

# Selecting Robust Strategies Based on Abstracted Game Models

## (Extended Abstract)

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### ABSTRACT

Game theory is a tool for modeling multi-agent decision problems and has been used successfully in problems such as poker, security, and trading agents. However, many real games are extremely large[4]. One approach for solving large games is to use abstraction techniques to shrink the game to a form that can be solved by removing detail and translating a solution back to the original. However, abstraction introduces error into the model. We study ways to analyze games that are robust to errors in the model of the game, including abstracted games. We empirically evaluate several solution methods to evaluate how robust they are for abstracted games.

### Categories and Subject Descriptors

Theory of Computation [Theory and algorithms for application domains]: Algorithmic game theory and mechanism design

### General Terms

Theory, Experimentation

### Keywords

Game Theory, Abstraction, Empirical Game Modeling

## 1. INTRODUCTION

Games that model real world interactions are often complex, with huge numbers of possible strategies and information states. We are interested in better understanding the effect of abstraction in game-theoretic analysis. In particular, we focus on the *strategy selection problem*: how should an agent choose a strategy to play in a game, based on an abstracted game model? This problem has three interacting components: (1) the method for abstracting the game, (2) the method for selecting a strategy based on the abstraction, and (3) the method for mapping this strategy back to the original game. This approach has been studied extensively for poker, which is a 2-player, zero-sum game. However, much less is known about how abstraction interacts with strategy selection in more general games.

The main contributions of our work are as follows. First, we specify a model of the strategy selection problem when players use asymmetric abstractions as a *meta-game*. In this model players can use different methods for abstracting the game, solving the game,

and reverse-mapping the solution. We introduce a collection of specific methods for abstracting and solving games; these are intended to be representative of the most popular methods used in the literature. Finally, we present the results of extensive simulation that evaluate the candidate abstraction and solution methods on different classes of games. Our results lead to several unique observations as well as identifying solution methods that are more robust than others to error introduced by abstraction.

**Appears in:** *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2015)*, Bordini, Elkind, Weiss, Yolum (eds.), May 4–8, 2015, Istanbul, Turkey.  
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## 2. ABSTRACTION META-GAMES

We first introduce a formal model that can be used to study the situation where players select strategies based on abstracted game models. Our model is based on the meta-game framework introduced by Kiekintveld et al. [2], which focused on situations where players received noisy observations of the same underlying game and had to select strategies based on these observations. The situation where players use abstractions is similar in that the players make strategy choices based on imperfect abstractions of the game. Opposing players may also use different abstractions which may cause problems for solution concepts that rely on coordination.

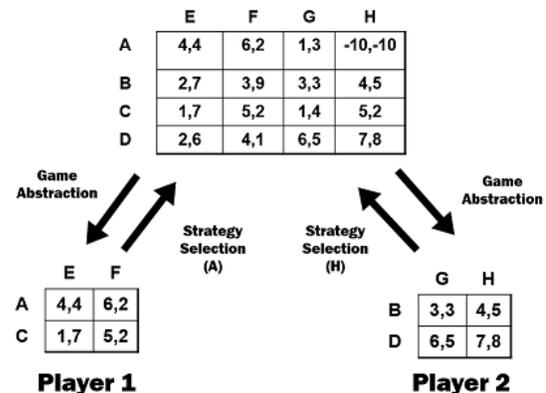


Figure 1: 2-players asymmetric abstractions

An example of an *abstraction meta-game* is shown in Figure 1. In this example, we have two players who are playing the one-shot normal form game shown at the top of the figure; this is the *base game*. Each player has four possible actions in the original game, and the payoffs are listed in the the matrix. Each player uses a different (unspecified) abstraction method to reduce the size of the game to only two actions for each player, as shown. Now the players analyze these smaller games to select a strategy to play. Here, both of the small games can be solved using dominance to

find a unique Nash equilibrium. The players select these strategies (A and H) to play. However, when these strategies are played in the base game they result in the outcome  $-10, -10$ , which is the worst possible payoff for both players!

### 3. ABSTRACTION

We define an abstraction method as a function that maps one normal-form game into a second (smaller) normal-form game. We identify two broad categories of abstractions that are common in the literature: *strategy elimination* and *strategy combination*. Strategy elimination abstractions remove some strategies completely to reduce the size of the game. Strategy combination simplify games by merging multiple strategies into a single representative strategy. Our goal in this paper is not to develop novel abstraction methods nor to exhaustively evaluate the many existing techniques.

The first abstraction method we consider is *TopN*, which is representative of strategy elimination. It creates a smaller game by selecting a subset of size  $N$  of the strategies for each player to form the abstracted game. For each strategy in the game, we calculate the expected payoff of the strategy against a uniform random opponent strategy. We then select the  $N$  strategies with the highest expected payoffs, breaking ties randomly. The abstracted game is the game where players are restricted to playing only the  $N$  selected strategies; the payoffs are unchanged. Since each strategy in the abstracted games is also a strategy in the original game the reverse mapping of the strategies back to the original game is trivial.

The second abstraction method we use is *KMeans*, which is representative of strategy combination. This method uses  $k$ -means clustering to group strategies into clusters based on the similarity of their payoffs. Strategies are re-assigned to the closest cluster based on the Euclidian distance between payoffs, ensuring that no cluster becomes empty. The payoffs for each outcome in the abstracted game are computed by averaging the payoffs for all of the outcomes in the cluster. The reverse mapping also assumes that players play strategies in the same cluster with uniform probability.

### 4. CANDIDATE SOLUTION METHODS

We consider several candidate solution methods for selecting strategies in abstracted games. All are based on known solution concepts or simple heuristics for playing games, and they are intended to provide a diverse pool of plausible strategies to evaluate.

**Uniform Random (UR):** Play each action with equal probability.

**Best Response to Uniform (BRU):** Pure-strategy best-response.

**Nash Equilibrium (MSNE):** We use the Gambit logit solver to calculate a sample Nash equilibrium.

**Epsilon-Nash Equilibrium (ENE):** For every pure-strategy profile, we first calculate the maximum that value ( $\epsilon$ ) that any player can gain by deviating to a different pure strategy and play the strategy profile with the smallest value of  $\epsilon$ .

**MaxMin:** Play the strategy that maximizes the worst-case payoff.

**Fair:** For every strategy profile we find the difference between the payoffs and select an outcome that minimizes this difference. Ties are broken in favor of outcomes with a higher sum of payoffs, and then randomly.

**Social:** Plays according to the outcome that maximizes the sums of the payoffs for all players. If there are ties the strategy plays a uniform random strategy over the strategies in the tied outcomes.

**Quantal Response Equilibria (QRE):** QRE [3] originated in behavioral game theory and uses a model of *noisy best-response* where players use a softmax decision rule instead of strict best-response.

**Cognitive Hierarchies (CH):** Cognitive Hierarchies [1] also originates in behavioral game theory. It models a recursive style of rea-

soning where level-0 agents play a uniform random strategy, level-1 agents play a best response to the level-0 agents, level-2 agents play a best response to a mixture over level 0 and 1 agents, etc.

**Quantal Level-k (QLK):** Combines the features of QRE and CH.

### 5. CONCLUSION

We conducted several experiments to identify the best parameter settings for QRE, CH, and QLK agents and then ran several round robin tournaments of over 300 games, and 18 unique agents, and 4 different levels of abstractions (including no abstraction) on 3 different classes of games.

Our results demonstrate that using abstraction to solve games is a complex endeavor, and the type of abstraction, the solution methods used to analyze the abstracted games, and the class of games all have a strong influence on the results. Many of the strongest results using abstraction to analyze large games (e.g., Poker) have focused on zero-sum games. One of the most interesting observations from our results is that abstraction often works very differently in zero-sum games than it does general-sum games or the more structured logical games. In particular, solution methods based on finding Nash equilibrium seem to work much better in zero-sum games than they do in the other classes of games in our experiments. Another important observation from our experiments is that Nash equilibrium often does not perform well in cases where abstraction is used as part of the solution process. It is still effective when the games are zero-sum, but in the other cases it was not robust to the introduction of error based on game abstraction.

We also found that the specific method used for generating abstractions has a strong impact on the results. One very interesting result was that TopN was sometimes able to *increase* the payoffs for the agents in comparison to the case without any abstraction. However, TopN is a symmetric abstraction when both players use it, while KMeans is asymmetric. Some methods like the social agent performed much better when using the symmetric TopN abstraction than when using the asymmetric KMeans abstraction. This kind of interaction is very important to understand in greater depth if we are to make effective use of abstraction as part of game-theoretic analysis. Our model of abstraction meta-games provides a formal model for studying this type of interaction, and our simulations have resulted in several interesting observations that provoke many additional questions about the use of abstraction in solving games.

### 6. ACKNOWLEDGMENTS

This material is based upon work supported by the National Science Foundation under Grant No. IIS-1253950.

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