How Hard Is It to Control a Group?*

JAAMAS Track

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ABSTRACT

We consider group identification models in which the aggregation of individual opinions concerning who is qualified in a given society determines the set of socially qualified individuals. In this setting, we study the extent to which social qualification can be changed when societies expand, shrink, or partition themselves. The answers we provide are with respect to the computational complexity of the corresponding control problems and fully cover the class of consent aggregation rules introduced by Samet & Schmeidler (2003) as well as procedural rules for group identification. We obtain both polynomial-time solvability results and NP-hardness results. For some NP-hard problems, we also derive fixed-parameter algorithms.

KEYWORDS

Group identification; Consent rules; Procedural rules; Computational complexity; Parameterized complexity; Control

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1 INTRODUCTION

Group decision-making plays an important role in multi-agent systems. Imagine for instance a set N of agents who have to determine those among them who are eligible or qualified to complete a task. In this paper, we study a model where each individual (agent) qualifies or disqualifies every individual in N, and then a *social rule* is applied to select the socially qualified individuals. This model has been extensively studied under the name *group identification* in the literature (see [4] for a survey). We particularly study the complexity of some control problems in this model. We focus on the consent rules, the consensus-start-respecting rule (CSR), and the liberal-start-respecting rule (LSR) [5, 11, 12, 17, 18]. Each consent rule is characterized by two positive integers s and t. If an individual qualifies herself, then this individual is socially qualified if and only if there are at least $s-1$ other individuals who also qualify her. However, if the individual disqualifies herself, then this individual is not socially qualified if and only if there are at least $t-1$ other individuals who also disqualify her. The CSR and the LSR social rules iteratively determine the socially qualified individuals. In the beginning, the set K^{L} of individuals each of whom qualifies herself are considered LSR socially qualified, while the set K^{C} of individuals each of whom is qualified by all individuals are considered CSR socially qualified. Then, in each iteration for the social rule LSR (resp. CSR), an individual a is added to K^{L} (resp. K^{C}) if there is an individual in K^{L} (resp. K^{C}) qualifying a. The iteration terminates when no new individual can be added to K^{L} (resp. K^{C}), and the socially qualified individuals are the ones in K^{L} (resp. K^{C}).

In general, in each of the control problems studied in this paper, there is an external (strategic) agent who has an incentive to change the results by either adding some individuals (GCAI), or deleting some individuals (GCDI), or partitioning the set of individuals (GCPI). In particular, in each problem there is a subset S of distinguished individuals and the external agent aims to make all individuals in S socially qualified by performing the corresponding control operations. We achieve both polynomial-time solvability results and NP-hardness results for these problems for the aforementioned social rules. Importantly, we obtain dichotomy results for all problems considered in this paper for the consent rules, with respect to the values of s and t. For some of the NP-hard problems, we also derive fixed-parameter algorithms with respect to $|S|$.

Group identification is related to the widely-studied Approval voting rule [2, 9, 10, 15, 16, 20], where each voter approves or disapproves each candidate and the winners are those with the most approvals. However, group identification and Approval are different in several aspects. First, Approval is often considered as a single-winner voting rule and thus is used along with some tie-breaking method. Recently, several variants of Approval voting have been studied as multi-winner voting rules. However, the number of winners is bounded by (or exactly equal to) an integer (see, e.g., [1, 14]). Second, the goal of voting is to find some outstanding candidates. Therefore, when voters and candidates coincide, it is natural to assume that every candidate approves herself, or we ask voters only to approve or disapprove other candidates except herself. However, in group identification, everyone can qualify or disqualify herself. Recently, approval-based multi-winner voting with a variable number of winners has also been studied (see, e.g., [3, 8, 13, 21]). However, these rules are completely different from what we study in the paper. Moreover, to the best of our knowledge, to date only the very recent papers [8] and [21] (appeared after the workshop version of our paper) considered such multi-winner voting from the complexity point of view and are concerned with different problems from ours.

2 PROBLEM FORMULATION

Let N be a set of individuals where each $a \in N$ has an opinion about who from the set N possesses a certain qualification and who

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Table 1: Here, "P" stands for "polynomial-time solvable", "I" stands for "immune", and "FPT" stands for "fixed-parameter tractable". FPT results are with respect to the size of S, the set of individuals the strategic agent wants to make socially qualified.

	consent rules $f^{(s,t)}$						f^{CSR}	f^{LSR}
	$s = 1$			$s \geq 2$				
	$t = 1$	$t = 2$	$t \geq 3$	$t = 1$	$t = 2$	$t \geq 3$		
GCAI	I	I	I	NP-hard (FPT)	NP-hard (FPT)	NP-hard (FPT)	NP-hard	NP-hard
GCDI	I	P	NP-hard (FPT)	I	P	NP-hard (FPT)	P	I
GCPI	I	NP-hard	NP-hard	I	NP-hard	NP-hard	Open	I

does not. For $a' \in N$, we write $\varphi(a, a') = 1$ to denote the fact that a qualifies a', and $\varphi(a, a') = 0$ to denote the fact that a disqualifies a'. The mapping $\varphi : N \times N \to \{0, 1\}$ is called a *profile* over N. A *social rule* is a function f assigning a subset $f(\varphi, T) \subseteq T$ to each pair (φ, T) consisting of a profile φ over N and a subset $T \subseteq N$. We call the individuals in $f(\varphi, T)$ the *socially qualified individuals* of T with respect to f and φ.

In what follows we focus our analysis on the class of consent rules introduced by Samet and Schmeidler [18] and two procedural rules axiomatically studied in [5].

Consent rules $f^{(s,t)}$. Each consent rule $f^{(s,t)}$ is specified by two positive integers s and t such that for every $T \subseteq N$ and every individual $a \in T$, it holds that

(1) if $\varphi(a, a) = 1$, then $a \in f^{(s,t)}(\varphi, T)$ if and only if $|\{a' \in T \mid \varphi(a', a) = 1\}| \geq s$, and
(2) if $\varphi(a, a) = 0$, then $a \notin f^{(s,t)}(\varphi, T)$ if and only if $|\{a' \in T \mid \varphi(a', a) = 0\}| \geq t$.

Consensus-start-respecting rule f^{CSR}. For every $T \subseteq N$, this rule determines the socially qualified individuals iteratively. First, all individuals who are qualified by everyone in the society are considered socially qualified. Then, in each iteration, all individuals who are qualified by at least one of the currently socially qualified individuals are added to the set of socially qualified individuals. The iterations terminate when no new individual is added. Formally, for every $T \subseteq N$, let $K_0^{\text{C}}(\varphi, T) = \{a \in T \mid \forall a' \in T,\ \varphi(a', a) = 1\}$. For each positive integer $\ell = 1, 2, \ldots$, let $K_\ell^{\text{C}}(\varphi, T) =$

$$K_{\ell-1}^{\text{C}}(\varphi, T) \cup \{a \in T \mid \exists a' \in K_{\ell-1}^{\text{C}}(\varphi, T),\ \varphi(a', a) = 1\}.$$

Then, $f^{\text{CSR}}(\varphi, T) = K_\ell^{\text{C}}(\varphi, T)$ for some ℓ such that

$$K_\ell^{\text{C}}(\varphi, T) = K_{\ell-1}^{\text{C}}(\varphi, T).$$

Liberal-start-respecting rule f^{LSR}. This rule is similar to f^{CSR} with the only difference that the initial socially qualified individuals are those who qualify themselves. Particularly, for every $T \subseteq N$, let $K_0^{\text{L}}(\varphi, T) = \{a \in T \mid \varphi(a, a) = 1\}$. For each positive integer $\ell = 1, 2, \ldots$, let $K_\ell^{\text{L}}(\varphi, T) =$

$$K_{\ell-1}^{\text{L}}(\varphi, T) \cup \{a \in T \mid \exists a' \in K_{\ell-1}^{\text{L}}(\varphi, T),\ \varphi(a', a) = 1\}.$$

Then, $f^{\text{LSR}}(\varphi, T) = K_\ell^{\text{L}}(\varphi, T)$ for some ℓ such that

$$K_\ell^{\text{L}}(\varphi, T) = K_{\ell-1}^{\text{L}}(\varphi, T).$$

Observe that due to the above definitions, when K_0^{C} (resp. K_0^{L}) is empty, we have that $f^{\text{CSR}}(\varphi, T) = \emptyset$ (resp. $f^{\text{LSR}}(\varphi, T) = \emptyset$).

For a social rule f, we define the following three problems.

Group Control by Adding Individuals (GCAI)

Input: A 5-tuple (N, φ, S, T, k) of a set N of individuals, a profile φ over N, two nonempty subsets $S, T \subseteq N$ such that $S \subseteq T$ and $S \nsubseteq f(\varphi, T)$, and a positive integer k.

Question: Is there a subset $U \subseteq N \setminus T$ such that $|U| \leq k$ and $S \subseteq f(\varphi, T \cup U)$?

Group Control by Deleting Individuals (GCDI)

Input: A 4-tuple (N, φ, S, k) of a set N of individuals, a profile φ over N, a nonempty subset $S \subseteq N$ such that $S \nsubseteq f(\varphi, N)$, and a positive integer k.

Question: Is there a subset $U \subseteq N \setminus S$ such that $|U| \leq k$ and $S \subseteq f(\varphi, N \setminus U)$?

Group Control by Partitioning of Individuals (GCPI)

Input: A 3-tuple (N, φ, S) of a set N of individuals, a profile φ over N, and a nonempty subset $S \subseteq N$ such that $S \nsubseteq f(\varphi, N)$.

Question: Is there a subset $U \subseteq N$ such that $S \subseteq f(\varphi, V)$ where $V = f(\varphi, U) \cup f(\varphi, N \setminus U)$?

A social rule is *immune* to a control type if it is impossible to make a socially disqualified individual socially qualified by carrying out the corresponding control operation.

3 COMPLEXITY RESULTS AND DISCUSSION

We refer to Table 1 for a summary of our main findings. We can see from the table that almost all social rules studied in this paper resist the three different control types, in the sense that either control problems for these rules are NP-hard or these rules are immune to the corresponding control types. Only GCDI for the consent rules $f^{(s,2)}$ and for f^{CSR} is polynomial-time solvable. From the parameterized complexity point of view, GCAI and GCDI for consent rules are fixed-parameter tractable (FPT). In addition, given that the procedural rule f^{LSR} is immune to GCDI and GCPI, we can conclude that f^{LSR} outperforms the consent rules and the f^{CSR} rule in terms of resistance to control behavior. Note that whether GCAI for the two procedural rules is FPT with respect to $|S|$ remains open. Moreover, whether GCPI for f^{CSR} is NP-hard remains open.

Recently, Erdélyi, Reger, and Yang studied the complexity of other strategic problems in the setting of group identification [6, 7].

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