1 leaves \( p \) unchanged). Moreover, since 2 knows that he controls \( q \), she knows that he can choose \( p \land \neg q \).

- \( F, \emptyset \models K_1(p \land q) \land \diamond_2(\neg p \land \neg q) \)

Agent 2 knows that actually \( p \land \neg q \) holds, and that 1 can choose a situation where \( p \) is false and in which agent 2 furthermore can set \( q \) to false.

- \( F, \emptyset \models K_1(\diamond_1(\neg p \land q) \land \diamond_2(\neg p \land \neg q)) \)

Agent 1 knows that together, the agents can always make the values of \( p \) and \( q \) different, but agent 2 even knows that, no matter which values 1 chooses for his variables, 2 can achieve a situation such that \( p \) and \( q \) are different.

Note that, by the same arguments as given for ECL-PC(PO), we may conclude that:

**Theorem 5.** The model checking and satisfiability problems for ECL-PC(UO) are both PSPACE-complete.

We give an axiomatization for ECL-PC(UO) in Figure 2. Developability \( \vdash \) in this section refers to that axiomatization. The following definitions and notations are useful.

**Definition 2.** Define \( \text{seeswho}(i, p) \) as \( \bigvee_{j \in N} K_i \text{ctrls}(j, p) \).

Let \( SW = \{\text{seeswho}(j, p) \mid j \in N, p \in A\} \). The elements of \( A \), CTL and \( SW \) are our new basic propositions. A controls observation description \( \omega \) is a full conjunction over \( SW \). We note \( \Omega \) the set of such controls observation descriptions. A new full description is a conjunction \( \pi \land \gamma \land \omega \), where \( \pi, \gamma \) and \( \omega \) are as explained above.

Let \( P \subseteq A \). We define \( \text{CTRL}(P) = \{\bigwedge_{i \in P} \text{ctrls}(i, p) \mid \emptyset \leq i \leq k \} \). Finally let \( \omega^i \) be of the form \( \bigwedge_{p \in A \cup \{\emptyset\}} \ell(\text{seeswho}(i, p)) \) and let the formula \( \omega^i \) be of the form \( \bigwedge_{p \in A \cup \{\emptyset\}} \ell(\text{seeswho}(j, p)) \) such that \( \omega^1 \land \omega^2 \) is a controls observation description.

As with ECL-PC(PO), a full description \( \pi \land \gamma \land \omega \) fully characterizes a situation: it specifies which atoms are true and which are false (this is \( \pi \)), it specifies which agents control which variables (through \( \gamma \)) and it specifies exactly which agent is aware of who owns which variables (through \( \omega \)). So semantically, it is immediately clear that any formula will be a disjunction of such full descriptions (namely, descriptions of those situations where \( \varphi \) is true), but our task is now to show that this is derivable in the logic.

**Figure 2:** Axiomatics of \( \Lambda_2 \). The meta-variable \( i \) ranges over \( N \), \( \varphi \) represents an arbitrary formula of ECL-PC(UO), \( p \) ranges over \( A \). Finally, \( \omega^i \), and \( \omega^i \) are as specified in Definition 2, and \( \gamma \in \text{CTRL}(P) \). Objective formulas have no modal operators.

**Lemma 5.** The axiomatization for \( \Lambda_2 \) in Figure 2 is sound.

We now prove that this axiomatization is complete.

**Theorem 6 (Normal form).** Every formula \( \varphi \) is provably equivalent to a disjunction of full descriptions, i.e., for every \( \varphi \) there exists a \( k \) and \( \pi_1, \gamma_1 \) and \( \omega_1 \) (1 \( \leq j \leq k \)) such that

\[
\vdash \varphi \iff \bigvee_{1 \leq j \leq k} \pi_j \land \gamma_j \land \omega_j
\]

The proof of Theorem 6 is omitted for reasons of space. We now define an alternative, possible worlds semantics for ECL-PC(UO). Given a frame \( F = (N, A_1, \ldots, A_n, \Omega_1, \ldots, \Omega_n) \), a corresponding pointed Kripke model for ECL-PC(UO) is a structure

\[ K, w_{(F, \theta)} = (W, R_1^\theta, \ldots, R_n^\theta, R_1^K, \ldots, R_n^K, \pi), w_{(F, \theta)} \]

where \( W = \Pi \times \Gamma \times \Omega \) is a set of worlds that correspond to a frame and a propositional valuation. For every \( w \in W \), we note \( w(\pi) \) the propositional description it contains, \( w(\gamma) \) the control description, and \( w(\omega) \) the controls observation description. Given two states \( w \) and \( w' \), a set of propositions \( X \), we have already defined \( w(\varphi) \equiv_X w'(\varphi) \). We define \( w(\varphi) \equiv_X w'(\gamma) \) to mean that for every \( P \in X \), \( w(\gamma) \equiv_X w'(\gamma) \) to mean that \( w(\omega) \equiv_X w'(\omega) \) to mean that for every \( P \in X \), \( w(\gamma) \equiv_X w'(\gamma) \). Similarly, we define \( w(\varphi) \equiv_X w'(\gamma) \) to mean that \( w(\omega) \equiv_X w'(\omega) \) to mean that for every \( P \in X \), \( w(\gamma) \equiv_X w'(\gamma) \). Finally, the world \( w_{(F, \theta)} \) is such that \( w_{(F, \theta)}(\pi) \) describes \( \theta \), \( w_{(F, \theta)}(\gamma) \) describes \( \Omega_1, \ldots, \Omega_n \), and \( w_{(F, \theta)}(\omega) \) describes \( \Omega_1, \ldots, \Omega_n \).

The relations \( R_1^\theta \subseteq W \times W \) and \( R_n^K \subseteq W \times W \) are defined as follows:

\[
R_1^\theta(w, w') \iff \begin{cases} w(\pi) \equiv_{A_1} w'(\pi) \\ w(\omega) = w'(\omega) \\ w(\gamma) = w'(\gamma) \end{cases}
\]

and

\[
R_n^K(w, w') \iff \begin{cases} w(\pi) \equiv_{A_n} w'(\pi) \\ w(\omega) \equiv_{\Omega_n} w'(\omega) \\ w(\gamma) \equiv_{\Omega_n} w'(\gamma) \end{cases}
\]

for all \( j \in N \).

Finally, \( \pi : W \to 2^A \) gives the set of Boolean variables true at each world. We can then define a Kripke semantics for our language,
with the key clauses defined via the satisfiability relation \( \models^k \) as follows:

\[
\begin{align*}
K, w \models^k p & \iff p \in \pi(w) \\
K, w \models^k \exists i \varphi & \iff \exists w' \in W \text{ s.t. } R^i_w(w, w') \text{ and } K, w', w' \models^k \varphi \\
K, w \models^k K_i \varphi & \iff \forall w' \in W \text{ s.t. } R^i_w(w, w') \text{ and } K, w', w' \models^k \varphi
\end{align*}
\]

The following is immediate.

Lemma 6. Let \( F, \theta \) be an ECL-PC(UO) frame and associated valuation, let \( K, w_{(\theta, F)} \) be the corresponding Kripke model and world, and let \( \phi \) be an arbitrary ECL-PC(UO) formula. Then:

\[
F, \theta \models^d \phi \iff K, w_{(\theta, F)} \models^k \phi
\]

The definition of a canonical model \( \hat{K} \) for the logic is as before (although the model of course will be different, since the axioms are different!), and the truth lemma holds for this language as well. But in this case, we do not need to restrict ourselves to a generated submodel.

Theorem 7. (\( \hat{K} \) SIMULATES AN ECL-PC(UO) FRAME.).

Let \( \hat{K} \) be as defined above. Define, for every \( i \in N \) and \( w \in W \), the sets \( \hat{A}_i = \{ p \mid \text{ctrls}(i, p) \in v \} \), and \( \hat{\Omega}_i = \{ p \mid \exists j \in N \} \in N, K_i \text{ctrls}(j, p) \in v \}. \) Then, in \( \hat{K} \), the accessibility relations satisfy the following properties:

1. \( \hat{R}^i_{\omega}(v, v') \iff \begin{cases} \pi(v) \equiv_{\hat{A}_i, \omega} \pi(v') \\ v(\omega) = v'(\omega) \\ v(\gamma) = v'(\gamma) \end{cases} \)

2. \( \hat{R}^k_i(v, v') \iff \begin{cases} \pi(v) \equiv_{\hat{A}_i} \pi(v') \\ v(\omega) = v'(\omega) \\ v(\gamma) \equiv_{\hat{\Omega}_i} v'(\gamma) \text{ for all } j \in N \end{cases} \)

Proof. We prove the second item. Suppose that \( \hat{R}^k_i(v, v') \). By definition, it means that for all \( \varphi, \psi \in v \) implies \( M_i \varphi \in v \). We now prove the three properties of the right side of the item. We first show that \( p \in v \) iff \( p \in v' \). Suppose that \( p \in v \). Then \( K_i p \in v' \) by Axiom 1. By hypothesis we obtain \( M_i K_i p \in v \), by which S5 yields \( p \in v \). The case \( p \notin v \) is similar.

Now we show that \( \text{ctrls}(i, p) \in v \) iff \( K_i \text{ctrls}(i, p) \in v' \). First, suppose that \( \text{ctrls}(i, p) \in v' \). Then by hypothesis, we have \( K_i \text{ctrls}(i, p) \in v \) and \( K_i \text{ctrls}(i, p) \in v \in S5 \). Second, suppose that \( \text{ctrls}(i, p) \notin v' \). Since \( v' \) is a m.c. set, \( \neg \text{ctrls}(i, p) \in v' \). Then, \( M_i \neg \text{ctrls}(i, p) \in v \) which is equivalent to \( M_i \neg \text{ctrls}(i, p) \in v \) and \( \neg K_i \text{ctrls}(i, p) \in v \). And since \( v \) is a m.c. set, we have \( K_i \text{ctrls}(i, p) \in v \).

Now, take any \( j \in N \) and any \( p \in \hat{A}_i \cap \hat{\Omega}_j \). We show that \( \text{ctrls}(j, p) \in v' \). By definition of \( \hat{\Omega}_j \), we have \( K_i \text{ctrls}(j, p) \in v' \). By hypothesis, we have \( M_i K_i \text{ctrls}(j, p) \in v \) which is equivalent to \( \text{ctrls}(j, p) \in v \). Second, suppose that \( \text{ctrls}(j, p) \notin v' \). Since \( v' \) is an m.c. set, \( \neg \text{ctrls}(j, p) \in v' \). Also, by definition of \( \hat{\Omega}_j \), we have \( \text{seeswho}(i, p) \in v' \). Hence, by Axiom A4 we have \( K_i \neg \text{ctrls}(j, p) \in v \). Hence, we have \( M_i K_i \neg \text{ctrls}(j, p) \in v \) which is equivalent to \( \neg \text{ctrls}(j, p) \in v \), and since \( v \) is an m.c. we obtain \( \text{ctrls}(j, p) \notin v' \).

We now prove the right to left direction of item 2. To do so, suppose that \( \text{hyp1} \pi(v) \equiv_{\hat{A}_i} \pi(v'), \text{hyp2} v(\omega) \equiv_{\hat{\Omega}_i} v'(\omega) \) and \( \text{hyp3} v(\gamma) \equiv_{\hat{\Omega}_i} v'(\gamma) \) for all \( j \in N \). We need to show that \( \hat{R}^k_i(v, v') \), that is, for all \( \varphi \) we have \( \varphi \in v' \) implies \( M_i \varphi \in v \).

Take an arbitrary \( \varphi \in v' \). By Theorem 6, we assume \( w \text{.l.o.g.} \) that for some \( k \) we have \( \varphi \leftrightarrow \bigwedge_{1 \leq i \leq k} (\pi_i \land \gamma_i \land \omega_i) \).

Since \( v' \) is an m.c. set, there is (uniquely) a full description \( \pi \land \gamma \land \omega \) such that \( (\pi \land \gamma \land \omega) \in v' \).

From (hyp1) we have \( \pi \in v \) and by Axiom 1 we obtain \( K_i \pi \in v \) \( \text{Axiom A3) \}

Let us write \( \omega \) as \( \omega_1 \land \omega_2 \) such that \( \omega_1 \) contains the \( \ell(\text{seeswho}(i, p)) \) literals (those concerning \( i \)’s observations) and \( \omega_2 \) contains all the other literals in \( \omega \). Since by (hyp2) we have \( v(\omega) \equiv_{\hat{\Omega}_i} v'(\omega) \), we have \( \omega_1 \in v \) and by Axiom A3 we get \( K_i \omega_1 \in v \). Hence \( K_i \omega_1 \in v \) \( \text{Axiom A6) \}

Finally, using Axiom A5 we obtain \( M_i (\omega_2 \land \gamma_2) \in v \) \( \text{Axiom A6) \}

Combing (8), (9), (10), and (11) we then obtain \( M_i (\pi \land \omega \land \gamma) \in v \), i.e., \( M_i \varphi \in v \). \( \square \)

Theorem 8. (Completeness of A2.). A2 is sound and complete with respect to the class of ECL-PC(UO) frames.

Let us finally sketch a general setup, in which:

1. not every atom \( p \in A \) needs to be in control of an agent;
2. agent \( i \) does not necessarily know what \( j \) sees (if \( i \neq j \)) and does not have complete ignorance either;
3. agent \( i \) does not necessarily know what \( j \) knows about control (if \( i \neq j \)) and does not have complete ignorance either.

To cater for this, let \( T_i = (\Omega_i, V_i) \), where \( \Omega_i \subseteq A \) and \( V_i \subseteq A \). The idea is that for every atom in \( \Omega_i \), agent \( i \) knows who controls it, and for every atom in \( V_i \), agent \( i \) knows what its truth value is. Now, a model \( M \) is of the form

\[
M = (N, S, R^\Delta, \equiv)
\]

1. \( S \) is a set of states \( \{A_1, \ldots, A_n, T_1, \ldots, T_n, \theta\} \);
   \[ (a) \cup_{i \in N} A_i \subseteq A \text{ and } A_i \cap A_j \neq \emptyset \]
   \[ (b) T_i = (\Omega_i, V_i) \text{ with } \Omega_i, V_i \subseteq A \]
2. \( R^\Delta : N \rightarrow S \times S \) is a binary relation. This relation satisfies the following: for every \( \{A_1, \ldots, A_n, T_1, \ldots, T_n, \theta\} \in S \), and every \( \theta' \) such that \( \theta \equiv_{A_1} \theta' \), there is a state \( t = (A_1, \ldots, A_n, T_1, \ldots, T_n, \theta') \);\]
3. Given two states \( s = (A_1, \ldots, A_n, T_1, \ldots, T_n, \theta) \) and \( s' = (A_1, \ldots, A_n, T_1', \ldots, T_n', \theta') \), define
   \[
   s \preceq_{i} s' \iff \begin{cases} T_i = T_i' \quad \forall p \in V_i \theta(p) = \theta'(p) \\ \forall p \in \Omega_i \forall j \in N (p \in A_j \iff p \in A_j')
   \end{cases}
   \]
The semantics is very general and allows for a number of specialisations. Examples of such specialisations are:

1. For all states \( s \) and every agent \( i, \Omega_i = \mathbb{A} \) (complete knowledge about control)
2. For all states \( s \) and \( t \), and every agent \( i \), the components \( \Omega_i \) and \( \Omega_t \) are the same.
3. For all states \( s \) and \( t \), and every agent \( i \), the components \( V_i \) and \( V_t \) are the same.

These properties entail some validities:

1. \( \models \text{ctrls}(j, p) \iff K_i \text{ctrls}(j, p) \)
2. \( \models K_i \text{ctrls}(j, p) \iff (K_h K_i \text{ctrls}(j, p) \land \square_N K_i \text{ctrls}(j, p)) \)
3. \( \models \text{sees}(i, p) \iff (K_h \text{sees}(i, p) \land \square_N \text{sees}(i, p)) \)

In fact, all those specialisations apply to ECL-PC(PO). Other natural assumptions would be for instance \( \mathbb{A}_i \subseteq \Omega_i \) (corresponding to \( \text{ctrls}(i, p) \rightarrow K_i \text{ctrls}(i, p) \)) and \( A \subseteq V_i \) (corresponding to \( \text{sees}(i, p) \rightarrow \text{sees}(i, p) \)).

We give one simple scenario that can be modelled in this setup, that of Voting. All agents either desire something \( (p_i) \) or not. They can reveal their preference through \( q_i \): if \( p_i \rightarrow q_i \), agent \( i \) is truthful, otherwise it lies. Here, \( A_i = \{ q_i \}, \Omega_i = \{ q_j \mid j \in N \} \) and \( V_i = \{ p_i \} \cup \{ q_j \mid j \in N \} \). In other words, we assume agents cannot control what they prefer, although what they can do is choose their vote. We have here

\[ \ell(p_i) \rightarrow K_i(\square_i(\ell(p_i) \land q_i) \land \square_i(\ell(p_i) \land \neg q_i)) \]

i.e., \( i \) knows that it can vote truthfully but it can also lie. We also get \( K_i q_i \rightarrow \neg (K_i p_i \lor K_i \neg p_i) \): even if \( i \) knows \( j \)'s vote, it does not know \( j \)'s real preference. Note that the information about what agents see and what they know about controls is still global, we have e.g. \( K_i K_i \text{ctrls}(h, q_i) \).

4. CONCLUSION

As noted before, we added an information component to the logic of propositional control CL-PC ([14]). From a technical perspective, like in [7], our logic ECL-PC(PO), even if we would require that all agents see all propositional variables, is an extension of CL-PC, since as presented in [14], the distribution of propositional variables \( \mathbb{A} \) over agents is assumed as given. In ECL-PC(PO), it is not given, but it is fixed, implying that a specification \( \psi \) may leave room for different distributions of the atoms, but once it is chosen, there is no way to refer to other distributions, not in terms of what agents can imagine, nor in terms of what they can achieve.

There are close connections between propositional logics of control and other logics that facilitate reasoning about the powers of coalitions, like Coalition Logic [11] and ATL [2]. In fact, CL-PC was partially motivated by the way the model checking system MOCHA for ATL [3] is designed, in which the system is divided in a number of modules (agents, in our terminology), each controlling its own set of Boolean variables. And indeed, there have been several attempts to add an epistemic component to ATL [13, 8, 1]. However, what those extensions all have in common is that the uncertainty of the agents is specified in an abstract way: in the Kripke models for the logics for cooperation and knowledge, the accessibility relations corresponding to knowledge are just given, abstract, equivalence relations. In our logic CL-PC(PO) the knowledge is determined by the variables of which the agent can see the truth value, and in ECL-PC(UO) this accessibility relation is determined by the variable of which the agent can see the ownership. In this sense, we provide a computationally grounded semantics [16] for knowledge, which brings our approach closer to the interpreted systems approach to epistemic logic [5, 6]. Interestingly enough, the key idea of interpreted systems (two states are the same for agent \( i \) if the atoms that it sees have the same value) does not only apply to the epistemic dimension in our logics, but also to the control dimension: two states are reachable in terms of \( i \)'s control, if the values of the atoms not in \( i \)'s control is the same.

Future work should study how to combine our two approaches, as suggested at the end of Section 3, and to weaken some of the underlying assumptions regarding the agents' knowledge. Related to this, we would like to provide a completeness proof for our systems that does not rely on a normal form (and on the assumption that the number of propositional atoms is finite). Doing this, one needs to find a way of juggling with the two types of definitions of ‘access’ we are dealing with here: on the one hand, the canonical model in modal logic defines this in terms of membership of formulas in the states, whereas the interpreted systems approach would to this in terms of ‘similarity’ of the states. We hope that work of Lomuscio [9], connecting general S5 semantics with that of interpreted systems, may give some first steps in this search. Another natural direction to be explored is to emphasize the group aspect of both dimensions: when forming a coalition \( C \) to bring about \( \varphi \), i.e., \( \Diamond \varphi \) gives rise to interesting questions from cooperative game theory, and epistemic logic provides the tools and results to combine this with interesting notions of group knowledge.

5. REFERENCES