The face of emotions: a logical formalization of expressive speech acts

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ABSTRACT
In this paper, we merge speech act theory, emotion theory, and logic. We propose a modal logic that integrates the concepts of belief, goal, ideal and responsibility and that allows to describe what a given agent expresses in the context of a conversation with another agent. We use the logic in order to provide a systematic analysis of expressive speech acts, that is, speech acts that are aimed at expressing a given emotion (e.g. to apologize, to thank, to reproach, etc.).

Categories and Subject Descriptors
1.2 [Artificial Intelligence]: Multiagent systems

General Terms
Theory

Keywords
Speech act theory, cognitive models, logic-based approaches and methods

1. INTRODUCTION
Since the works of Austin [2] and Searle [20] on speech acts, there has been a lot of work on illocutionary acts\footnote{Searle distinguishes several types of speech acts: utterance acts (using for uttering words); propositional acts (for referring and predicating); illocutionary acts (for stating, questioning, commanding, promising, etc.). See [20, Section 2.1] for more details.} and on their use for the formal specification of an agent communication language (see, e.g., [6, 27, 23, 9, 10]). Searle has defined five classes of illocutionary acts [21, Chapter 1], and every utterance realizes the performance of one (or more) illocutionary act(s) of these classes. Thus, Searle’s classification is a taxonomy. These five classes of illocutionary acts are:

- assertives (for describing facts, e.g. “It rains”),
- directives (for representing order or request for instance, e.g. “Open the door, please”),
- commissives (for representing commitment, e.g. “I will help you”),
- declarations (for representing institutional illocutionary acts, e.g. “I name this ship the Queen Elizabeth”),
- expresses (for representing psychological attitudes, e.g. “I congratulate you” or “I thank you”).

Existing literature on speech acts is mainly about the first three classes of illocutionary acts and, to a lesser extent, about the fourth. Thus, as far as we know, there is no work about the last class of illocutionary acts, that is, expresses. As Searle says:

The illocutionary point of this class is to express the psychological state specified in the sincerity condition about a state of affairs specified in the propositional content. The paradigms of expressive verbs are “to thank”, “to congratulate”, “to apologize”, “to deplore”, and “to welcome”. [21, Chapter 1]

In this paper we propose a first formalization of expressive speech acts in a BDI-like logic where utterances are represented by the mental states they express. The logic, which is presented in Section 2, has specific modal operators that allow us to represent expressed psychological mental states.

We focus on particular psychological states that are emotional states. Emotions that we consider are either basic emotions (only defined from beliefs and goals) or complex emotions (based on complex reasoning about norms, responsibility, etc.). For instance, joy and sadness are basic emotions, whereas guilt or regret are complex emotions requiring a complex form counterfactual reasoning about responsibility where reality is compared to an imagined view of what might have been [12, 15]. Basic and complex emotions are studied in Section 3. In the paper we only consider the cognitive structure of emotion rather than emotion as a complex psychological phenomenon including cognitive and somatic aspects (i.e. feeling). Indeed the cognitive structure of emotion is sufficient for our needs, as we only consider the mental states that can be expressed by use of language.

In Section 4, expressive speech acts are defined as public expressions of emotional states.
2. LOGICAL FRAMEWORK

MLC (Modal Logic of Communication) is a BDI-like logic [7, 17] that allows us to represent agents’ mental states (beliefs, desires and ideals) as well as the overt and social aspect of communication. It has modal operators that describe the conversational state of an agent $i$ with respect to another agent $j$ in front of an audience $H$, i.e. what agent $i$ expresses to agent $j$ in front of the audience $H$. A conversational state is a static description of the utterances that are performed by the participants in a dialogue, and is similar to the commitment store of Walton & Krabbe [28].

2.1 Syntax

Assume a finite non-empty set $AGT = \{1, \ldots, n\}$ of agents, a countable set $ATM = \{p, q, \ldots\}$ of atomic propositions denoting facts. The language $L$ of the logic MLC is the set of formulas defined by the following BNF:

$$\varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid Bel_i \varphi \mid Goal_i \varphi \mid Ideal_i \varphi \mid Cd_i \varphi \mid \text{Exp}_{i,j,H} \varphi$$

where $p$ ranges over $ATM$, $i, j$ range over $AGT$ and $H$ ranges over $2^{AGT}$. The other Boolean constructions $\top, \bot, \lor, \rightarrow$ and $\leftrightarrow$ are defined in the standard way.

Operators $Bel_i$ and $Goal_i$ are used to represent agent $i$’s beliefs and goals. Given an arbitrary formula $\varphi$ of the logic, $Bel_i \varphi$ has to be read ‘agent $i$ believes that $\varphi$', whereas $Goal_i \varphi$ has to be read ‘agent $i$ has the goal that $\varphi$’ or ‘agent $i$ wants $\varphi$ to be true’. Following [8], we consider goals the most basic class of motivational attitudes. The concept of goal is more general than the concept of desire (therefore, the former class includes the latter). Desires are intrinsically endogenous, while goals might originate from external inputs.\footnote{See [22] for a detailed analysis of how an agent may want something without desiring it and on the problem of reasons for acting independent from desires.}

For instance, an agent might have a goal because of norm compliance or because it adopted this goal from another agent (e.g. agent $i$ has the goal to close the door because agent $j$ asked it to do so and $i$ accepted $j$’s request). Moreover, differently from a desire, a goal is not necessarily associated with a pleasant state of mind (i.e. goals do not necessarily have a hedonistic component).

As the class of goals includes desires, we assume that goals can be incompatible with beliefs. For instance, a person may wish to become multimillionaire even though she believes that her aspiration will never be satisfied.

The operators $Ideal_i$ are used to represent an agent’s moral attitudes, after supposing that agents are capable to discern what (from their point of view) is morally right from what is morally wrong. This is a necessary step towards an analysis of social emotions such as guilt and shame which involve a moral dimension. The formula $Ideal_i \varphi$ means ‘$\varphi$ is an ideal state of affairs for agent $i$’. More generally, $Ideal_i \varphi$ expresses that agent $i$ thinks that it ought to promote the realization of $\varphi$, that is, agent $i$ conceives a demanding connection between itself and the state of affairs $\varphi$. When agent $i$ endorses the ideal that $\varphi$ (i.e. $Ideal_i \varphi$ is true), it means that $i$ addresses a command to itself, or a request or an imperative to achieve $\varphi$ (when $\varphi$ is actually false) or to maintain $\varphi$ (when $\varphi$ is actually true) [4]. In this sense, $i$ feels morally responsible for the realization of $\varphi$.

There are different ways to explain how a state of affairs $\varphi$ becomes an ideal state of affairs of an agent. A plausible explanation is based on the hypothesis that ideals are just social norms internalized (or adopted) by an agent (see [8] for a general theory of norm internalization). Suppose that an agent believes that in a certain group (or institution) there exists a certain norm (e.g. an obligation) prescribing that a state of affairs $\varphi$ should be true. Moreover, assume that the agent identifies itself as a member of this group. In this case, the agent adopts the norm, that is, the external norm becomes an ideal of the agent. For example, since I believe that in Italy it is obligatory to pay taxes and I identify myself as an Italian citizen, I adopt this obligation by imposing the imperative to pay taxes to myself.

The operators $Cd_i$ are used to talk about agents’ choices and actions, and will be later used in order to define a basic notion of responsibility. Formula $Cd_i \varphi$ has to be read ‘given what the other agents have done, agent $i$ could have ensured $\varphi$ to be true’ or ‘given what the other agents have decided to do, agent $i$ could have ensured $\varphi$ to be true’. Similar operators have been studied in [15] in the framework of STIT logic (the logic of Seeing it to that) [11] in order to provide an analysis of counterfactual emotions such as regret and disappointment.

Finally, formula $Exp_{i,j,H} \varphi$ has to be read ‘agent $i$ expressed to agent $j$ that $\varphi$ is true in front of group $H$’. Given a formula $Exp_{i,j,H} \varphi$, we call $i$ the speaker, $j$ the addressee, $H$ the audience and $\varphi$ the content of the speaker’s expression. For example, we can represent the sentence “John told to Mary: I have a new car,” by the formula $Exp_{John, Mary, H} \text{newCar}$ where $H$ are the agents who can hear John’s speech act. The basic function of modalities $Exp_{i,j,H}$ is to keep trace of the information that agent $i$ has communicated to agent $j$ in front of an audience $H$.

Further concepts.

We define a basic concept of responsibility as follows:

$$\text{Resp}_{i, \varphi} \overset{\text{def}}{=} \varphi \land Cd_i \neg \varphi$$

According to this definition, ‘agent $i$ is responsible for $\varphi$’ (noted $\text{Resp}_{i, \varphi}$) if and only if, $\varphi$ is true and, given what the other agents have done, $i$ could have ensured $\varphi$ to be false’ which is the same thing as saying ‘$\varphi$ is true and $i$ could have prevented $\varphi$ to be true’. In other words, agent $i$ is responsible for $\varphi$ only if, there is a counterfactual dependence between the state of affairs $\varphi$ and agent $i$’s choice.\footnote{This view of responsibility is close to that of [15, 5]. A stronger view of responsibility requires that agent $i$ is responsible for $\varphi$ only if it brings about $\varphi$, no matter what the other agents do.}

The concept of inevitability is defined as the dual of the operator $Cd_i$:

$$\text{Inev}_{i, \varphi} \overset{\text{def}}{=} \neg Cd_i \neg \varphi$$

Thus, ‘$\varphi$ is inevitable for agent $i$’ (noted $\text{Inev}_{i, \varphi}$) if and only if, it is not the case that, given what the other agents have done, $i$ could have ensured $\varphi$ to be false.

We define one more concepts which will be useful for the analysis of expressive speech acts such as to sympathize, to apologize and to be sorry for proposed in Section 4. We say that ‘agent $i$ is willing to adopt agent $j$’s goal that $\varphi$’ or ‘agent $i$ is cooperative about $\varphi$ with regard to agent $j$’ (noted $AdoptGoal_{i,j, \varphi}$) if and only if, if $i$ believes that $j$
AdoptGoal\textsubscript{i,j}ϕ \overset{df}{=} \text{Bel}_i \text{ Goal}_j \varphi \rightarrow \text{Goal}_i \varphi

2.2 Semantics

We use a standard possible worlds semantics where accessibility relations are used to interpret the modal operators of our logic. MLC-models are tuples \( M = \langle W, B, G, I, O, E, V \rangle \) defined as follows:

- \( W \) is a nonempty set of possible worlds or states;
- \( B : \text{AGT} \rightarrow 2^{W \times W} \) maps every agent \( i \in \text{AGT} \) to a serial,\(^5\) transitive,\(^6\) and Euclidean\(^7\) relation \( B_i \) over \( W \);
- \( G : \text{AGT} \rightarrow 2^{W \times W} \) maps every agent \( i \in \text{AGT} \) to a serial relation \( G_i \) over \( W \);
- \( I : \text{AGT} \rightarrow 2^{2^{W \times W}} \) maps every agent \( i \in \text{AGT} \) to a serial relation \( I_i \) over \( W \);
- \( O : \text{AGT} \rightarrow 2^{2^{W \times W}} \) maps every agent \( i \in \text{AGT} \) to an equivalence (i.e. reflexive,\(^8\) transitive and symmetric\(^9\)) relation \( O_i \) over \( W \);
- \( E : \text{AGT} \times \text{AGT} \times 2^{\text{AGT}} \rightarrow 2^{W \times W} \) maps every pair of agents \( i, j \in \text{AGT} \) and set of agents \( H \in 2^{\text{AGT}} \) to a transitive relation \( E_{i,j,H} \) over \( W \);
- \( V : \text{ATM} \rightarrow 2^W \) is a valuation function.

Moreover, we write \( B_i(w) = \{ v \mid (w, v) \in B_i \} \), \( G_i(w) = \{ v \mid (w, v) \in G_i \} \), \( I_i(w) = \{ v \mid (w, v) \in I_i \} \), \( O_i(w) = \{ v \mid (w, v) \in O_i \} \) and \( E_{i,j,H}(w) = \{ v \mid (w, v) \in E_{i,j,H} \} \).

The set \( B_i(w) \) is the information state of agent \( i \) at world \( w \): the set of worlds that agent \( i \) considers possible at world \( w \). The fact that every \( B_i \) is serial means that an agent has always consistent beliefs. Moreover, the transitivity and Euclideanity of \( B_i \) mean that an agent’s beliefs are positively and negatively introspective.

The set \( G_i(w) \) is the goal state of agent \( i \) at world \( w \): the set of worlds that agent \( i \) wants to reach (or prefers) at world \( w \). The fact that every \( G_i \) is serial means that an agent has always at least one state that it wants to reach.

The set \( I_i(w) \) is the ideal state of agent \( i \) at world \( w \): the set of worlds that agent \( i \) considers ideal (from a moral point of view) at world \( w \). The fact that every \( I_i \) is serial means that an agent has always at least one ideal state.

The set \( O_i(w) \) is the outcome state of agent \( i \) at world \( w \): \( O_i(w) \) is the set of outcomes that agent \( i \) could have ensured at \( w \), given what the other agents have done (at \( w \)). Therefore, the fact that \( O_i \) is reflexive means that the actual world is an outcome that agent \( i \) could have ensured,

\(^4\)We are aware that some form of conditional rather than material implication would be more suited to express entailment in the notion of goal adoption.
\(^5\)A given relation \( R \) on \( W \) is serial if and only if for every \( w \in W \) there is \( v \) such that \( (w, v) \in R \).
\(^6\)A given relation \( R \) on \( W \) is transitive if and only if, if \((u, v) \in R \) and \((v, w) \in R \) then \((u, w) \in R \).
\(^7\)A given relation \( R \) on \( W \) is Euclidean if and only if, if \((w, v) \in R \) and \((w, u) \in R \) then \((v, u) \in R \).
\(^8\)A given relation \( R \) on \( W \) is reflexive if and only if for every \( w \in W \), \((w, w) \in R \).
\(^9\)A given relation \( R \) on \( W \) is symmetric if and only if, if \((w, v) \in R \) then \((v, w) \in R \).
2.3 Axiomatization

All KD45-principles for the operators $\text{Bel}_i$ (KD45$_\text{Bel}$)
All KD-principles for the operators $\text{Goal}_i$ (KD$_\text{Goal}$)
All KD-principles for the operators $\text{Ideal}_i$ (KD$_\text{Ideal}$)
All S5-principles for the operators $\text{Cd}_i$ (S5$_\text{Cd}$)
All K4-principles for the operators $\text{Exp}_{i,j,H}$ (K4$_\text{Express}$)

$\text{Goal}_i \varphi \rightarrow \text{Bel}_i \varphi$ (Pl$_\text{Goal}$)
$\neg \text{Goal}_i \varphi \rightarrow \text{Bel}_i \neg \varphi$ (Nl$_\text{Goal}$)
$\text{Ideal}_i \varphi \rightarrow \text{Bel}_i \text{Ideal}_i \varphi$ (Pl$_\text{Ideal}$)
$\neg \text{Ideal}_i \varphi \rightarrow \text{Bel}_i \neg \text{Ideal}_i \varphi$ (Nl$_\text{Ideal}$)

$\text{Exp}_{i,j,H} \varphi \rightarrow \text{Bel}_i \text{Exp}_{i,j,H} \varphi$
$\neg \text{Exp}_{i,j,H} \varphi \rightarrow \text{Bel}_i \neg \text{Exp}_{i,j,H} \varphi$

Figure 1: Axiomatization of MLC

Figure 1 contains the axiomatization of the logic MLC. We have all principles of the normal modal logic KD45 for every belief operator $\text{Bel}_i$. Thus, an agent cannot have inconsistent beliefs (i.e., $\neg(\text{Bel}_i \varphi \land \text{Bel}_i \neg \varphi)$), and it has positive and negative introspection over its beliefs (i.e., $\text{Bel}_i \varphi \rightarrow \text{Bel}_i \text{Bel}_i \varphi$ and $\neg \text{Bel}_i \varphi \rightarrow \text{Bel}_i \neg \text{Bel}_i \varphi$).

We have all principles of the normal modal logic KD for every operator $\text{Goal}_i$, and for every operator $\text{Ideal}_i$ (i.e., $\neg (\text{Goal}_i \varphi \land \text{Goal}_i \neg \varphi)$ and $\neg (\text{Ideal}_i \varphi \land \text{Ideal}_i \neg \varphi)$).

We have all principles of the normal modal logic S5 for every operator $\text{Cd}_i$, taking it as a ‘Diamond’ operator. Thus, for example, if $\varphi$ is true then an agent could have ensured $\varphi$ (i.e., $\varphi \rightarrow \text{Cd}_i \varphi$).

Moreover, we have all principles of the normal modal logic K4 for every communication operator $\text{Exp}_{i,j,H}$. Thus, it’s action of expressing to $j$ that $\varphi$ entails $i$’s action of expressing to $j$ that $\varphi$ (i.e., $\text{Exp}_{i,j,H} \varphi \rightarrow \text{Exp}_{i,j,H} \text{Exp}_{i,j,H} \varphi$). In other words, the action of expressing something to someone has a self-referential nature. We do not include Axiom D for the operator $\text{Exp}_{i,j,H}$. Thus, we accept that an agent may express inconsistent things to another agent (even though it cannot believe them), that is, we accept formula $\text{Exp}_{i,j,H} \bot \bot$ to be satisfiable in our logic.

Axioms (Pl$_\text{Goal}$) and (Nl$_\text{Goal}$) are standard axioms of positive and negative introspection for goals [14], while Axioms (Pl$_\text{Ideal}$) and (Nl$_\text{Ideal}$) are corresponding principles for ideals.

Finally, Axioms (Pl$_\text{Express}$) and (Nl$_\text{Express}$) are corresponding principles of positive and negative introspection for communication: if an agent $i$ expressed (resp. did not express) something to another agent $j$ in front of an audience $H$, then this is public for the group $H \cup \{i,j\}$ including the speaker, the addressee, and all agents in the audience.

Note that we did not include a general inclusion principle of the form:

$\text{Exp}_{i,j,H} \varphi \rightarrow \text{Exp}_{i,i,H} \varphi$ for $I \subseteq H$

In fact, we want to be able to model situations in which an agent $i$ expressed something in secret to another agent $j$ (while all other agents were not hearing), and it expressed the contrary to $j$ in front of a larger group including $j$, without expressing an inconsistency.

For example, Bill might express in secret to Mary that he loves Ann, i.e., $\text{Exp}_{\text{Bill},\text{Mary}} \text{BillLovesAnn}$, and express to Mary that he does not love Ann when he is in front of Bob, i.e., $\text{Exp}_{\text{Bill},\text{Mary},\text{Bob}} \neg \text{BillLovesAnn}$, without expressing an inconsistency in front of Mary, i.e., $\neg \text{Exp}_{\text{Bill},\text{Mary},\text{Bob}} \bot$.

**Theorem 1.** The axiomatization in Figure 1 is sound and complete with respect to the class of MLC-models.

**Proof (Sketch).** It is a routine task to check that the axioms of the logic MLC correspond one-to-one to their semantic counterparts on the models.

In particular, (KD45$_\text{Bel}$) corresponds to the fact that every $B_i$ is serial, transitive and Euclidean. (KD$_\text{Goal}$) and (KD$_\text{Ideal}$) correspond to the fact that every $G_i$ (resp. $I_i$) is serial. (S5$_\text{Cd}$) corresponds to the fact that every $O_i$ is an equivalence relation, while (K4$_\text{Express}$) corresponds to the transitivity of every $E_{i,j,H}$. Axioms (Pl$_\text{Goal}$) and (Nl$_\text{Goal}$) together correspond to the Constraint $S_1$, Axioms (Pl$_\text{Ideal}$) and (Nl$_\text{Ideal}$) together correspond to the Constraint $S_2$. Axioms (Pl$_\text{Express}$) and (Nl$_\text{Express}$) together correspond to the Constraint $S_3$. It is routine, too, to check that all axioms of the logic MLC are in the Sahlqvist class. This means that the axioms are all expressible as first-order conditions on models and that they are complete with respect to the defined model classes, cf. [3, Th. 2.42].

We write $\vdash_{\text{MLC}} \varphi$ if $\varphi$ is a MLC-theorem. The following are examples of MLC-theorems. For every $i, j \in \text{AGT}$ and for every $H \subseteq 2^{\text{AGT}}$ we have:

$\vdash_{\text{MLC}} \text{Exp}_{i,j,H} \varphi \leftrightarrow \bigwedge_{z \in H \cup \{i,j\}} \text{Bel}_i \text{Exp}_{i,j,H} \varphi$

$\vdash_{\text{MLC}} \neg \text{Exp}_{i,j,H} \varphi \leftrightarrow \bigwedge_{z \in H \cup \{i,j\}} \text{Bel}_i \neg \text{Exp}_{i,j,H} \varphi$

According to former formula, agent $i$ has expressed that $\varphi$ to $j$ in front of the audience $H$ if and only if $i, j$ and every agent in the audience believes this. According to the latter, agent $i$ did not express that $\varphi$ to $j$ in front of the audience $H$ if and only if $i, j$ and every agent in the audience believes this.

3. FORMALIZATION OF EMOTIONS

As said in Section 1, Searle says that expressives are expressions of psychological states. Vanderveken agree with this and says that such psychological states have the logical form $m(p)$ where $m$ is the psychological mode and $p$ “the propositional content which represents the state of affairs to which the act is directed” [26, p. 213]. Here, emotions are viewed as particular mental states that have the logical form $m(p)$. Thus, emotion is here always about a state of affairs. When it is not the case, we consider such feeling to be a mood rather than an emotion. We are not concerned here by mood.

Following dimensional theories of emotion [18], the difference between two close labels in a multi-dimensional space may be a difference of intensity of the same emotion. It means that their cognitive structure is the same. In this paper we do not deal with intensity of emotions and we only formalize cognitive structures of emotions rather than
emotions themselves. Following appraisal theories [19, 13], the cognitive structure of an emotion is the configuration of mental states that an agent has in mind when feeling this emotion and that is responsible for this feeling. It is just a part of the entire affective phenomenon.

In the rest of this article, we use the term emotion to refer to the cognitive structure of emotion. The definitions of emotions will be written in italic in order to distinguish them from the definitions of expressive speech acts given in Section 4.

### 3.1 Cognitive structure of basic emotions

Basic emotions concern emotions built from belief, and goals or ideals. When agent $i$ believes that $\varphi$ is true, if it aims at $\varphi$ then it feels joy about the fact that $\varphi$ is true; if it aims at $\neg\varphi$ then it feels sadness about the fact that $\varphi$ is true; if it thinks that $\varphi$ is an ideal state of affairs then it feels approval; finally, if it thinks that $\neg\varphi$ is an ideal state of affairs then it feels disapproval. These emotions are summarized in the following table.

<table>
<thead>
<tr>
<th>$\wedge$</th>
<th>Goal$_i\varphi$</th>
<th>Goal$_i\neg\varphi$</th>
<th>Ideal$_i\varphi$</th>
<th>Ideal$_i\neg\varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bel$_i\varphi$</td>
<td>Joy$_i\varphi$</td>
<td>Sadness$_i\varphi$</td>
<td>Approval$_i\varphi$</td>
<td>Disapproval$_i\varphi$</td>
</tr>
</tbody>
</table>

Agent $i$ feels joy about $\varphi$ if and only if, $i$ believes that $\varphi$ is true and wants $\varphi$ to be true:

$$Joy_i\varphi \overset{def}{=} \text{Bel}_i\varphi \land \text{Goal}_i\varphi$$

For example, agent $i$ feels joy for having passed the exam because $i$ believes that it has passed the exam and wants to pass the exam. In this sense, $i$ is pleased by the fact that it believes to have achieved what it wanted to achieve. This means that joy has a positive valence, that is, it is associated with goal achievement.\(^{10}\)

Consider now sadness:

$$Sadness_i\varphi \overset{def}{=} \text{Bel}_i\varphi \land \text{Goal}_i\neg\varphi$$

That is, agent $i$ feels sadness about $\varphi$ if and only if $i$ believes that $\varphi$ is true and wants $\neg\varphi$ to be true. For instance, agent $i$ feels sad for not having passed the exam because $i$ believes that it has not passed the exam and wants to pass the exam. In this sense, $i$ is displeased by the fact that it believes not to have achieved what it was committed to achieve. This means that sadness has a negative valence, that is, it is associated with goal frustration.

When $\varphi$ concerns ideals, agent $i$ approves $\varphi$ or $i$ disapproves $\varphi$, depending respectively on the fact that $\varphi$ is ideal or not ideal for it. Thus:

$$\begin{align*}
\text{Approval}_i\varphi & \overset{def}{=} \text{Bel}_i\varphi \land \text{Ideal}_i\varphi \\
\text{Disapproval}_i\varphi & \overset{def}{=} \text{Bel}_i\varphi \land \text{Ideal}_i\neg\varphi
\end{align*}$$

Note that we refer here to the expressive part of approval and of disapproval. In fact, approval and disapproval are both expressives and declarations in Speech Act theory. There also exists a normative sense (like in: The judge says "I disapprove your release on parole [and thus, you come back to the jail"] that corresponds to a declaration in accordance with law (and not necessary with the internal psychological state of the judge). Here we focus on the expressive sense.

### 3.2 Cognitive structure of complex emotions

As said in the introduction, the cognitive structures of complex emotions include complex reasoning about norms, responsibility, etc. In the following, we suppose that agent $i$ feels an emotion related to its own responsibility or related to the responsibility of agent $j$ (supposed to be different from agent $i$) about $\varphi$. At the same time, when $\varphi$ (respectively $\neg\varphi$) is a goal or an ideal of agent $i$, thus we can expect that agent $i$ feels an emotion about $\varphi$.

There are many psychological models of emotions in the literature. One of the most widely accepted model in AI is that of Ortony, Clore and Collins [16], which defines emotions such as reproach, shame and anger that have already been formalized in logic (e.g. [1, 24]). However this model does not define emotions such as guilt or regret that are based on the concept of responsibility about actions and choices. Indeed, several psychologists (e.g. [13]) showed that guilt involves the conviction of having injured someone or of having violated some norm or imperative, and the belief that this could have been avoided. Similarly, many psychologists (e.g. [29, 12]) agree in considering regret as a negative, cognitively determined emotion that we experience when re-alizing or imagining that our present situation would have been better, had we acted differently. Our formalization of complex emotions such as regret and guilt follows this latter work in the area of psychology of emotions. (See also [15] a logical formalization of regret and [25] for a logical formalization of guilt.)

For instance, when agent $i$ believes that it is responsible for $\varphi$ while it has $\neg\varphi$ as a goal, agent $i$ feels regret, and vice versa. Formally:

$$\text{Regret}_{i}\varphi \overset{def}{=} \text{Goal}_i\neg\varphi \land \text{Bel}_i\text{Resp}_i\varphi$$

Imagine a situation in which there are only two agents $i$ and $j$, that is, $AGT = \{i, j\}$. Agent $i$ decides to park its car in a no parking area. Agent $j$ (the policeman) fines agent $i$ 100 €. Agent $i$ regrets for having been fined 100 € (noted Regret$_{fine}$). This means that, $i$ wants not to be fined (noted Goal$_i\neg\text{fine}$) and believes that it is responsible for having been fined (noted Bel$_i\text{Resp}_i\text{fine}$). That is, agent $i$ believes that it has been fined 100 € and believes that it could have avoided to be fined (by parking elsewhere).

As Bel$_i\text{Resp}_i\varphi \rightarrow \text{Bel}_i\varphi$, we have the following theorem.

**Theorem 2.**

$$\text{Regret}_{i}\varphi \rightarrow \text{Sadness}_i\varphi$$

This means that if agent $i$ regrets for $\varphi$, then it feels sad about $\varphi$. In the previous example, agent $i$ regrets for having been fined 100 € which entails that it is sad for having been fined 100 €.

When agent $i$ believes that agent $j$ is responsible for $\varphi$, and $i$ has $\neg\varphi$ as a goal, $i$ is disappointed about $\varphi$. Formally:

$$\text{Disappointment}_{i,j}\varphi \overset{def}{=} \text{Goal}_i\neg\varphi \land \text{Bel}_i\text{Resp}_j\varphi$$

Note that disappointment may have different degrees of intensity. Thus, a strong disappointment is closer to anger.

In a similar way, agent $i$ feels guilty for $\varphi$ (noted Guilt$_{i}\varphi$) if and only if $\neg\varphi$ is an ideal state of affairs for $i$ (noted Ideal$_i\neg\varphi$) and $i$ believes that it is responsible for $\varphi$. Formally:

$$\text{Guilt}_{i}\varphi \overset{def}{=} \text{Ideal}_i\neg\varphi \land \text{Bel}_i\text{Resp}_i\varphi$$
Thus, regret concerns goals whereas guilt concerns ideals. For example, imagine a situation in which there are only two agents $i$ and $j$ (that is $AGT = \{i, j\}$). Agent $i$ decides to shoot with a gun and accidentally kills agent $j$. Agent $i$ feels guilty for having killed someone (noted $\text{Ideal} \neg \text{killedSomeone}$). This means that, $i$ addresses an imperative to itself not to kill other people (noted $\text{Ideal} \neg \text{killedSomeone}$) and agent $i$ believes that it is responsible for having killed someone (noted $\text{Resp} \neg \text{killedSomeone}$).

We do not give more details about the cognitive structure of complex emotions. All these emotions are summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Bel, $\text{Resp}_{i,j} \varphi$</th>
<th>Bel, $\text{Resp}_{i,j} \neg \varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Goal}_{i} \varphi$</td>
<td>$\text{Rejoicing}_{i,j} \varphi$</td>
<td>$\text{Gratitude}_{i,j} \varphi$</td>
</tr>
<tr>
<td>$\text{Goal}_{i} \neg \varphi$</td>
<td>$\text{Regret}_{i,j} \varphi$</td>
<td>$\text{Disappointment}_{i,j} \varphi$</td>
</tr>
<tr>
<td>$\text{Ideal}_{i} \varphi$</td>
<td>$\text{MoralSatisfaction}_{i,j} \varphi$</td>
<td>$\text{Admiration}_{i,j} \varphi$</td>
</tr>
<tr>
<td>$\text{Ideal}_{i} \neg \varphi$</td>
<td>$\text{Guilt}_{i,j} \varphi$</td>
<td>$\text{Reproach}_{i,j} \varphi$</td>
</tr>
</tbody>
</table>

4. **EXPRESSION SPEECH ACTS**

As Searle says [20, Section 3.4]: “Wherever there is a psychological state specified in the sincerity condition, the performance of the act counts as an expression of that psychological state. This law holds whether the act is sincere or insincere, that is whether the speaker actually has the specified psychological state or not. (...) To thank, welcome or congratulate counts as an expression of gratitude, pleasure (at H’s arrival) or pleasure (at H’s good fortune)”.

This is true for every class of illocutionary acts not only for expressions.

The sincerity condition of expressions is that the speaker has the psychological states that he/she expresses when he/she performs an expressive act. In others words, when agent $i$ congratulates agent $j$ about some $\varphi$ related to $j$, the sincerity condition is that $i$ is pleased about $\varphi$. “To congratulate” is nothing but the expression of its sincerity condition [20, Section 3.4].

Formally, if we note $\mu(\varphi)$ an emotion about the proposition $\varphi$, we characterize the performance of an expressive as the expression of $\mu(\varphi)$ from a speaker $i$ to an addressee $j$ in front of a group of agents $H$ as follows: $\text{Exp}_{i,j,H} \mu(\varphi)$.

Note that the expression of a proposition (of the form $\text{Exp}_{i,j,H} \varphi$) and the expression (of the form $\text{Exp}_{i,j,H} \{\varphi\}$) should not be mixed up: an expressive is the expression of a particular proposition (that is, a psychological state, an emotion) but the expression of a proposition is not necessarily an expressive. For instance, we can express a commitment and the corresponding illocutionary act is a commissive; or we can express our intention that the speaker does something, and the corresponding act is a directive.

When every action is publicly performed, $H$ represents the set of all agents $AGT$. In this case, if an agent says something, everybody knows that. The parameter $H$ in the formula $\text{Exp}_{i,j,H} \mu(\varphi)$ becomes useful in case of a private conversation within a group, where illocutionary acts are not publicly performed. For instance, suppose that a group of friends are together at a party. Suppose also that John is sad because he lost his cat. He wants to share his sorrow with Beth but not with the rest of the group. In this case, $H$ is reduced to the empty set. Thus, the formula characterizing this situation is: $\text{Exp}_{\text{John}, \text{Beth}} \text{Sadness}_{\text{John}} \text{catDeath}$.

4.1 Expression of basic emotions

We propose to represent expressive speech acts as particular assertive speech acts where the propositional content is about a psychological state. More precisely, it is the emotion that the speaker wants to express. For instance, when agent $i$ wants to express to agent $j$ its joy about $\varphi$ (we call this act: to be delighted about $\varphi$), $i$ asserts to $j$ that it feels joy about the fact that $\varphi$ is true. In the same way, to express sadness about the fact that $\varphi$ is true, it is to be saddened by the fact that $\varphi$ is true. In the expressive sense, to express his/her (dis)approval is to (dis)approve of. Thus, formally:

$$
\text{IsDelighted}_{i,j,H} \varphi \overset{\text{def}}{=} \text{Exp}_{i,j,H} \text{Joy}_{i,j} \varphi
$$

$$
\text{IsSaddened}_{i,j,H} \varphi \overset{\text{def}}{=} \text{Exp}_{i,j,H} \text{Sadness}_{i,j} \varphi
$$

$$
\text{ApprovesOf}_{i,j,H} \varphi \overset{\text{def}}{=} \text{Exp}_{i,j,H} \text{Approval}_{i,j} \varphi
$$

$$
\text{DisapprovesOf}_{i,j,H} \varphi \overset{\text{def}}{=} \text{Exp}_{i,j,H} \text{Disapproval}_{i,j} \varphi
$$

Note that in the case of disapproval, and following Vandeveken [26, p. 216], “it is not presupposed that the hearer is responsible for the state of affairs”. Thus, we do not necessarily have that agent $j$ is responsible for $\varphi$.

We say that agent $i$ expresses to agent $j$ that it is sorry for $\varphi$ if and only if, $i$ expresses to agent $j$ that it is sad about the fact that $j$ did not achieve its goal that $\neg \varphi$ (i.e. agent $j$ has $\neg \varphi$ as a goal and $\varphi$ is true):

$$
\text{IsSorryFor}_{i,j,H} \varphi \overset{\text{def}}{=} \text{IsSaddened}_{i,j,H}(\text{Goal}_{j} \neg \varphi \land \varphi)
$$

$$
\text{IsSaddened}_{i,j,H} \varphi \overset{\text{def}}{=} \text{Exp}_{i,j,H} \text{Sadness}_{i,j}(\text{Goal}_{j} \neg \varphi \land \varphi)
$$

The expressive to sympathize adds to the expressive to be sorry for an aspect of goal adoption. More precisely, agent $i$ sympathizes with agent $j$ for the fact that $\varphi$ is true if and only if, $i$ expresses sadness about the fact that agent $j$ did not achieve its goal that $\neg \varphi$ (i.e. $i$ expresses to $j$ that it is sorry for $\varphi$) and $i$ expresses that it is willing to adopt $j$’s goal that $\neg \varphi$:

$$
\text{Sympathizes}_{i,j,H} \varphi \overset{\text{def}}{=} \text{IsSorryFor}_{i,j,H} \varphi \land \text{Exp}_{i,j,H} \text{AdoptGoal}_{i,j} \neg \varphi
$$

This definition logically entails the following theorem.

**Theorem 3.**

$$
\text{Sympathizes}_{i,j,H} \varphi \rightarrow \text{IsSaddened}_{i,j,H} \varphi
$$

Thus, when agent $i$ sympathizes with agent $j$ about $\varphi$, it expresses that it is sad about $\varphi$.

4.2 Expression of complex emotions

In this section, we focus on expression of complex emotions (see Section 3.2). To express rejoicing is just to rejoice and to express gratitude is to thank (what corresponds to Vandeveken’s definitions):

$$
\text{Rejoices}_{i,j,H} \varphi \overset{\text{def}}{=} \text{Exp}_{i,j,H} \text{Rejoicing}_{i,j} \varphi
$$

$$
\text{Thanks}_{i,j,H} \varphi \overset{\text{def}}{=} \text{Exp}_{i,j,H} \text{Gratitude}_{i,j} \varphi
$$
To rejoice and to thank both entail to be delighted.

Theorem 4.

\*

To express regret is just to regret:

\[
\text{Regrets}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{Regret}_{i} \varphi
\]

Following Vanderveken, to deplore is to express discontent with a high degree of strength and with a deep discontent or a deep sorrow. As we do not deal with degrees, to deplore is here just the expression of disappointment:

\[
\text{Deplores}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{Deproval}_{i} \varphi
\]

We can prove the following theorem.

Theorem 5.

\[
\text{Regrets}_{i,j,H} \varphi \rightarrow \text{IsSaddened}_{i,j,H} \varphi
\]

\[
\text{Deplores}_{i,j,H} \varphi \rightarrow \text{IsSaddened}_{i,j,H} \varphi
\]

It means that if we regret for \( \varphi \) or if we deplore it, we are sad about the fact that \( \varphi \) is true.

Sometimes, we can also express some set of regret where the speaker is responsible for and where the consequence is bad for someone else. In this case, to express regret corresponds to to apologize. More precisely, agent \( i \) apologizes to agent \( j \) for \( \varphi \) if and only if, \( i \) expresses sadness about the fact that \( j \) did not achieve its goal that \( \neg \varphi \) and \( i \) expresses that it believes to be responsible for \( \varphi \):

\[
\text{Apologizes}_{i,j,H} \varphi = \text{IsSaddened}_{i,j,H} \varphi (\text{Goal}_{j} \neg \varphi \land \varphi)
\]

\[
\land \text{Exp}_{i,j,H} \text{Bel}_{i} \text{Resp}_{j} \varphi
\]

This definition entails the following theorem.

Theorem 6.

\[
\text{Apologizes}_{i,j,H} \varphi \rightarrow \text{Regrets}_{i,j,H} \varphi (\text{Goal}_{j} \neg \varphi \land \varphi)
\]

Thus, when agent \( i \) apologizes to agent \( j \) for \( \varphi \), \( i \) expresses regret about the fact that \( j \) has \( \neg \varphi \) as a goal and \( \varphi \) is true.

The expression of moral satisfaction is defined as follows:

\[
\text{IsMorallySatisfied}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{MoralSatisfaction}_{i} \varphi
\]

To express admiration is to compliment. Vanderveken says that “Complimenting does not necessarily relate to something done by the hearer, since we can compliment someone on his intelligence, musical ability (...).” But in these cases we can object that complimenting is more about the use of this intelligence or of this ability than about the intelligence itself or the ability itself. In any case, the following definition applies only to the case in which the hearer is responsible for \( \varphi \):

\[
\text{Compliments}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{Admiration}_{i} \varphi
\]

We can prove the following theorem.

Theorem 7.

\[
\text{IsMorallySatisfied}_{i,j,H} \varphi \rightarrow \text{ApprovesOf}_{i,j,H} \varphi
\]

\[
\text{Compliments}_{i,j,H} \varphi \rightarrow \text{ApprovesOf}_{i,j,H} \varphi
\]

To express guilt is to express that one feels guilty, and to express reproach is just to reproach:

\[
\text{FeelsGuilty}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{Guilt}_{i} \varphi
\]

\[
\text{Reproaches}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{Reproach}_{i} \varphi
\]

These definitions entail the following theorem.

Theorem 8.

\[
\text{FeelsGuilty}_{i,j,H} \varphi \rightarrow \text{DisapprovesOf}_{i,j,H} \varphi
\]

\[
\text{Reproaches}_{i,j,H} \varphi \rightarrow \text{DisapprovesOf}_{i,j,H} \varphi
\]

In other words, if agent \( i \) expresses that it feels guilty about the fact that \( \varphi \) is true, or if agent \( i \) reproaches agent \( j \) for \( \varphi \), then agent \( i \) also expresses its disapproval for \( \varphi \).

To accuse is not an expressive (but an assertive —see[26, p. 179]). It is however interesting to give a name to the expression of a speaker’s belief about the hearer’s responsibility:

\[
\text{Accuses}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{Bel}_{i} \text{Resp}_{j} \varphi
\]

We are now able to formalize the expressive to protest. Following Vanderveken, to protest is nothing but to express his/her disapproval together with the fact that the addressee of the act is responsible for the present state of affairs. The latter is what we call to accuse. Thus:

\[
\text{Protests}_{i,j,H} \varphi = \text{Exp}_{i,j,H} \text{Bel}_{i} \text{Resp}_{j} \varphi \land \text{Accuses}_{i,j,H} \varphi
\]

4.3 Remark

When the performance of an expressive entails the performance of another expressive — this is typically the case in the previous theorems —, it means that each time we express some psychological attitude, we also express some other psychological attitude. This relation exists in speech act theory through the semantic tree of expressives (see [26, p. 218]). In this tree, the success conditions of to express are a subset of the success conditions of to approve, and the success conditions of to approve are themselves a subset of success conditions of to praise, for instance. This means that, from an illocutionary point of view, to praise entails to approve, and to approve entails to express.

If we suppose that the speaker has the psychological attitudes that he/her expresses, then the previous theorems suggest that feeling some emotions entails feeling some others. For example, Theorem 5.1 says that feeling regret entails feeling sadness. This is in accordance with the literature in psychology according to which we can feel several emotions at the same time (see [13] for more details).

5. CONCLUSION

In this article we have presented the logic MLC that allows us to represent the cognitive structure of basic emotions (such as joy or sadness) and more complex emotions (such as regret or guilt), and their expression in front of a group of people.
agents. Recall that a cognitive structure of emotion corresponds to the mental states that an agent must necessarily have for feeling the corresponding emotion.

Our work is based on the assumption that the performance of an illocutionary act consists in the expression of some mental states by the speaker. The logic MLC includes a novel modal operator formalizing what is expressed by performing a speech act. This operator allows us to formalize every class of illocutionary act. In this work, we only presented expressive speech acts because this class is less studied than the others (assertives, directives, commissives and declaratives). In future work, we will present a generalization of this work by including other classes of illocutionary acts.

By means of the logic MLC we have proved some intuitive theorems highlighting the relationships between different emotions (e.g. regret entails sadness) and between different expressive speech acts (e.g. to apologize entails to regret).

Note that we did not exploit in detail the argument \( H \) (the audience) in our formalization of expressive speech acts. However, as we have briefly shown in Section 4, the argument \( H \) becomes useful when we want to describe a private conversation within a group discussion. For instance, if a lecturer tells to the chairman that he/she has stage fright, there is no reason to suppose that everyone who is present at the conference hears that. The argument \( H \) in the modal operator \( \exp_{p,j,H} \) allows us to represent such cases.

6. ACKNOWLEDGMENTS

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7. REFERENCES