Improving Fairness in Nonwasteful Matching with Hierarchical Regional Minimum Quotas

(Extended Abstract)

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ABSTRACT

This paper considers matching problems with hierarchical regional minimum quotas. Although such quotas are relevant in many real-world settings, it is known that nonwastefulness and fairness (which compose stability) are incompatible when minimum quotas are imposed. We develop a new strategy-proof nonwasteful mechanism called Adaptive Deferred Acceptance with Regional minimum Quotas (ADA-RQ), which is fairer than an existing nonwasteful mechanism called Multi-Stage DA with Regional minimum Quotas (MSDA-RQ).

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems*; J.4 [**Social and Behavioral Sciences**]: Economics

Keywords

Two-sided matching, Deferred acceptance algorithm, Minimum quotas

1. INTRODUCTION

The theory of matching has been extensively developed for markets in which the agents (students/schools, hospitals/residents, workers/firms) have individual *maximum* quotas, i.e., the number of students assigned to a school cannot exceed a certain limit [4]. In many real-world markets, however, *minimum* quotas may also be relevant [1]. For example, school districts may need at least a certain number of students in each school in order for the school to operate. Furthermore, these minimum quotas can be imposed on a set of schools (region) rather than on individual schools. For example, when allocating students to labs, it is common that labs are classified into several sub-departments (courses). Achieving a good balance of the total number of allocated students for each course can be important. In this paper, we concentrate on the case where regions have a hierarchical structure and minimum quotas are imposed on these regions. Goto et al. [3] show that handling a more general case is hard, since checking whether a feasible matching exists or not is NP-complete.

In the standard model (with only maximum quotas), the Deferred Acceptance mechanism (DA) [2] is well known as a stable (i.e., fair and nonwasteful) and strategy-proof mechanism. When minimum quotas are imposed, fairness and nonwastefulness are incompatible [1]. Our work is based on [3], which develop a fair (but wasteful) mechanism and nonwasteful (but not fair) mechanisms with regional minimum quotas. Their results show that achieving complete fairness is costly; the welfare of students of a fair mechanism tends to be very low compared to non-fair mechanisms. Thus, in this paper, we set our goal to develop a nonwasteful mechanism, which is *fairer* than a nonwasteful mechanism called Multi-Stage Deferred Acceptance mechanism for Regional minimum Quotas (MSDA-RQ) [3].

2. MODEL

A market is a tuple $(S, C, R, (p_r)_{r\in R}, (q_c)_{c\in C}, (\succ_s)_{s\in S}, (\succ_c)_{c\in C}, \succ_{ML})$. $S = \{s_1, s_2, \ldots, s_n\}$ is a set of students, $C = \{c_1, c_2, \ldots, c_m\}$ is a set of schools, and $R = \{r_1, r_2, \ldots\}$ is a set of regions. Each student s has a strict preference relation $\succ_s$ over the schools, while each school c has idiosyncratic strict priority relation $\succ_c$ over the students. In addition to the idiosyncratic school priorities, our mechanism assumes the existence of a separate *master list* (ML). ML may correspond to GPA or TOEFL scores, which induce a common ranking across all students. Without loss of generality, we assume $s_1 \succ_{ML} s_2 \succ_{ML} \cdots \succ_{ML} s_n$.

A region $r \in R$ is a subset of schools, i.e., $r \in 2^C \setminus \{\emptyset\}$. We assume the set of regions R is hierarchical and forms a tree as follows: (i) root node C is the region that contains all schools, (ii) leaf node $\{c\}$ is a region that contains only one individual school $c \in C$, and (iii) for each node $r \in R$, where $r \neq C$, its parent node $r' \in R$ is a region that is the proper inclusion-minimal superset of r.

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$(q_c)_{c \in C}$ is an individual maximum quota vectors. $(p_r)_{r \in R}$ are regional minimum quota vectors. We assume $0 \leq p_r \leq \sum_{c \in r} q_c$ holds for all $r \in R$. We assume that all schools are acceptable to all students and vice versa.

Goto et al. [3] show that if $p_r \leq \sum_{c \in r} q_c$ for each region r, as well as some minor conditions on minimum/maximum quotas hold, a feasible matching always exists. In this paper, we assume these conditions hold.

A matching is mapping $\mu : S \cup C \to 2^{S \cup C}$ that satisfies the following conditions: (i) $\mu(s) \in C$ for all $s \in S$, (ii) $\mu(c) \subseteq S$ for all $c \in C$, and (iii) for any s and c, we have $\mu(s) = c$ if and only if $s \in \mu(c)$. A matching μ is *feasible* if $\forall r, p_r \leq |\mu(r)| \leq \sum_{c \in r} q_c$ holds, where $\mu(r) = \bigcup_{c \in r} \mu(c)$. Also, we say a matching μ is *semi-feasible* if it is a subset of any feasible matching.

Given matching μ, student s has justified envy toward s', if s' is matched to school c, which is more preferable than her current match $\mu(s)$, although she has a higher priority ranking than s' at c. We say matching μ is *fair* if no student has justified envy.

Given matching μ, student s claims an empty seat of c, which is more preferable than her current match c', if the matching obtained by moving her from c' to c is feasible. We say matching μ is *nonwasteful* if no student claims an empty seat.

We say a mechanism is nonwasteful if it produces a nonwasteful matching for every possible profile of the preferences and priorities. Similarly, a mechanism is fair if it produces a fair matching for every possible profile of the preferences and priorities. We say a mechanism is strategy-proof if no student ever has any incentive to misreport her preference, no matter what the other students report.

3. ADAPTIVE DEFERRED ACCEPTANCE MECHANISM WITH REGIONAL MINIMUM QUOTAS (ADA-RQ)

The MSDA-RQ determines the number of students who are allocated in each stage using a pessimistic, static estimation. In our new mechanism, which we call Adaptive Deferred Acceptance mechanism with Regional minimum Quotas (ADA-RQ), instead of relying on a pessimistic, static estimation, we adaptively determine the number based on actual allocations that are obtained by using students' preferences. Although this idea is appealing to increase the number of allocated students, it seems incompatible with strategy-proofness. Since the number of allocated students depends on students' preferences, a student might have an incentive to manipulate this number. In particular, reducing this number can be beneficial since by doing so, the student can reduce the number of her rivals. Surprisingly, the ADA-RQ is guaranteed to be strategy-proof, and it is shown to be *fairer* than the MSDA-RQ.

We say school c is *forbidden* if we allocate one more student to c, the current matching does not satisfy semi-feasibility, even though its maximum quota is strictly positive. If the current matching is not semi-feasible (although all individual maximum quotas are satisfied), then the minimum quota of a certain region cannot be satisfied by any matching that is obtained by extending the current matching. We say c is *active* if c is not forbidden.

Each **Stage** k can contain multiple rounds. In **Round** t, we run the standard DA for t students, who have the highest priority ranking according to $\succ_{ML}$, and obtain matching μ^k. Note that this matching μ^k is tentative and will be recalculated in the next round. If the set of forbidden schools does not change, the mechanism proceeds to the next round (for $k+1$ students) without finalizing μ^k. In this case, we can safely allocate one more student to any active school without violating minimum quotas. On the other hand, when the set of forbidden schools expands according to q^{k+1} and $\mu^1, \ldots, \mu^k$, if we continue to add one more student, there is a chance that a student is allocated to a forbidden school and some minimum quota will be violated. Thus, in this case, the mechanism finalizes μ^k and proceeds to the next stage.

The ADA-RQ is described in Mechanism 1.

Mechanism 1 Adaptive Deferred Acceptance mechanism with Regional minimum Quotas (ADA-RQ)

Let $L := (s_1, \ldots, s_n)$, $q^1 := q$. Also, let $\hat{C}^1 = \emptyset$. Proceed to **Stage** 1.

Stage k**:** Proceed to **Round** 1.

Round t**:** Select top t students from L. Let μ^k be the matching that is obtained by the standard DA for the selected students under q^k and active schools according to $\hat{C}^k$. Let q_c^{k+1} be $q_c^k - |\mu^k(c)|$ for all $c \in C$, and $\hat{C}^{k+1}$ be the forbidden schools based on q^{k+1} and $\mu^1, \ldots, \mu^k$.

- **(i)** If all students in L are already selected, then finalize μ^k and terminate the mechanism.
- **(ii)** If $\hat{C}^{k+1} = \hat{C}^k$, then proceed to **Round** $t+1$.
- **(iii)** Otherwise, finalize μ^k. Remove top t students from L, and proceed to **Stage** $k+1$.

4. CONCLUSIONS

This paper dealt with the matching problem with hierarchical regional minimum quotas and developed a new strategy-proof, nonwasteful mechanism (ADA-RQ), which is fairer than the MSDA-RQ. In the future, we would like to check whether a similar idea to the ADA-RQ can be applied to handle different types of distributional constraints.

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