Schulze and Ranked-Pairs Voting Are Fixed-Parameter Tractable to Bribe, Manipulate, and Control*

(Extended Abstract)

Lane A. Hemaspaandra
Dept. of Computer Science
University of Rochester
Rochester, NY 14627
lane@cs.rochester.edu

Rahman Lavaee
Dept. of Computer Science
University of Rochester
Rochester, NY 14627
rlavaee@cs.rochester.edu

Curtis Menton
Dept. of Computer Science
University of Rochester
Rochester, NY 14627
menton@cs.rochester.edu

ABSTRACT

Schulze and ranked-pairs elections have received attention recently, with the former having quickly become a widely used election system. For many cases these systems have been proven resistant to bribery, control, and manipulation, with ranked pairs being particularly praised for being NP-hard for all three of those. Nonetheless, this work shows that with respect to the number of candidates, both Schulze and ranked-pairs elections are fixed-parameter tractable to bribe, control, and manipulate: we can obtain uniform, polynomial-time algorithms whose degree does not depend on the number of candidates.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Algorithms, Theory

Keywords

computational social choice, elections, fixed-parameter tractability

1. INTRODUCTION

Schulze voting [4], though relatively recently proposed, has quickly been rather widely adopted. Designed in part to well-handle candidate cloning, its users include the Wikimedia foundation, the Pirate Party in a dozen countries, Debian, KDE, the Free Software Foundation Europe, and dozens of other organizations.

Although the winner-choosing process in Schulze voting is a bit complicated to describe, involving minima and maxima and comparisons of paths in the so-called weighted majority graph (WMG), finding who won a Schulze election nonetheless is polynomial-time computable. However, Parkes and Xia [3], followed by Menton and Singh [2], showed that for Schulze elections bribery is NP-hard, 15 of the 22 benchmark control attacks are NP-hard, and the complexity of manipulation is an open question (except it is in P if there is at most one manipulator).

Parkes and Xia also note that, by the work of [3, 6, 5], the ranked-pairs election system, which is not widely popular but like Schulze has a polynomial-time winner-determination problem and like Schulze is based on the weighted majority graph, is resistant to (basically, NP-hard with respect to) bribery, control (of each of the control types they study in their paper), and manipulation. Based on that discovery of ranked pairs being more broadly resistant to attacks than Schulze, and the fact that Schulze itself "is in wide use," and the fact that there is "broad axiomatic support for both Schulze and ranked pairs," Parkes and Xia quite reasonably conclude that "there seems to be good support to adopt ranked pairs in practical applications."

However, in this paper we show that the resistances-to-attack of Schulze and ranked pairs are both quite fragile. For each of the bribery/control/manipulation cases studied by Parkes and Xia, and Menton and Singh, for which they did not already prove Schulze voting to be in P (that is, they either proved the case NP-hard or left it as an open research issue), we can establish that Schulze voting is fixed-parameter tractable (with respect to the number of candidates). Fixed-parameter tractable (FPT) means there is an algorithm for the problem whose running time is $f(j)I^{O(1)}$, where j is the number of candidates and I is the input's size. This of course implies that for each fixed number of candidates, the problems are in polynomial time, but it says much more; it implies that there is a global bound on the degree of the polynomial running time, regardless of what the fixed number of candidates is.

That result might lead one to even more strongly suggest the adoption of ranked pairs as an attractive alternative to Schulze. However, although for ranked pairs Parkes and Xia proved all the types of bribery, control, and manipulation they studied to be NP-hard, we can establish that every one of those cases is fixed-parameter tractable (with respect to the number of candidates) for ranked pairs. So even ranked pairs does not offer a safe haven from fixed-parameter tractability.

Due to the space limitations in this extended abstract, in the presentation below of our key idea we assume that the reader is familiar with the notion of a WMG and the def-

*Supported in part by grants NSF-CCF-{0915792,1101479}.

Appears in: *Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2013)*, Ito, Jonker, Gini, and Shehory (eds.), May, 6–10, 2013, Saint Paul, Minnesota, USA.

inition of Schulze elections. For a detailed presentation of those, of our winner certification structure for ranked pairs, of our entire detailed presentation of the important issue of related work, of our proofs applying the certification structures to prove FPT results for bribery, manipulation, and control, and of additional results, we urge the reader to see the technical report version [1], which is over 30 pages long.

2. PRESENTATION OF THE KEY IDEA

The central technical contribution of this work is to show how a certain small needle can be threaded for Schulze and ranked-pair elections. In particular, we need to, for each of those election systems, find a (winner set certification) structure that on one hand is rich enough that for each structure instance we can within an integer linear programming feasibility problem check whether the given manipulative action can lead to success in a way consistent with the case of which the particular instance of the structure is speaking. Yet on the other hand, the structure must be so restrictive that the number of such structures is bounded purely as a function of the number of candidates (independent of the number of voters). In brief, we need to find, if one exists, a "sweet spot" that meets both these competing needs.

We achieve this with structures we call Schulze winner-set certification frameworks (SWCFs) and ranked pairs winner-set certification frameworks (RPWCFs). A Schulze winner-set certification framework contains a "pattern" for how we can prove that a given set of candidates is the winner set of a Schulze election. To do that, the structure for each winner a specifies, for each other candidate b, a "strong path" γ_{ab} from a to b in the WMG (victory in Schulze elections is based on having strong paths), and then—to establish that the other candidate b has no stronger path back to a—for every simple path from b back to our candidate a the structure identifies a "weak link" (a directed edge on that path) that will keep the path from being too strong; to be more specific, we mean an edge on that path in the WMG such that its weight is less than or equal to that of every edge in our allegedly quite strong path γ_{ab}. (Now, keep in mind, at the time we are looping through the structure, we will not even know how strong each link is, as the manipulation/bribery/control will not yet even have happened; rather, the structure is specifying a particular pattern of victory, and the integer linear programming feasibility problem will have to check whether the given type/amount of manipulation/bribery/control can bring to life that victory pattern.) And the structure for each candidate a it claims is not a winner will specify what rival b eliminates that candidate from the winner set and then outlines a pattern for a proof that that is the case, in particular giving a "strong path" from b to a and for each simple path from a to b our structure specifies a "weak link," i.e., an edge on that path from a to b whose weight in the WMG we hope will be *strictly* less than the weight of all edges in the selected strong path from b to a; if all our hopes of this sort turn out to be true (and that is what the integer linear program will be testing, for each of our certification framework's structures), this proves that b eliminates a. We stress that the certification framework does not itself have its hands on the weights of the WMG, and so the paths and edges it specifies are all given in terms of the self-loop-free graph, on nodes named $1, 2, \ldots, \|C\|$, that between each pair of distinct nodes has edges in both directions. (Since the candidate names are irrelevant in Schulze voting, we can change to those canonical names, so that our Schulze structures are always in terms of those names.)

Crucially, the number of structures (in that Schulze winner-set certification framework), though large, is bounded as a function of the number of candidates. Yet, also crucially, this approach provides enough structure to allow a polynomial-sized integer linear programming feasibility problem to do the "rest" of the work, namely, to see whether by a given type of attack we can bring to life the proof framework that a given instance of the structure sets out, as to who the winners/nonwinners are in the Schulze election and why.

For ranked pairs, the entire approach is just as described above, except the certification framework we use is completely different than that used for Schulze. Ranked pairs is a method that is defined in highly sequential terms, through successive rounds some of which add a relationship between two candidates, and so our certification framework will be making extensive guesses about what happens in each round (and about a number of other things). But again, we will ensure that the number of such certification structures is bounded as a function of the number of candidates (independent of the number of voters), yet each structure will give enough information that the rest of the work can be done by an integer linear programming feasibility problem.

We mention again that for a detailed presentation of how to translate the above key idea into detailed fixed-parameter tractability results for bribery, control, and manipulation, using the integer programming framework of Lenstra, and for all the definitions, results, explanations, discussion of related work, and open questions that space does not allow here, we urge the reader to see the technical report version [1].

3. REFERENCES

[1] L. Hemaspaandra, R. Lavaee, and C. Menton. Schulze and ranked-pairs voting are fixed-parameter tractable to bribe, manipulate, and control. Technical Report arXiv:1210.6963 [cs.GT], arXiv.org/corr/, Dec. 2012.

[2] C. Menton and P. Singh. Manipulation and control complexity of Schulze voting. Technical Report arXiv:1206.2111v3 [cs.GT], arXiv.org, Sept. 2012.

[3] D. Parkes and L. Xia. A complexity-of-strategic-behavior comparison between Schulze's rule and ranked pairs. In *Proceedings of the 26th AAAI Conference on Artificial Intelligence*, pages 1429–1435. AAAI Press, Aug. 2012.

[4] M. Schulze. A new monotonic, clone-independent, reversal symmetric, and Condorcet-consistent single-winner election method. *Social Choice and Welfare*, 36:267–303, 2011.

[5] L. Xia. Computing the margin of victory for various voting rules. In *Proceedings of the 13th ACM Conference on Electronic Commerce*, pages 982–999, June 2012.

[6] L. Xia, M. Zuckerman, A. Procaccia, V. Conitzer, and J. Rosenschein. Complexity of unweighted manipulation under some common voting rules. In *Proceedings of the 21st International Joint Conference on Artificial Intelligence*, pages 348–353, July 2009.