Representing and Reasoning about Communicative Conditional Commitments

(Extended Abstract)

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ABSTRACT

Social commitments provide a powerful representation for modeling multi-agent interactions without relinquishing part of agents autonomy and flexibility. However, distinguishing between different but related types of conditional commitments, a natural frame of social commitments, is not considered yet. In this paper, we define a new logical language, CTL^{cc}, which extends CTL with modalities to represent conditional commitments and their fulfillments using the formalism of interpreted systems. Such a language excludes the paradox that plagues the semantics of fulfilling commitments in the literature. We present a set of rules to reason about conditional commitments and their fulfillments.

Categories and Subject Descriptors

F.4.1 [**Mathematical Logic**]: Temporal Logic

General Terms

Design; Languages

Keywords

Strong (Weak) Conditional Commitments; Reasoning

1. INTERPRETED SYSTEMS AND CTL^{CC}

Assuming the formalism of interpreted systems [3] comprises of a set $\mathcal{A} = \{1, \ldots, n\}$ of n agents. Each agent i is associated with countable sets L_i and Act_i of local states and actions. The local protocol $\mathcal{P}_i : L_i \to 2^{Act_i}$ is a function giving the set of enabled actions that may be performed by i in a given state. Let $g = (l_1, \ldots, l_n)$ be a global state, the set of all global states $G = L_1 \times \ldots \times L_n$ is the Cartesian product of all local states of n agents. We use $l_i(g)$ to represent the local state of i in the global state g. The global (resp. local) evolution function is defined as follows: $\tau : G \times ACT \to G$ (resp. $\tau_i : L_i \times Act_i \to L_i$), where $ACT = Act_1 \times \ldots \times Act_n$ and each component $a \in ACT$ is a "joint action" [3].

We advocate the extended version of the interpreted system formalism introduced in [1, 2] to account for agent communication. In [1, 2], the authors associated with each $i \in \mathcal{A}$ a countable set Var_i of at most $n-1$ local Boolean variables to represent communication channels through which messages are sent and received. The value of a variable x in Var_i at local state $l_i(g)$ is denoted by $l_i^x(g)$. If $l_i(g) = l_i(g')$, then $l_i^x(g) = l_i^x(g')$ for all $x \in Var_i$. The idea is that a communication channel between i and j does exist iff $Var_i \cap Var_j \neq \emptyset$. For $x \in Var_i \cap Var_j$, $l_i^x(g) = l_j^x(g')$ means the values of x in $l_i(g)$ for i and in $l_j(g')$ for j are the same. As commitments are established through communication among agents, we call them *communicative conditional commitments*.

Definition 1. *A model of communicative conditional commitments generated from interpreted systems is a tuple $\mathcal{M} = (S, R_t, \{\sim_{i\to j} \mid (i,j) \in \mathcal{A}^2\}, I, \mathcal{V})$ where $S \subseteq L_1 \times \ldots \times L_n$ is a set of global states; $R_t \subseteq S \times S$ is a transition relation defined by $(s, s') \in R_t$ iff there exists a joint action $(a_1, \ldots, a_n) \in ACT$ s.t. $\tau(s, a_1, \ldots, a_n) = s'$; for each pair $(i,j) \in \mathcal{A}^2$, $\sim_{i\to j} \subseteq S \times S$ is a serial accessibility relation defined by $s \sim_{i\to j} s'$ iff 1) $l_i(s) = l_i(s')$; 2) $(s, s') \in R_t$; 3) $Var_i \cap Var_j \neq \emptyset$ and $\forall x \in Var_i \cap Var_j$ we have $l_i^x(s) = l_j^x(s')$; and 4) $\forall y \in Var_j - Var_i$ we have $l_j^y(s) = l_j^y(s')$; $I \subseteq S$ is a set of initial global states; and $\mathcal{V} : \mathcal{PV} \to 2^S$, where $\mathcal{PV}$ is a set of atomic propositions, is a valuation function.*

Definition 2. *The syntax of CTL^{cc}, an extension of CTL with modalities for communicative conditional commitments and their fulfillments, is defined as follows:*

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid E\bigcirc\varphi \mid E\Box\varphi \mid E(\varphi\ U\ \varphi) \mid \mathcal{CC} \mid Fu(\mathcal{CC})$$
$$\mathcal{CC} ::= CC^w_{i\to j}(\psi, \varphi) \mid CC^s_{i\to j}(\psi, \varphi)$$

where $p \in \mathcal{PV}$ and E is the existential quantifier on paths. $\bigcirc$, $\Box$, and U are CTL path connectives standing for "next", "globally", and "until" respectively. $\mathcal{CC}$ and Fu stand for conditional commitments and their fulfillments respectively.

A weak (strong) conditional commitment $CC^w_{i\to j}(\psi, \varphi)$ ($CC^s_{i\to j}(\psi, \varphi)$) is read as "agent i weakly (strongly) commits towards agent j that φ when the condition ψ holds". The main difference between our two types of conditional commitments

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(weak and strong) is that weak conditional commitments can be established even if the condition will never be satisfied, while strong conditional commitments are only established when there is a possibility to satisfy their conditions. Concretely, only weak commitments are considered in the literature to model commitment protocols. $Fu(CC^w_{i\to j}(\psi,\varphi))$ (resp. $Fu(CC^s_{i\to j}(\psi,\varphi))$) is read as "the weak (resp. strong) commitment $CC^w_{i\to j}(\psi,\varphi)$ (resp. $CC^s_{i\to j}(\psi,\varphi)$) is fulfilled".

Excluding the commitments and their fulfillments, the semantics of CTL^{cc} state formulae is defined in the model $\mathcal{M}$ as usual (semantics of CTL). The state formula $CC^w_{i\to j}(\psi,\varphi)$ is satisfied in the model $\mathcal{M}$ at s iff the content φ holds in every state satisfying ψ and accessible via $\sim_{i\to j}$. This would be formalized as follows:

$(\mathcal{M},s) \models CC^w_{i\to j}(\psi,\varphi)$ *iff* $\forall s' \in S : s \sim_{i\to j} s'$ *and* $s' \in [\![\psi]\!]$, *we have* $(\mathcal{M},s') \models \varphi$

where $[\![\psi]\!]$ denotes the set of states satisfying the formula ψ, i.e., $[\![\psi]\!] = \{s \in S \mid (\mathcal{M},s) \models \psi\}$. The semantics of the strong commitment $CC^s(\psi,\varphi)$ is similar, but we add condition 1 to ensure that there is at least one accessible state satisfying the condition ψ. Formally:

$(\mathcal{M},s) \models CC^s_{i\to j}(\psi,\varphi)$ *iff* 1) $\exists s' \in S : s \sim_{i\to j} s'$ *and* $s' \in [\![\psi]\!]$; *and* 2) $\forall s' \in S : s \sim_{i\to j} s'$ *and* $s' \in [\![\psi]\!]$, *we have* $(\mathcal{M},s') \models \varphi$

The state formula $Fu(CC^w_{i\to j}(\psi,\varphi))$ is satisfied in the model $\mathcal{M}$ at s iff s satisfies the content φ and the negation of the weak commitment $CC^w_{i\to j}(\psi,\varphi)$ and there exists a state s' satisfying the weak commitment from which s is "seen" via $\sim_{i\to j}$. Formally:

$(\mathcal{M},s) \models Fu(CC^w_{i\to j}(\psi,\varphi))$ *iff* $\exists s' \in S : s' \sim_{i\to j} s$ *and* $(\mathcal{M},s') \models CC^w_{i\to j}(\psi,\varphi)$ *and* $(\mathcal{M},s) \models \varphi \wedge \neg CC^w_{i\to j}(\psi,\varphi)$

The semantics of the strong fulfillment $Fu(CC^s(\psi,\varphi))$ is similar, but the focus is on checking the satisfiability of the condition ψ. This would be formalized as follows:

$(\mathcal{M},s) \models Fu(CC^s_{i\to j}(\psi,\varphi))$ *iff* $\exists s' \in S : s' \sim_{i\to j} s$ *and* $(\mathcal{M},s') \models CC^s_{i\to j}(\psi,\varphi)$ *and* $(\mathcal{M},s) \models \psi \wedge \neg CC^s_{i\to j}(\psi,\varphi)$

Our semantics solves the *fulfillment paradox* in [1, 2] where the state labelled by the fulfillment is also labelled by the commitment that is then marked unresolved. As conditional commitment is a first class citizen in our approach, propositional commitment $C_{i\to j}(\varphi)$ can be abbreviated as: $C_{i\to j}(\varphi) \triangleq CC^{\xi}_{i\to j}(\top,\varphi)$ where $\xi \in \{w,s\}$ and $\top \triangleq (p \vee \neg p)$.

2. REASONING RULES

We consider here several reasoning rules that are supported in our logic. The proofs are straightforward from the proposed semantics. Similar rules were first presented by Singh [4]. The idea is to: 1) capture the semantic characteristics of commitments and their fulfillments; and 2) show how two types of commitments and their fulfillments are related to each other. When the debtor i and creditor j are understood from the context, we simply write $CC^{\xi}(\psi,\varphi)$ instead of $CC^{\xi}_{i\to j}(\psi,\varphi)$ and $C(\varphi)$ instead of $C_{i\to j}(\varphi)$.

$\mathbf{R}_1$. Fulfillment Necessity. $Fu(CC^{\xi}(\psi,\varphi)) \supset \varphi$
`Meaning`: when a commitment is fulfilled, its content holds.

$\mathbf{R}_2$. Fulfillment. $Fu(CC^{\xi}(\psi,\varphi)) \supset \neg CC^{\xi}(\psi,\varphi)$
`Meaning`: the commitment is discharged and no longer active once it is fulfilled.

$\mathbf{R}_3$. Partially Detach. $CC^{\xi}(\psi_1 \wedge \psi_2,\varphi) \wedge A\bigcirc\psi_1 \supset CC^{\xi}(\psi_2,\varphi)$
`Meaning`: when part of the condition (i.e., ψ_1) of a commitment holds next, the commitment with the remainder of the condition (i.e., ψ_2) and the same content comes into being.

As special case of $\mathbf{R}_3$, we obtain the following rule ($\mathbf{R}_4$): $CC^{\xi}(\psi,\varphi) \wedge A\bigcirc\psi \supset C(\varphi)$, i.e., detaching into propositional commitment in one shot.

$\mathbf{R}_5$. L-Disjoin. $CC^{\xi}(\psi_1,\varphi) \wedge CC^{\xi}(\psi_2,\varphi) \supset CC^{\xi}(\psi_1 \vee \psi_2,\varphi)$
`Meaning`: if i commits that φ if ψ_1 and commits that the same content holds if ψ_2, then i commits that φ if ψ_1 or ψ_2.

$\mathbf{R}_6$. R-Conjion. $CC^{\xi}(\psi,\varphi_1) \wedge CC^{\xi}(\psi,\varphi_2) \supset CC^{\xi}(\psi,\varphi_1 \wedge \varphi_2)$
`Meaning`: when ψ holds, an agent i would become committed to bring about φ_1 and φ_2 if i double commits to bring about φ_1 if ψ and to bring about φ_2 if the same condition holds.

$\mathbf{R}_7$. Consistency (Strong Commitment). $\neg CC^s(\psi,\bot)$ where $\bot$ is abbreviated as $\bot \triangleq \neg\top$
`Meaning`: an agent cannot strongly commit to false.

The following rule holds: $CC^w(\bot,\bot)$, or in general $CC^w(\bot,\varphi)$ as there is no accessible state that satisfies $\bot$. For weak commitments, the following holds:

$\mathbf{R}_8$. Consistency (Weak Commitment). $A\bigcirc\psi \supset \neg CC^w(\psi,\bot)$

$\mathbf{R}_9$. Consistency (Fulfilment). $\neg Fu(CC^{\xi}(\psi,\bot))$
`Meaning`: A commitment to false cannot be fulfilled.

$\mathbf{R}_{10}$. Strong Consistency. $CC^s(\psi,\varphi) \supset \neg CC^s(\psi,\neg\varphi)$
`Meaning`: when a strong commitment holds, then there is no possibility for committing to the negation of its content subject to the same condition.

This rule is not valid in the case of weak commitments since $CC^w(\bot,\varphi)$ holds. Thus, for weak commitments, it is easy to prove the following:

$\mathbf{R}_{11}$. Weak Consistency. $A\bigcirc\psi \wedge CC^w(\psi,\varphi) \supset \neg CC^w(\psi,\neg\varphi)$

$\mathbf{R}_{12}$. Chain. $(CC^{\xi}(\psi_1,\varphi_1) \wedge (\varphi_1 \supset \psi_2) \wedge CC^{\xi}(\psi_2,\varphi_2)) \supset CC^{\xi}(\psi_1,\varphi_2)$
`Meaning`: commitments are close under implication.

$\mathbf{R}_{13}$. Weaken. $CC^{\xi}(\psi,\varphi_1 \wedge \varphi_2) \supset CC^{\xi}(\psi,\varphi_1)$
`Meaning`: if i commits to a conjunction subject to a condition, i is also committed to each part of the conjunction.

$\mathbf{R}_{14}$. Nonexistence. $A\bigcirc(\psi \wedge \neg\varphi) \supset \neg CC^{\xi}(\psi,\varphi)$
`Meaning`: if the condition is brought about but the content does not hold, then the commitment does not hold too.

To `conclude`, Singh [4] introduced constraints that correspond to these rules in the sense of *Benthem's correspondence theory* and proved that any logic generated by subsets of those rules is sound and complete w.r.t. models that satisfy those constraints. Soundness and completeness of our logic follow then from this result w.r.t. the same models.

3. REFERENCES

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