Voter Dissatisfaction in Committee Elections
(Extended Abstract)

Dorothea Baumeister*
Institut für Informatik
Heinrich-Heine-Universität Düsseldorf
40225 Düsseldorf, Germany
baumeister@cs.uni-duesseldorf.de

Sophie Dennisen
Institut für Informatik
Technische Universität Clausthal
38678 Clausthal-Zellerfeld, Germany
sophie.dennisen@tu-clausthal.de

ABSTRACT
The minisum and the minimax rules are two different rules for the
election of a committee considered by Brams et al. [2]. As input
they assume approval ballots from the voters. The first rule elects
those committees which minimize the sum of the Hamming dis-
tances to the votes, the second one elects those committees with
the smallest maximum Hamming distance to an individual vote. We
extend this approach of measuring the dissatisfaction in commit-
tee elections to different forms of ballots, i.e., trichotomous votes,
complete and incomplete linear orders. To measure the dissatisfac-
tion we use a modified Hamming distance, ranksums, and a modi-
fied Kemeny distance.

Categories and Subject Descriptors
I.2.11 [Artificial Intelligence]: Distributed Artificial Intelligence—
Multiagent Systems;
J.4 [Computer Applications]: Social and Behavioral Sciences—
Economics

General Terms
Economics, Theory

Keywords
committee elections, social choice theory, computational social
choice, winner determination

where a certain number of alternatives or objects must be chosen
from a given set. Hence, committee elections may also be applied
in artificial intelligence and multi agent systems, for example in the
design of recommender systems, where the task is to choose a fixed
number of products to recommend to a user (see [11]). The algo-

rithmic aspects of several committee voting rules have been studied
in [10, 1, 4], where the focus is mostly on approval-based rules.

A widely used rule for committee elections for approval votes is
the minisum rule, where a winning committee minimizes the sum
of the Hamming distances to the individual votes. This corresponds
the minimax rule, which tries to minimize the voter dissatisfaction
measured by the Hamming distance. Here, a committee with the
smallest maximum Hamming distance to an individual vote will
be elected, which corresponds to an egalitarian approach. We ex-
tend these approaches to trichotomous votes, complete and incom-
plete linear orders. Closely related to the minimization of voter
dissatisfaction in committee elections are systems of proportional
representation. The main difference is that in a proportional repre-
sentation scheme each voter is represented by a candidate (i.e., the
political party she voted for), and the dissatisfaction is computed
for this candidate and not for the committee as a whole.

2. DEFINITIONS AND NOTATIONS
An election is a pair \((C, V)\), where \(C = \{c_1, \ldots, c_m\}\) is the set of
candidates and \(V\) is a list of voters represented by their vote. We
study four different forms of votes: approval votes represented as
\(\{0, 1\}^m\) vectors, where a 1 stands for approval and a 0 for disap-
pearance of the candidate; trichotomous votes represented as
\([-1, 0, 1\]^m\) vectors, where a 1 stands for approval, a -1 for disappro-
approval of the candidate; complete linear orders represented as a total,
transitive, and asymmetric binary relation over the set of candidates;
icomplete linear orders represented as a transitive and asymmet-
tric (not necessarily total) binary relation over the set of candidates.

We study elections where the winner is a committee of \(k \leq m\)
candidates. Note that the size of the committee is fixed in advance.
Since the winning set may consist of several committees, one has
to use a tie-breaking rule to obtain a single winning committee, if
necessary. In the case of approval votes, a committee may be repre-
sented as a \([1, 0]^m\) vector having \(k\) ones, where the approved can-
didates are members of the committee. Analogously, committees
for trichotomous votes may be denoted by \([1, -1]^m\) vectors hav-
ing \(k\) ones. For complete and incomplete linear orders, we denote
committees as a set \(K \subseteq C\) of \(k\) candidates.

We study how the dissatisfaction of a single voter with a com-
mittee may be measured, for each of the different types of votes intro-
duced above. In a single winner approval election, the candidates

\*This work was supported in part by a grant for gender-sensitive
universities and the project “Online Participation”, both funded by
the NRW Ministry for Innovation, Science, and Research.

Appears in: Proceedings of the 14th International
Conference on Autonomous Agents and Multiagent
Systems (AAMAS 2015), Bordini, Elkind, Weiss, Yolum
(eds.), May 4–8, 2015, Istanbul, Turkey.
Copyright © 2015, International Foundation for Autonomous Agents
and Multiagent Systems (www.ifaamas.org). All rights reserved.
with the highest number of approvals are the winners. The most obvious way of measuring the dissatisfaction in case of committee elections for approval votes is the Hamming distance, as it is used by Brams et al. [2]. The disagreement between \( v, w \in \{1,0\}^m \) is then formally defined by: \( HD(v,w) = \sum_{1 \leq i \leq n} |v(i) - w(i)| \). Single winners for trichotomous votes may be elected by Combined Approval voting, see [6]. The combined approval score of a candidate \( c_i \) is the sum of the \( i \)-th entries of all trichotomous votes, and the candidates with the highest score wins. For committee elections, the Hamming distance is adapted to trichotomous votes such that a complete disagreement adds two points, and an abstention in one vote and an approval/disapproval in the other one adds one point. Formally, it is defined analogously to the Hamming distance, but to avoid confusion we denote it by \( \delta \).

Since there is no direct way to extend the Hamming distance to linear orders, we use other metrics in this case. For complete linear orders, we follow the approach of the Wilcoxon rank-sum test [12]. We take the sum of the ranks of the committee members in a vote to measure the dissatisfaction. Let \( p(c,v) \) denote the position of candidate \( c \) in vote \( v \), where the most liked candidate is on the first position. The normalized ranksum for voter \( v \) and committee \( K \) is

\[
RS(K,v) = \frac{\sum_{c \in K} p(c,v) - \frac{t(v) + 1}{2}}{\frac{t(v)}{2}}.
\]

The last type of votes we consider are incomplete linear orders. Here, we use a modified version of the Kemeny distance [7]. The usual Kemeny distance is defined to compare two linear orders that may contain indifferences. The disagreement between two votes \( v \) and \( w \) is defined as \( \sum_{a,b \in C} d_{v,w}(a,b) \), where the distance \( d_{v,w} \) between the votes for an unordered pair of candidates \( a,b \) is 1, if in one vote \( a \) and \( b \) are considered equal and in the other vote one of them is preferred. The distance is 2, if in one vote it holds \( a > b \) and in the other one \( b > a \), in all other cases the distance is 0. The winners in a Kemeny election are those which are on a first position in a linear order for which the sum of the distances to the votes is minimum. Dwork et al. [5] consider a variant of Kemeny with incomplete linear orders, but they only define the case where a vote is a linear order over a subset of the candidates, whereas we consider arbitrary incomplete linear orders. We adapt the Kemeny distance in a slightly different way to measure the disagreement between incomplete linear orders and a committee. For two candidates \( a \) and \( b \) the distance \( d_{v,K}(a,b) \) between a vote and a committee is 2, if only one of them is in the committee but in the vote the other one is preferred. The distance is 1, if only one of them is in the committee and in the vote the relation between both is unknown, in all other cases the distance is 0. This can be seen as a slightly more pessimistic variant, since in our model the distance also increases if the relation between both candidates is unknown in the vote. The disagreement between a vote \( v \) and a committee \( K \) is then

\[
Dist(K,v) = \sum_{a,b \in C} d_{K,v}(a,b).
\]

We follow the approach of Brams et al. [2] by defining minsum and minimax rules to elect a committee based on the different measures proposed above. They defined the winning committees in minsum-approval to be those which minimize the sum of the Hamming distances to all votes. Whereas in minimax-approval the committees with the smallest maximum Hamming distance to the votes are chosen as winning committees. Brams et al. [8] propose two possibilities for varying the minsum and minimax rules, namely the use of count weights and proximity weights. When determining the winning committee using count weights, the distance will always be multiplied by the number of times the ballot has been cast. Note that the use of count weights has only an effect for the minmax approach, since in the minsum approach in both definitions the sum of the distances is taken over all ballots. Let \( F_2(C) \) denote all committees of size \( k \). Now, we combine the minsum and minimax rules with the above defined measures of disagreement to define committee election systems.

Formally, the set of winning committees in the minsum rule are

\[
\arg\min_{K \subseteq F_2(C)} \sum_{v \in V} \Delta(K,v),
\]

and in the minmax rule

\[
\arg\min_{K \subseteq F_2(C)} \max_{v \in V} \Delta(K,v),
\]

where \( \Delta \in \{HD,BS,RS,Dist\} \) for the corresponding type of vote. The resulting voting systems are minsum/minimax-CAV for trichotomous votes, minsum/minimax-ranksum for complete linear orders, and minsum/minimax-Kemeny for incomplete linear orders.

**Theorem 1.** The set of winning committees in a minisum/minimax-ranksum election and in a minisum/minimax-Kemeny election with complete linear orders are always equal.

This is true, since \( Dist(K,v) = 2 \cdot RS(K,v) \), for any committee \( K \in F_2(C) \) and any complete linear order \( v \) over \( C \). It is known that winner determination for minsum-approval is in \( P \) (see [2]) and for minmax-approval it is NP-hard (see [9]). It is trivial that winner determination for minsum-CAV is also in \( P \), the complexity of all other winner determination problems is open.

**REFERENCES**


