Coalition Formation in Structured Environments (Doctoral Consortium)

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ABSTRACT

We study coalition formation games in which cooperation among the players is restricted by some combinatorial structures. We investigate the existence and computational issues related to stable outcomes in such games. In particular, we show that acyclicity is often sufficient for several notions of stability.

Keywords

Hedonic games, social networks, acyclic graphs, kernels

1. INTRODUCTION

Coalition formation arises everywhere in human activities: families are groups of people closely related by blood, clubs are social gatherings for people who share the common interest, political parties are organizations of people with the same political purpose, and companies are large entities of people who unite in order to gain more profits. Individuals form a coalition in order to achieve the objectives that they cannot accomplish on their own.

Many relevant aspects of this setting are captured by *hedonic games* [1]. The basic premise of hedonic games is that players are *selfish*. An outcome of the game is therefore considered to be *stable* if there are no profitable deviations. Based on this presumption, various types of deviations have been considered, which gives rise to different notions of stability concepts, including *individual stability* and *core stability*. In real life settings, however, a deviation should not only be profitable but also be feasible; for instance, it seems unlikely that the rightwing and the leftwing parties are able to form a coalition without the cooperation from intermediary parties. In situations like this, we do not have to expect all possible deviations.

In cooperative transferable utility games, such restrictions of communication structure have been considered by Myerson [8], who proposed a cooperative game constrained by graphs. Under his model, a subset of players can form a coalition if and only they are connected in the underlying graph structure. Since Myerson [8], several types of cooperative games have been defined. Nevertheless, few studies have so far been made on coalition formation games where communication structure between players is restricted.

The aims of this thesis are two-fold: first, to describe how communication structures affect the existence of stable outcomes; and second, to discuss computational issues that arise when considering games with a restricted communication structure.

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2. HEDONIC GRAPH GAMES

We first propose a hedonic coalition formation game where communication between players occurs through networks. Typical examples of such settings include social networks, gas pipeline networks across countries, and commercial airline networks.

DEFINITION 2.1. *A* hedonic graph game *is a triple* $(N, (\succeq_i)_{i\in N}, L)$ *where* $(N, (\succeq_i)_{i\in N})$ *is a hedonic game, and* $L \subseteq \{\, \{i, j\} \mid i \neq j, i, j \in N \,\}$ *is the set of communication links between players. A coalition* $S \subseteq N$ *is said to be* feasible *if it is connected in* (N, L).

Under this model, the stability notion can be relaxed to the ones such that no feasible coalitions of players wish to deviate. In particular, a partition π of players is said to be *core stable* if no connected subset can strictly improve the utilities of all the members of the coalition; and to be *individually stable* if no player can profitably deviate to her neighboring coalition without hurting the member of the coalition.

EXAMPLE 2.2. Consider the coalition formation problem in a parliament consisting of three parties: left-wing (1), centrist (2), and right-wing (3). Then 1 and 3 cannot form a coalition without 2. We describe this scenario as a hedonic game $(N, (\succeq_i)_{i\in N}, L)$ on a path where $N = \{1, 2, 3\}$, $L = \{\{1, 2\}, \{2, 3\}\}$, and the preference profile is given by

$$\begin{aligned}
1 &: \{1,2\} \succ_1 \{1\} \succ_1 \{1,2,3\} \succ_1 \{1,3\} \\
2 &: \{1,2,3\} \sim_2 \{1,2\} \succ_2 \{2,3\} \sim_2 \{2\} \\
3 &: \{1,2,3\} \sim_3 \{2,3\} \succ_3 \{1,3\} \sim_3 \{3\}
\end{aligned}$$

The core stable and individually stable partition of this game is $\pi = \{\{1, 2\}, \{3\}\}$.

Without any restrictions on games, unfortunately stable outcomes may not exist [1]. Nevertheless, if the underlying network is acyclic, stable outcomes exist and some of the problems known to be computationally hard become polynomial-time solvable. In the context of NTU-games on acyclic graphs, Demange [3] provides a constructive algorithm that finds a core stable outcome in time polynomial in the number of connected coalitions. For individual stability, Igarashi and Elkind [5] show that such a partition always exists for an arbitrary hedonic game on an acyclic graph and can be computed in time polynomial in the number of players.

3. GROUP ACTIVITY SELECTION

We will next consider coalition formation problems from a more centralized aspect, called *group activity selection problems on social networks* [6, 4]. Now players have preferences over the pairs of activities and group sizes; an assignment of players to activities is called *feasible* if each group of the same activity is connected in the underlying social network. We obtain a similar existence and computational result to hedonic games, but only if several groups of agents can simultaneously engage in the same activity, i.e., if the activities are *copyable*. In contrast, if each activity can be assigned to at most one coalition, finding a stable outcome turns out to be hard even if the underlying network is very simple. The hardness lies in the fact that acyclicity does not necessarily guarantees the existence of stable outcomes even if the network is a path or a star. We thus investigate the parameterized complexity of finding such outcomes. For acyclic networks, we show that the problem is fixed parameter tractable with respect to the number of activities. For general cases, we obtain a W[1]- hardness result even when the social network is a clique.

4. MORE GENERAL FRAMEWORK

The graph-restricted model is sufficiently general to encompass theoretical insights about stable outcomes; however, it might not be fully satisfactory in certain settings. Such examples include stable marriage problems whose feasible coalitions can not be represented by connected subsets of an undirected graph. Now we propose a more general framework of hedonic coalition formation games. A family $\mathcal{F}$ of subsets of a finite set N is called a *feasible coalition system* on N if $\{i\} \in \mathcal{F}$ for all $i \in N$ and $\emptyset \notin \mathcal{F}$. The combination of a hedonic game and a feasible coalition system results in a hedonic game with a restricted communication structure.

DEFINITION 4.1. *A hedonic game with a restricted communication structure is a triple* $\Gamma = (N, (\succeq_i)_{i\in N}, \mathcal{F})$ *where* $(N, (\succeq_i)_{i\in N})$ *is a hedonic game and* $\mathcal{F}$ *is a feasible coalition system on* N.

The existence result regarding core stable outcomes for hedonic games on acyclic graphs can be generalized further. To see this, one can associate core stable partitions with graph-theoretical concepts, called *kernels*. A *kernel* of a digraph (V, A) is a subset K of the vertices such that K is dominating and independent. It is known that stable matchings can be seen as kernels of specially oriented digraphs. This relation also holds for hedonic games. Given a hedonic game Γ, we define an orientation $D^\Gamma = (\mathcal{F}, A^\Gamma)$ where $(S, T) \in A^\Gamma$ if and only if there exists a player $i \in S \cap T$ such that $S \succeq_i T$. The core stable feasible partitions of a hedonic game Γ are precisely the set of kernels of D^Γ. Hence, finding a core stable partition of a hedonic game can be reduced to finding a kernel in a special digraph. We note that although the size of the digraph D^Γ can be exponentially large, this approach is useful if the number of feasible coalitions is bounded by a polynomial in the number of players.

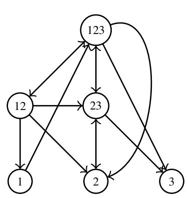

Figure 1: The digraph D^Γ corresponding to Example 2.2.

Boros and Gurvich [2] proved that the existence of a kernel is guaranteed for every clique-acyclic orientation of a graph if the underlying graph is perfect. This means that if the underlying communication structure $\mathcal{F}$ forms a normal family, i.e., $\mathcal{F}$ satisfies the Helly property and the intersection graph of $\mathcal{F}$ is perfect, core stable partitions always exist irrespective of preference profiles.

However, the question has not been settled as to whether computing a kernel of a perfect graph is polynomially solvable since no known proofs give a polynomial-time algorithm to find it. In [7], we have developed a polynomial-time algorithm to find a kernel for a certain class of graph families. The result strengthens Demange's result [3] concerning the core to a more general hedonic game.

5. CONCLUSION AND FUTURE WORK

Incorporating the aspects of communication structure into a coalition formation problem helps us understand what leads to stability and what causes instability. I believe that our work opens up an interesting line of research, with many problems left open to explore. The following is a list of specific problems that are worth investigating.

1. Computational complexity of determining the existence of core stable outcomes for almost acyclic graphs
   It would be interesting to see whether the tractability results in hedonic games on acyclic graphs can be extended to graphs that are "almost" acyclic. In particular, it remains open whether deciding the existence of core stable partitions is polynomial-time solvable when the set of feasible coalitions are the connected subsets of a single cycle. I expect that the crucial obstacle is how to detect a vicious cycle that prevents the existence of core stable outcomes.

2. Dynamics of coalition formation
   We have seen that some classes of hedonic games admit a stable partition; however, it is unclear how players actually reach stability. Namely, is there any natural convergence process to arrive at stable outcomes? One could, for instance, ask whether a better response dynamics on trees can converge to individual stability.

3. Empirical research on real-life networks
   It would be interesting to analyze what kind of deviations can occur in real-life networks. For instance, different academic communities may have different collaboration culture. One could ask the question "which deviation criterion explains the stability of some community structures."

REFERENCES

[1] A. Bogomolnaia, and M. O. Jackson. The stability of hedonic coalition structures. *Games and Economic Behavior* 38:201–230, 2002.

[2] E. Boros, and V. Gurvich. Perfect graphs are kernel solvable. *Discrete Mathematics*, 159:35–55, 1996.

[3] G. Demange. On group stability in hierarchies and networks. *Journal of Political Economy*, 112(4):754–778, 2004.

[4] A. Igarashi, R. Bredereck, and E. Elkind. On parameterized complexity of group activity selection problems on social networks. *AAMAS 2017*, 2017 (Extended Abstract). Long version available as arXiv:1703.01121 [cs.GT].

[5] A. Igarashi and E. Elkind. Hedonic games with graph-restricted communication. *AAMAS 2016*, pages 242–250, 2016.

[6] A. Igarashi, D. Peters, and E. Elkind. Group activity selection on social networks. *AAAI 2017*, 2017.

[7] A. Igarashi, F. Meunier, and A. Pass-Lanneau. Computing kernels in graphs with a clique-cutset, under submission.

[8] R. B., Myerson. Graphs and cooperation in games. *Mathematics of Operations Research*, 2(3), 225–229 (1977).