Deliberating about Voting Dimensions
(Extended Abstract)

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ABSTRACT

It has been claimed that deliberation is capable of overcoming social choice theory impossibility results, by bringing about single-peakedness. Our aim is to better understand the relationship between single-peakedness and collective justifications of preferences.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent Systems*; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Theory, Economics

Keywords

Judgment Aggregation, Single-peakedness, Deliberation

1. INTRODUCTION

Arrow's theorem shows that it is not possible to aggregate individual preferences by means of an aggregation procedure that balances fairness and efficiency. Among the well-known escape routes to Arrow's result, Black's restriction of possible preferences to single-peaked profiles [2, 1] is significant because it has been associated with a convincing intuitive interpretation: it amounts to assuming that individuals agree on a common *dimension* that structures the decision problem at issue. This is an important point: any restriction of individual preferences has to be justified, as it is somehow contradicting the rationale of an aggregative view of democratic decisions: individual preferences are not matter of normative judgement, it is the aggregation procedure that carries the burden of the normative justification of choices. By contrast, *deliberative democracy* [4] stresses the role of public justifications of policies rather than the conditions on the aggregation of preferences. Deliberation is a discursive situation among rational and equal agents and what in principle makes a collective choice fair lies in collective justification that deliberation can bring about. In the last thirty odd years, the connection between deliberative and aggregational models have been investigated and discussed within the normative theory of democracy [4, 3]. An interesting problem is to understand to what extent the two models are compatible. A proposal of integration relies on the idea of a deliberation that is capable of promoting agents' awareness of the relevant dimensions involved in a decision problem and that may bring about a collective justification of the elected policy [3, 7]. It is worth noting that there is a gap between the intuitive notion of a dimension and the formal condition of single-peakedness, that merely states a structural property of preference profiles. Following [3], we shall distinguish between a formal notion of a dimension, the *formal dimension*, that is the one in the formal definition of single-peakedness, and a semantic notion of a dimension, a *semantic dimension*, that is the criterion that agents use to make their choices. In this paper, we view semantic dimensions as public justifications of preferences and we discuss how formal dimensions are related to semantic dimensions. Moreover, we discuss whether single-peakedness may provide justifications of collective choices.

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2. SINGLE-PEAKEDNESS

Let N be a set of agents and A a set of alternatives. For $i \in N$, a (strict) preference ordering $>_P$ is an *irreflexive*, *transitive* and *complete* relation $>_P \subseteq A \times A$. A *preference profile* **P** is a list of preference orderings $(>_1, \ldots, >_n)$. Let $\mathcal{L}(A)$ denote the set of all preference ordering. A *social welfare function* $F : \mathcal{L}(A)^n \to \mathcal{L}(A)$ maps preference profiles to preference orderings. E.g. the majority rule is defined as $F(\mathbf{P}) = \{(x, y) \text{ s.t. } |\{i \in N \mid (x, y) \in >_i \}| > n/2\}$. Single-peakedness is defined as follows. Given a linear order $>$, we say that *y is between x and z* iff $x > y > z$ or $z > y > x$. The *peak* of preference order $\text{PEAK}(>_i)$ is the maximal element wrt $>_i$. A preference profile is *single-peaked* if and only if there exists a linear order $>_\Omega$ of the alternatives (a *formal dimension*) such that for every $>_i$ and every alternative y such that $y \neq \text{PEAK}(>_i)$, i prefers any alternative that is between $\text{PEAK}(>_i)$ and y (wrt $>_\Omega$) to y. Let $>'_\Omega$ denote the opposite dimension. If there is an odd number of $n + 1$ voters and we order the agents' peaks according to $>_\Omega$, the *median voter*'s peak, namely the option that has $n/2$ peaks on the right and $n/2$ peaks on the left is elected by majority [2].

3. EXAMPLE

We rephrase an example discussed in [6]. Suppose agents 1, 2, and 3 have to elect a collective policy among alternatives a, b and c. Their preference profile is single-peaked, e.g. wrt $c > a > b$ (Tab. 1). Thus, there is a winning policy, i.e. a. Suppose that agents justify their preferences by appealing to three relations that express the extent to which the alternatives promote *productivity* P, *cost* C, or *fairness* F (Tab. 2).

1:	$c >_1 a >_1 b$	aPb, cPb, cPa	aPb, cPb, cPa
2:	$a >_2 b >_2 c$	aCb, bCc, aCc	aPb, bFc, aFc
3:	$b >_3 a >_3 c$	bFa, bFc, aFc	bFa, bFc, aFc
maj.	$a > b > c$	-	-
	Tab. 1	Tab. 2	Tab. 3

E.g. 1 prefers a over b and 1's justification is that a is more productive than b. We want to discuss what may provide a collective justification of the chosen policy (i.e. a). In order vote on justifications, agents have to agree on a common agenda. Suppose that 2 agrees to give up his justifications in terms of cost (C) and to use P and F. Agent 2 can do so without revising his preferences. We obtain a profile of justifications with just P and F (Tab. 3). Moreover, P and F refer to opposite rankings, i.e. xPy iff yFx. By voting and reasoning about the judgments in Tab. 3, we obtain the following discursive dilemma [5].

	aPb	aFc	$aPb \wedge aFc$
1	yes	no	no
2	yes	yes	yes
3	no	yes	no
maj	yes	yes	no

Here, discursive dilemmas have the following interpretation. By majority, a can be justified by saying that it is more productive than b (aPb). However, a is not chosen *because of productivity*, as it is dominated by c along the productivity axis. Nor a is chosen on the ground of fairness, as it is dominated by b on the axis of fairness. The actual justification refers to the fact that a is *both* more productive than b *and* more fair than c. However, the dilemma shows that agents cannot elect the conjunction of the two. In the next sections, we shall generalise this example.

4. MODEL AND RESULTS

Given a preference ordering $>_P$, a *justification* of $>_P$ is a *transitive* and *irreflexive* relation D such that $D \subseteq >_P$. A set of justifications $J = \{D_1, ..., D_m\}$ *justifies* a preference ordering $>_P$ if J is a partition of $>_P$. For example, $a > b > c$ can be justified by two relations as in $J = \{\{(a,b)\}, \{(b,c), (a,c)\}\}$. Our assumptions on sets of justifications have the following meaning. Each pair of alternatives $(x, y) \in >_P$ is justified by some D_j in J. Moreover, agents cannot have both aDb and bDa. The D_is are not necessarily complete, therefore some pair (a, b) can be justified by D, whereas some other pair (c, d) is justified by D'. For example, an agent can prefer a over b because "a promotes GDP's growth better that b" and c over d because "c is more liberal than d ". The transitivity of Ds means that if an agent justifies $a >_P b$ on the ground of D (e.g. "a promotes fairness more than b") and justifies $b >_P c$ on the same ground of D, then he is committed to justify also $a >_P c$ on the same ground. We assume that each agent i has a set of justifications J_i of $>_i$. A profile of justifications is a list $\mathbf{J} = (J_1, \ldots, J_n)$. Single-peakedness can be easily generalized to profiles of justifications as follows: J_i is compatible with a dimension $>_\Omega$ iff, whenever $x >_\Omega y >_\Omega z$ or $x >'_\Omega y >'_\Omega z$, if, for $D_j \in J_i$, xD_jy, then for no $D_k \in J_i$, zD_ky.

Next, we introduce a fragment of first order logic in order to model *judgments* that agents use to express their justifications. Assume a set of constants $\mathcal{A}$, one for each alternative, and a set of *justification predicates* $\mathcal{D} = \{\bar{D}_1, ..., \bar{D}_m\}$, where $\bar{D}_j$ is a binary relational symbol. Let $\mathcal{L}_D$ be the language containing all the atomic judgments $a\bar{D}b$. The model of a set of judgment is defined as follows. The domain is the set of the alternatives A, the interpretation $\mathcal{I}$ of the individual constants is fixed, and relations are interpreted by $\mathcal{I}(D_j) \subseteq A \times A$. A *judgment set* $\bar{J}$ is a subset of $\mathcal{L}_D$ that satisfies some easy properties that ensure that the model of $\bar{J}$, $\mathcal{I}(\bar{J})$ is a set of justifications. Thus, single-peakedness for profiles of judgments sets $\bar{\mathbf{J}} = (\bar{J}_1, \ldots, \bar{J}_n)$ is defined by saying that there exists a dimension $>_\Omega$ such that the models of the judgments sets are compatible with $>_\Omega$. We say that a judgments profile *justifies* a preference profile $\mathbf{P}$ iff the each $\mathcal{I}(\bar{J}_i)$ justifies $>_i$. A *semantic dimension* is a set of judgments that justify a preference ordering.

At least in principle, agents can agree on the agenda of justifications, without revising their preferences. It is enough to assume a language that contains exactly two relations $\bar{D}$ and $\bar{D}'$ that denote opposite orders, $\mathcal{L}_{\{\bar{D},\bar{D}'\}}$. Thus, any justification can be expressed by $\bar{D}$ or $\bar{D}'$. We say that $\bar{J}_1$ is *consistent* with $\bar{J}_2$ if for every $x\bar{D}_1y$ in $\bar{J}_1$, there exists a $x\bar{D}_2y \in \bar{J}_2$ and *vice versa*. Accordingly, a profile $\bar{\mathbf{J}}$ is consistent with $\bar{\mathbf{J}}$' if every $\bar{J}_i$ is consistent with $\bar{J}'_i$.

PROPOSITION 1. *For every single-peaked profile $\bar{\mathbf{J}}$, there exists a single-peaked profile $\bar{\mathbf{J}}'$ that is consistent with $\bar{\mathbf{J}}$ and that is defined in $\mathcal{L}_{\{\bar{D},\bar{D}'\}}$, where $\bar{D}$ denotes $>_\Omega$ and $\bar{D}'$ denotes $>'_\Omega$.*

We show that majority on judgments is inconsistent, provided we endow agents with minimal reasoning capabilities. Assume agents express judgments in $\mathcal{L}^{\neg,\wedge}_{\{\bar{D},\bar{D}'\}}$, i.e. $\mathcal{L}_{\{\bar{D},\bar{D}'\}}$ closed under negations and conjunctions. Assume that $\bar{J}_i$ satisfies the usual rationality requirements [5]. Let $\mathbf{P}$ be a preference profiles that is single-peaked wrt $>_\Omega$ and such that there are $n/2$ agents with $>_i = >_\Omega$, $n/2$ agents with $>_i = >'_\Omega$, and one agent j whose top differs from the top of $>'_\Omega$ and $>_\Omega$. By construction, the winning alternative, say x, is the peak of j. Thus, there is a y such that there is a majority for $x\bar{D}y$, as $\bar{D}$ denotes $>_\Omega$, and there is a z such that there is a majority for $x\bar{D}'z$, as $\bar{D}'$ denotes $>'_\Omega$. However, the conjunction $xDy \wedge xD'z$ is not supported by majority. Thus, the voting pattern on the judgments profile returns a discursive dilemma, as in the previous example.

PROPOSITION 2. *For every profile of judgments $\bar{\mathbf{J}}$ that is defined in $\mathcal{L}^{\neg,\wedge}_{\{\bar{D},\bar{D}'\}}$ and that justifies $\mathbf{P}$, $M(\bar{\mathbf{J}})$ is inconsistent.*

5. CONCLUSION

Although single-peakedness provides a solution in preference aggregation, it is problematic in case of collective reasoning about justifications. We believe that this points shed some doubts on the compatibility of a deliberative model based on public justifications with aggregative models based on fair aggregation procedures, at least in case we understand agreement on collective justifications as voting.

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