Representing and Reasoning about Communicative Conditional Commitments
(Extended Abstract)

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ABSTRACT
Social commitments provide a powerful representation for modeling multi-agent interactions without relinquishing part of agents autonomy and flexibility. However, distinguishing between different but related types of conditional commitments, a natural frame of social commitments, is not considered yet. In this paper, we define a new logical language, CTL\textsuperscript{CC}, which extends CTL with modalities to represent conditional commitments and their fulfillments using the formalism of interpreted systems. Such a language excludes the paradox that plagues the semantics of fulfilling commitments and their fulfillments respectively. We use \(\text{CC}_i \varphi\) to represent the value of \(\varphi\) in \(l_i\) for \(i\) and in \(l_j(\varphi')\) for \(j\) are the same. As commitments are established through communication among agents, we call them communicative conditional commitments.

DEFINITION 1. A model of communicative conditional commitments generated from interpreted systems is a tuple \(\mathcal{M} = (S, R, \{\sim_{i \rightarrow j} \mid (i, j) \in \mathcal{A}^2\}, I, \forall)\) where \(S \subseteq L_1 \times \ldots \times L_n\) is a set of global states; \(R_i \subseteq S \times S\) is a transition relation defined by \((s, s') \in R_i\) iff there exists a joint action \((a_1, \ldots, a_n) \in \text{ACT}\) s.t. \(\tau(s, a_1, \ldots, a_n) = s'\); for each pair \((i, j) \in \mathcal{A}^2, \sim_{i \rightarrow j} \subseteq S \times S\) is a serial accessibility relation defined by \(s \sim_{i \rightarrow j} s'\) iff \(1) \ l_i(s) = l_i(s'); 2) \ (s, s') \in R_i; 3) \ V_{\mathcal{A}} \land V_{\mathcal{A}} \neq \emptyset\) and \(\forall \varphi \in \text{Var}, \forall \varphi \forall \varphi \forall \varphi \forall \varphi \forall \varphi \forall \varphi\) we have \(l_i(s) = l_i(s')\); and 4) \(\forall y \in V_{\mathcal{A}} \land V_{\mathcal{A}}\), we have \(l_i(s) = l_i(s')\); \(I \subseteq S\) is a set of initial global states; and \(\forall \varphi \in \mathcal{PV} \rightarrow 2^{\mathcal{PV}}\), \(\mathcal{PV}\) is a set of atomic propositions, is a valuation function.

DEFINITION 2. The syntax of CTL\textsuperscript{CC}, an extension of CTL with modalities for communicative conditional commitments and their fulfillments, is defined as follows:
\[
\varphi ::= p \mid \neg \varphi \mid \varphi \lor \varphi \mid E \bigcirc \varphi \mid E \boxdot \varphi \mid E(\varphi U \varphi) \mid CC \mid Fu(CC)
\]

where \(p \in \mathcal{PV}\) and \(E\) is the existential quantifier on paths. \(\bigcirc, \boxdot, \text{ and } U\) are CTL path connectives standing for “next”, “globally”, and “until” respectively. \(CC\) and \(Fu\) stand for conditional commitments and their fulfillments respectively.

A weak (strong) conditional commitment \(CC^w_{i \rightarrow j}(\psi, \varphi)\) (\(CC^w_{i \rightarrow j}(\psi, \varphi)\) is read as “agent \(i\) weakly (strongly) commits towards agent \(j\) that \(\varphi\) when the condition \(\psi\) holds”. The main difference between our two types of conditional commitments
(weak and strong) is that weak conditional commitments can be established even if the condition will never be satisfied, while strong conditional commitments are only established when there is a possibility to satisfy their conditions. Concretely, only weak commitments are considered in the literature to model commitment protocols. \( Fu(CC_{ij}(\psi, \varphi)) \) (resp. \( Fu(\neg CC_{ij}(\psi, \varphi)) \)) is read as “the weak (resp. strong) commitment \( CC_{ij}(\psi, \varphi) \) (resp. \( CC_{ij}(\psi, \varphi) \)) is fulfilled”.

Excluding the commitments and their fulfillments, the semantics of CTL\(^c\) state formulae is defined in the model \( M \) as usual (semantics of CTL). The state formula \( CC_{ij}(\psi, \varphi) \) is satisfied in the model \( M \) at \( s \) iff the content \( \varphi \) holds in every state satisfying \( \psi \) and accessible via \( s_{\sim ij} \). This would be formalized as follows:

\[
(M, t) \models CC_{ij}(\psi, \varphi) \iff \forall s' \in S : s_{\sim ij} s' \text{ and } s' \in [\![\psi]\!]_M,
\]

where \([\![\cdot]\!]_M\) denotes the set of states satisfying the formula \( \psi \), i.e., \([\![\psi]\!]_M = \{ s \in S | (M, s) \models \psi \} \). The semantics of the strong commitment \( CC_*(\psi, \varphi) \) is similar, but we add condition 1 to ensure that there is at least one accessible state satisfying the condition \( \psi \).

Formally:

\[
(M, s) \models CC_*(\psi, \varphi) \iff 1 \exists s' \in S : s_{\sim ij} s' \text{ and } s' \in [\![\psi]\!]_M,
\]

and \( 2 \forall s' \in S : s_{\sim ij} s' \text{ and } s' \in [\![\psi]\!]_M \), we have \((M, s') \models \varphi\).

The state formula \( Fu(CC_{ij}(\psi, \varphi)) \) is satisfied in the model \( M \) at \( s \) iff \( s \) satisfies the content \( \varphi \) and the negation of \( CC_{ij}(\psi, \varphi) \) holds. There exists a state \( s' \) satisfying the weak commitment from which \( s \) is “seen” via \( s_{\sim ij} \).

Formally:

\[
(M, s) \models Fu(CC_{ij}(\psi, \varphi)) \iff \exists s' \in S : s_{\sim ij} s \text{ and } (M, s') \models CC_{ij}(\psi, \varphi)
\]

\[
(M, s') \models CC_{ij}(\psi, \varphi) \text{ and } (M, s) \models \psi \land \neg CC_{ij}(\psi, \varphi).
\]

The semantics of the strong commitment \( Fu(\neg CC_*(\psi, \varphi)) \) is similar, but the focus is on checking the satisfiability of the condition \( \psi \). This would be formalized as follows:

\[
(M, s) \models Fu(\neg CC_*(\psi, \varphi)) \iff \exists s' \in S : s_{\sim ij} s \text{ and } (M, s') \models CC_*(\psi, \varphi)
\]

\[
(M, s') \models CC_*(\psi, \varphi) \text{ and } (M, s) \models \psi \land \neg CC_*(\psi, \varphi).
\]

Our semantics solves the fulfillment paradox in [1, 2] where the state labelled by the fulfillment is also labelled by the commitment that is then marked unresolved. As conditional commitment is a first class citizen in our approach, propositional commitment \( C_{ij}(\varphi) \) can be abbreviated as \( C_{ij}(\varphi) \triangleq CC_{ij}(\top, \varphi) \) where \( \xi \in \{ w, s \} \) and \( \top \triangleq (p \lor \neg p) \).

2. REASONING RULES

We consider here several reasoning rules that are supported in our logic. The proofs are straightforward from the proposed semantics. Similar rules were first presented by Singh [4]. The idea is to: 1) capture the semantic characteristics of commitments and their fulfillments; and 2) show how two types of commitments and their fulfillments are related to each other. When the debtor \( i \) and creditor \( j \) are understood from the context, we simply write \( CC_*(\psi, \varphi) \) instead of \( CC_{ij}(\psi, \varphi) \) and \( C_*(\varphi) \) instead of \( C_{ij}(\varphi) \).

R\(_1\). Fulfillment Necessity. \( Fu(CC_*(\psi, \varphi)) \supset \varphi \)

**Meaning:** when a commitment is fulfilled, its content holds.

R\(_2\). Fulfillment. \( Fu(CC_*(\psi, \varphi)) \supset \neg CC_*(\psi, \varphi) \)

**Meaning:** the commitment is discharged and no longer active once it is fulfilled.

R\(_3\). Partially Detach. \( CC_*(\psi_1 \land \neg \psi_2, \varphi) \land A \supset CC_*(\psi_2, \varphi) \)

**Meaning:** when part of the condition (i.e., \( \psi_1 \)) of a commitment holds next, the commitment with the remainder of the condition (i.e., \( \psi_2 \)) and the same content comes into being.

As special case of \( R_3 \), we obtain the following rule (\( R_4 \)):

\( CC_*(\psi, \varphi) \land A \supset C_*(\varphi) \), i.e., detaching into propositional commitment in one shot.

R\(_4\). L-Disjoin. \( CC_*(\psi, \varphi) \land CC_*(\psi_2, \varphi) \supset CC_*(\psi_1 \lor \psi_2, \varphi) \)

**Meaning:** if \( i \) commits that \( \psi \) if \( \psi_1 \) and commits that the same content holds if \( \psi_2 \), then \( i \) commits that \( \varphi \) if \( \psi_1 \) or \( \psi_2 \).

R\(_5\). R-Conjoin. \( CC_*(\psi, \varphi) \land CC_*(\psi_2, \varphi) \supset CC_*(\psi_1 \land \neg \psi_2, \varphi) \)

**Meaning:** when \( \psi \) holds, an agent \( i \) would become committed to bring about \( \psi_1 \land \neg \psi_2 \) if \( i \) double commits to bring about \( \psi_1 \) if \( \psi \) and to bring about \( \psi_2 \) if the same condition holds.

R\(_6\). Consistency (Strong Commitment). \( \neg CC_*(\psi, \bot) \) where \( \bot \) is abbreviated as \( \bot \equiv \bot \).

**Meaning:** an agent cannot strongly commit to false.

The following rule holds: \( CC_*(\bot, \bot) \) or in general \( CC_*(\bot, \varphi) \) as there is no accessible state that satisfies \( \bot \).

For weak commitments, the following holds:

R\(_7\). Consistency (Weak Commitment). \( A \supset \neg CC_*(\psi, \bot) \)

R\(_8\). Consistency (Null fulfillment). \( \neg Fu(CC_*(\psi, \bot)) \)

**Meaning:** a commitment to false cannot be fulfilled.

R\(_9\). Strong Consistency. \( CC_*(\psi, \varphi) \supset \neg CC_*(\psi, \neg \varphi) \)

**Meaning:** when a strong commitment holds, then there is no possibility for committing to the negation of its content subject to the same condition.

This rule is not valid in the case of weak commitments since \( CC_*(\bot, \varphi) \) holds. Thus, for weak commitments, it is easy to prove the following:

R\(_{10}\). Weak Consistency. \( A \supset \neg CC_*(\psi, \neg \varphi) \)

R\(_{11}\). Chain. \( CC_*(\psi_1, \varphi_1) \land (\varphi_1 \supset \psi_2) \land CC_*(\psi_2, \varphi_2) \supset CC_*(\psi_1, \varphi_2) \)

**Meaning:** commitments are close under implication.

R\(_{12}\). Weaken. \( CC_*(\psi, \varphi_1 \land \varphi_2) \supset CC_*(\psi, \varphi_1) \)

**Meaning:** if \( i \) commits to a conjunction subject to a condition, \( i \) is also committed to each part of the conjunction.

R\(_{13}\). Nonexistence. \( A \supset \neg CC_*(\psi, \varphi) \)

**Meaning:** if the condition is brought about but the content does not hold, then the commitment does not hold too.

To conclude, Singh [4] introduced constraints that correspond to these rules in the sense of Bentham’s correspondence theory and proved that any logic generated by subsets of those rules is sound and complete w.r.t. models that satisfy those constraints. Soundness and completeness of our logic follow then from this result w.r.t. the same models.

3. REFERENCES


