Bribery in k-Approval and k-Veto Under Partial Information (Extended Abstract)

Dirk Briskorn
Schumpeter School of Business and Economics
Bergische Universität Wuppertal
Wuppertal, Germany
briskorn@uni-wuppertal.de

Gábor Erdélyi and Christian Reger
School of Economic Disciplines
University of Siegen
Siegen, Germany
{erdelyi, reger}@wiwi.uni-siegen.de

ABSTRACT

In bribery an external agent tries to alter the outcome of an election by changing some voters' votes. Usually, when investigating bribery problems, full information is assumed, i.e., that the manipulative agent knows the set of candidates, each voter's votes and the voting rule used. In this paper, we formally introduce different structures of partial information, we show the connections between them and existing notions, define bribery under partial profiles, and examine the complexity of bribery under partial information for the k-Approval and k-Veto rules.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Algorithms, Theory, Economics

Keywords

Computational Social Choice, Voting Rules, Bribery

1. INTRODUCTION

Voting provides a useful method for collective decision-making and preference aggregation, and as such has applications in politics, economics, and computer science. In many applications in computer science we are dealing with huge data volumes. Thus, it is worth studying the computational aspects of problems related to voting. For over two decades now, many have investigated the complexity of voting problems (such as the winner problem, manipulation, bribery, or electoral control) in different settings. Traditionally, the complexity of voting problems is studied under the full information assumption; in the bribery problem for instance, it is assumed that the briber knows the set of candidates, the set of voters, each voter's full preferences over the candidates, and the voting rule used. However, recognizing that in many real-world settings we simply lack full information, a new research line is concerned with the complexity analysis of voting problems under some kind of uncertainty. Following this approach, in this paper we present a systematic study of the complexity of bribery in k-Approval and k-Veto in settings where the briber has only partial information regarding the voters' preferences. Furthermore, we introduce three new notions of partial information and show the connections between these and existing models from the literature.

Appears in: *Proceedings of the 15th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2016), J. Thangarajah, K. Tuyls, C. Jonker, S. Marsella (eds.), May 9–13, 2016, Singapore.*

2. OUR RESULTS

In this section we introduce and motivate different types of partial information models followed by a discussion on how these types relate to each other. At the end of the section we present our results on bribery under partial information.

For each of the types of partial information introduced in the following we specify the structure of data given and how the set of potential rankings is specified. We let $m = |C|$ be the number of candidates.

Gaps (GAPS): Our first partial information model covers the case, where the briber only knows fractions of each vote, i.e., there are some blocks in each vote that are fully ranked and there are some blocks, where the briber knows which candidates there are in that block, but has no information on how they are ranked. Formally, for each vote v we have a partition $C_1^v, \dots, C_{2m+1}^v$ of the set of candidates and a total order for each C_k^v with k even. Note that possibly $C_k^v = \emptyset$ for some k. A ranking of candidates is in the information set if and only if for each (c, c') with $c \in C_k^v$ and $c' \in C_{k'}^v$, $k' > k$, c is preferred to c' and candidates in C_k^v, k even, are ranked according to the total order given for C_k^v. Note that if $C_k^v = C_{k+1}^v = \emptyset$ we can drop both partite sets without changing the information set. Therefore, we can restrict ourselves to at most $2m + 1$ partite sets.

One Gap (1GAP): A similar model was introduced by Baumeister et al. [1] as doubly-truncated preferences, where in each vote there are subsets of candidates ranked at the top and at the bottom of the votes, and there is a gap between the top and bottom ranked candidates. We adopt this notion and extend it in a way that we allow the top or bottom ranked candidate set to be empty. Formally, 1GAP refers to the special case of GAPS with $C_k^v = \emptyset$, for each $k \in \{1, 5, 6, \dots, 2m + 1\}$, for each voter v.

Top-truncated Orders (TTO): TTO was introduced by Baumeister et al. [1] and refers to the special case of 1GAP where $C_4^v = \emptyset$ for each voter v.

Bottom-truncated Orders (BTO): BTO was also introduced by Baumeister et al. [1] and refers to the special case of 1GAP where $C_2^v = \emptyset$ for each voter v.

Complete or empty votes (CEV): As suggested by Konczak and Lang [4], we introduce CEV as a special case of

Voting rule	FI	Gaps	FP	TOS	PC	CEV	1TOS	1Gap	TTO	BTO
Plurality	P	**NPC**	**NPC**	**NPC**	**NPC**	**P**	**P**	**NPC**	**P**	**NPC**
2-Approval	P	**NPC**	**NPC**	**NPC**	**NPC**	**P**	**P**	**NPC**	**P**	**NPC**
k-Approval, $k \geq 3$	NPC	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
3-Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
k-Veto, $k \geq 4$	NPC	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*

Table 1: Summary of the complexity results for bribery under partial information. P stands for element of P, NPC stands for NP-complete.

TTO with either $C_2^v = \emptyset$ or $C_3^v = \emptyset$ for each voter v. Note that this is equivalent to the special case of BTO with either $C_1^v = \emptyset$ or $C_2^v = \emptyset$ for each voter v.

Fixed Positions (FP): For each vote v we have a subset of candidates C^v with distinct positions in range between 1 and m assigned. A ranking of candidates is in the information set if and only if each candidate in C^v has the assigned position.

Pairwise Comparisons (PC): PC is probably the most natural way of displaying partial preferences. It has been introduced by Konczak and Lang [4] and has been used in many papers since. Formally, for each vote v we have a subset Π^v of $C \times C$. A ranking of candidates is in the information set if and only if for each $(c, c') \in \Pi^v$ c is preferred to c'. Note that we may restrict Π^v to be asymmetric and transitive for matters of convenience.

Totally Ordered Subset of Candidates (TOS): For each voter the briber has the information in a form of a totally ordered subset (for each voter a possibly different subset). Formally, for each vote v we have a subset C^v of candidates and a total order for C^v. A ranking of candidates is in the information set if and only if c is preferred to c' for each pair of candidates (c, c') with $c, c' \in C^v$ and c is preferred to c' according to the given order.

Unique Totally Ordered Subset of Candidates (1TOS): 1TOS was first suggested by Konczak and Lang [4] and formally defined by Chevaleyre et al. [2]. 1TOS refers to the special case of TOS where each voter v ranks the same subset of candidates, i.e., $C^v = C'$ for each voter v.

The following theorem shows the relations between the partial information models discussed in this paper.

THEOREM 2.1. *The following relations hold:*

(1) 1TOS ⊂ TOS.
(2) CEV ⊂ TOS.
(3) CEV ⊂ TTO.
(4) CEV ⊂ BTO.
(5) TTO ⊂ 1GAP.
(6) BTO ⊂ 1GAP.
(7) 1GAP ⊂ GAPS.
(8) 1GAP ⊂ FP.
(9) TOS ⊂ PC.
(10) GAPS ⊂ PC.

This list is complete in the following sense: Relations that are not listed here and that do not follow from transitivity do not hold in general.

An *n-voter profile* P on C consists of n strict linear orders $P = (v_1, \ldots, v_n)$. Let P' be a partial profile and let $I(P')$ denote the *information set*, which is the set of all complete n-voter profiles which are not contradicted by P'.

In the classic bribery problem (with full information about votes and a voting rule given) the question is whether a briber can change a given number of votes such that his favorite candidate is a winner. We carry over this idea to partial information models. Given an election, a designated candidate c, a non-negative integer ℓ and a partial profile P' according to one of the partial information models. We ask if it is possible to make c a winner of the election under a given voting rule for each complete profile in $I(P')$ by changing up to ℓ votes?

Table 1 shows the results on the complexity of bribery under partial information in k-Approval and k-Veto. Column FI displays the results for the case with full information due to Faliszewski et al. [3] and Lin [5]. Results in italic are hardness results that follow from already existing hardness results for full information. Results in boldface are new. Each NP-hardness result was achieved by a reduction from the NP-complete problem EXACT COVER BY 3-SETS.

3. CONCLUSIONS

We introduced three new partial information models (Fixed Positions, Gaps, and Totally Ordered Subsets of Candidates) and studied six known models. We showed the relations of all nine partial information models discussed in this paper. Furthermore, we defined bribery under partial information. We obtained a number of results on the complexity of this problem with respect to two important families of voting rules, namely k-Approval and k-Veto. We refer the reader to Table 1 for an overview.

Acknowledgments

The second and third author were supported in part by the DFG under grant ER 738/2-1.

REFERENCES

[1] D. Baumeister, P. Faliszewski, J. Lang, and J. Rothe. Campaigns for lazy voters: Truncated ballots. In *Proceedings of the 11th International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 577–584. IFAAMAS, June 2012.

[2] Y. Chevaleyre, J. Lang, N. Maudet, and J. Monnot. Possible winners when new candidates are added: The case of scoring rules. In *Proceedings of the 24th AAAI Conference on Artificial Intelligence*, pages 762–767. AAAI Press, July 2010.

[3] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra. How hard is bribery in elections? *Journal of Artificial Intelligence Research*, 35:485–532, 2009.

[4] K. Konczak and J. Lang. Voting procedures with incomplete preferences. In *Proceedings of IJCAI-05 Multidisciplinary Workshop on Advances in Preference Handling*, pages 124–129, 2005.

[5] A. Lin. The complexity of manipulating k-approval elections. In *Proceedings of the 3rd International Conference on Agents and Artificial Intelligence*, pages 212–218, 2011.