

# DYCOM: A Dynamic Truthful Budget Balanced Double-sided Combinatorial Market

## (Extended Abstract)

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### ABSTRACT

Recently, there has been increased attention on finding solutions for double-sided markets with strategic buying and selling agents.

We present and evaluate the first dynamic double-sided combinatorial market, named DYCOM, that allows truthful and individually-rational behavior for both buying and selling agents, keeps the market budget balanced and approximates social welfare efficiency. We experimentally examine the allocative efficiency of DYCOM. DYCOM performs well by all benchmarks and in many cases improves on previous mechanisms.

### 1. INTRODUCTION

One-sided combinatorial auctions where multiple commodities are offered have been studied for over a decade in economics and computer science. Agents bid on bundles of commodities and the auction aims to find a high social welfare (SWF) (an efficient) allocation of commodities to agents, while ensuring that truthful reporting of the agents' input is their best strategy ([7, 1, 14]).

Recent years have brought increased attention to the problems that arise in double-sided markets, in which the set of agents is composed of buying and selling agents. In double-sided markets the commodities are initially held by the set of selling agents, who have costs for the commodities they hold and are expected to behave strategically. The market maker's role is to match buying and selling agents as well as to determine the prices paid by/to the buying/selling agents respectively.

The Vickrey-Clarke-Groves (VCG) mechanism [15, 6, 8] is the cornerstone method in auction theory for high-SWF (efficient) allocation and incentivizing agents' truth-telling strategy. VCG is also individually rational (IR) in many settings. IR requires that no agent can lose by participating in the mechanism. In double-sided markets, another important requirement is budget-balance (BB), meaning that the market does not end up with a loss. VCG is not BB except in special cases [9]. It is well known from [13] that maximizing SWF while maintaining IR and truthfulness perform

runs a deficit (is not BB) even in the bilateral trade setting, i.e., when there are just two agents trading with each other. Well known circumventions of [13]'s impossibility in the setting of a double sided auction with a single commodity (unit demand/supply) are [10, 11], which relax efficiency in return for maintaining the properties of truthfulness, IR and BB. Other circumventions of [13] include relaxing determinism in addition to efficiency, i.e. randomized solutions in the combinatorial market setting[3].

Despite the growing interest in double-sided markets researchers have yet to find a sufficient mechanism that is both dynamic and combinatorial. The complexity of the dynamic design has been circumvented by employing an iterative process [12]; however, to our knowledge, the dynamic double-sided market literature focuses on a single-unit single-commodity for sale [16, 4, 2].

We present and evaluate the first **DY**namic double-sided **CO**mbinatorial Market, named DYCOM, that allows truthful and IR behavior for both buying and selling agents, keeps the market BB and approximates SWF efficiency. Our setting has multiple commodities, each with multiple units, that are bought and sold in different bundles by agents that arrive over time.

The main idea behind DYCOM is the transformation of the double-sided combinatorial market into a one-sided combinatorial auction. The transformation of the market into an auction makes use of a novel principle, that each selling agent is a buying agent of his own commodities. DYCOM is a primal-dual sequential posted-price mechanism that builds upon a combinatorial auction studied in the literature [5].

We experimentally examine the allocative efficiency of our solution. DYCOM performs well by all benchmarks and in many cases improves on previous mechanisms.

The paper's contributions are threefold. First, we provide the first dynamic double-sided combinatorial market that is truthful, IR and BB for all agents that approximates SWF efficiency. Second, our experimental tests show that our solution is a general and practical platform as it performs as well as the known McAfee [11]'s which is non dynamic with single-commodity, unit-demand and performs better than the randomized non dynamic combinatorial market with limited (subadditive) valuations and cost domains [3].

### 2. DYNAMIC COMBINATORIAL MARKET

Consider a dynamic market in which agents arrive over time and prices increase with demand. Agents are either

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buyers or sellers which arrive once and are faced with a vector of prices. Agents can demand/supply a bundle of their choice for the given prices or leave. We assume the demand and supply of each commodity's units are bounded by  $(\theta, \Theta)$ . For every arriving agent  $t$  we construct a virtual agent  $i$  that is interested in buying some of selling agent  $t$ 's commodities. In order to simulate a virtual buying agent  $i$  that represents selling agent  $t$ 's interests we allow virtual agent  $i$  to buy the commodities that are not beneficial for selling agent  $t$  to sell. Selling agent  $t$ 's commodities that were not bought by its virtual buying agent are offered to the "regular" (non virtual) buying agents that arrive in the time periods that follow.

We assume a priori knowledge of the values  $v_{\max}$  and  $c_{\min}$  such that  $v_{\max} > c_{\min}$  where  $v_{\max}$  is an upper bound for the buying agents valuation of any bundle and  $c_{\min}$  is a lower bound for the selling agents cost of any bundle. It is easy to verify that  $v_{\max}$  and  $c_{\min}$  knowledge is necessary in order to obtain non-trivial approximation ratio.

DYCOM handles dynamically arriving agents. The prices of all commodities are updated for every arriving agent. DYCOM queries the arriving agent for his demand or supply given the current prices. Each arriving selling agent is converted into a virtual buying agent. Payment to the arriving selling agent is made every time his commodities are bought by future arriving buying agents at the prices presented to him at arrival time. Each arriving buying agent is allocated his requested bundle at current prices and is charged according to them.

The following two Lemmas characterize DYCOM's theoretical properties:

LEMMA 2.1. *DYCOM is a truthful, IR and BB market*

LEMMA 2.2. *DYCOM approximates the SWF with in  $O(\Theta[(1 + s_{\max}(v_{\max} - c_{\min}))^{\frac{1}{\Theta-1}} - 1] + \theta)$ .*

### 3. EXPERIMENTAL RESULTS

We experimentally benchmarked the SWF efficiency of DYCOM under a variety of agents' bid distributions and agents' demand against ([2, 11, 3] and simplex). The most notable of DYCOM's results were when compared with: (1) An optimal non dynamic and non-truthful allocation algorithm (simplex), where DYCOM's approximation approaches 0.5 of the market SWF. (2) McAfee [11]'s non dynamic single commodity unit demand market. Here DYCOM's approximation approaches 1 though DYCOM is tailored for a completely general combinatorial setting and it is dynamic unlike [11]. (3) [3]'s randomized non dynamic combinatorial market. In this comparison DYCOM's approximation approaches 10 times that of [3]'s SWF in large markets even though DYCOM is deterministic and dynamic unlike [3]. More specifically Figure 1 shows that even if each agent's demand/supply is bounded by at least 1/200 of total market units, DYCOM performs better. Figure 1 shows its finding under subadditive valuations and costs. When we generate data removing this assumption DYCOM performs even better with respect to [3] under the same size market's demand/supply bounds.

All results were averaged over 1000 trials. In all the experiments we found minimal to no qualitative differences between the use of different distributions. We note that the theoretical approximation ratio claimed in Lemma 2.2 converges in large markets to 0.06 in the runs we performed. We

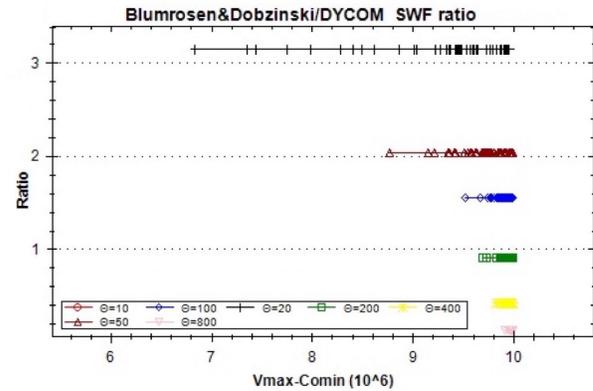


Figure 1: *DYCOM's SWF approx ratio vs. [3]'s SWF approximation ratio. When demand/supply of each agent is at most 1/200 of total market units DYCOM's SWF approximation ratio is better than [3] even if subadditive valuations and costs are assumed.*

also note that we perform a comparison between DYCOM's theoretical approximation ratio claimed in Lemma 2.2 and the other known combinatorial double sided market by [3]. [3]'s approximation in a randomized mechanism (assuming all valuations and costs are subadditive) is  $8H_{s_{\max}}$  where  $H_{s_{\max}}$  is the  $s_{\max}$  harmonic number. [3] assumes distributional knowledge of the median value of each selling agent's  $\Theta, \theta$  bounds. We found that for large markets DYCOM achieves a better theoretical approximation ratio than [3] even though [3]'s solution is randomized non-dynamic and the approximation ratio is only guaranteed for the cases where valuations and costs are subadditive and not generated for the general case as ours.

### 4. CONCLUSION AND DISCUSSION

We present and evaluate DYCOM the *first dynamic double-sided combinatorial market* that is truthful, IR and BB for buying and selling agents and approximates SWF efficiency.

DYCOM is a primal-dual sequential posted-price mechanism that builds upon a combinatorial auction studied in the literature [5]. The main idea behind our DYCOM solution is a transformation of the double-sided combinatorial market into a one-sided combinatorial auction. The proof of its economic properties as well as its theoretical approximation guarantee are omitted due to space limitations.

To validate the performance of DYCOM, we experimentally benchmarked DYCOM's SWF efficiency under variety of agents' bid distributions and demand. The results show that DYCOM is a general and practical platform as 1) it performs as well as the known McAfee [11]'s non dynamic single-commodity unit-demand double-sided market and 2) its approximation approaches 10 times that of [3]'s market's SWF in large markets though DYCOM is deterministic, dynamic and a completely general combinatorial setting.

In addition to providing a practical solution to the important dynamic double-sided combinatorial market problem, we believe that our double-sided combinatorial market transformation into a one-sided combinatorial auction is of independent interest for future work on reducing other forms of multi-sided exchanges to the well studied form of one-sided auctions.

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