Truthful Mechanisms for Location Games of Dual-Role Facilities

Xujin Chen
Acad. Math. & Syst. Sci. (AMSS)
Chinese Acad. Sci. (CAS);
University of CAS
China
xchen@amss.ac.cn

Minming Li
City University of Hong Kong &
City University of Hong Kong
Shenzhen Research Institute
China
minming.li@cityu.edu.hk

Changjun Wang
Beijing University of Technology
China
wcj@bjut.edu.cn

Chenhao Wang*
AMSS, CAS;
University of CAS;
City University of Hong Kong
China
chenhwang4-c@my.cityu.edu.hk

Yingchao Zhao
Caritas Institute of Higher Education
China
zhaoyingchao@gmail.com

ABSTRACT

This paper is devoted to the facility location games with payments, where every agent plays a dual role of facility and customer. In this game, each selfish agent is located on a publicly known location in a metric space, and can allow a facility to be opened at his place. But the opening cost is his private information and he may strategically report this opening cost. Besides, each agent also bears a service cost equal to the distance to his nearest open facility. We are concerned with designing truthful mechanisms for the game, which, given agents' reports, output a set of agents whose facilities could be opened, and a payment to each of these agents who opens a facility. The objective is to minimize (exactly or approximately) the social cost (the total opening and service costs) or the maximum agent cost of the outcome.

We characterize the normalized truthful mechanisms for this game. Concerning the minimum social-cost objective, we give an optimal truthful mechanism without regard to time complexity, and show a small gap between the best known approximation ratio of polynomial-time truthful mechanisms for the game and that of polynomial-time approximation algorithms for the counterpart of pure optimization. For the minimum maximum-cost objective, we provide an optimal truthful mechanism which runs in polynomial time. We also investigate mechanism design for the game under a budget on the total payment.

KEYWORDS

Truthful mechanism design; Facility location game; Payments

ACM Reference Format:

Xujin Chen, Minming Li, Changjun Wang, Chenhao Wang, and Yingchao Zhao. 2019. Truthful Mechanisms for Location Games of Dual-Role Facilities. In *Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), Montreal, Canada, May 13–17, 2019,* IFAAMAS, 9 pages.

*Corresponding author: Chenhao Wang.

Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), N. Agmon, M. E. Taylor, E. Elkind, M. Veloso (eds.), May 13–17, 2019, Montreal, Canada.

1 INTRODUCTION

The classic facility location games without payments model the scenario where the government plans to build some public facilities in a street or a general metric space, where some self-interested customers are located. Each customer has a *service* (*connection*) *cost* equal to the distance to the nearest facility. As strategic agents, the customers hold their locations as private information and can strategically report them to minimize their own service costs. After receiving the reported information, the government will then use a mechanism to map it to some facility locations. The purpose of the mechanism is to optimize a certain objective, such as minimizing the total cost or minimizing the maximum agent cost, while guaranteeing that truthfully reporting is every customer's optimal strategy.

In most studies of the facility location games without payments, the government can build facilities anywhere in the space and only the customers are strategic players. However, in reality the potential facility locations are usually limited and pre-given; each potential facility might have an *opening cost*. Archer and Tardos [1] studied a facility location game with payments where *only* facilities are selfish players and can strategically report their opening costs, while the customers' locations are public information.

In this paper, we study a game model merging the customers with the location owners, *i.e.*, facilities are only allowed to open at the locations of some customers and the customers are also the location owners (in the following, we refer to them as agents, or equivalently facilities). The agents report their opening costs as private information, while their locations are publicly known. Once receiving the reports (referred to as bids), the government uses a mechanism to decide which facilities to be opened and how much to pay to the corresponding agents. Each agent is incurred either an opening cost if a facility is open at his location, or a service cost if not; he wishes to maximize his utility: the difference between the payment he receives and the cost he is incurred.

The studied problem models the scenario where the authority of a city wants to build some public facilities (e.g., libraries and supermarkets) for some communities. Due to land use restrictions, the facilities can only be built in the communities, while other lands are urban green space. Each community has a private opening-cost, which contains many components (e.g., the cost for demolition, construction, renovation, and daily management), and the community has its own connections to different parties (and therefore can deal on different prices) who can take care of these components. Therefore, the authority has no way to access these values, and the opening-costs for facilities by communities can be considered private. In some cases, the authority has a budget and cannot pay too much.

Our goal is to design mechanisms, which satisfy one or more desirable properties, for this facility location game, where each agent plays a dual role of both a facility and a customer. Usually, a mechanism is required to be *truthful*, that is, for every agent reporting his true opening cost is the optimal strategy to maximize his utility. In addition, the mechanism is expected to have a good performance guarantee (w.r.t. a certain objective function) and to satisfy *individual rationality*, that is, every agent benefits from participating the game.

Related Work

Facility Location Problem. The metric uncapacitated facility location problem (UFL) studies the optimization problem of selecting a set of facilities in a metric space to minimize the sum of facility opening costs and customer service (connection) costs. This is a well-known NP-hard problem. It is proved that any polynomial-time algorithm cannot be 1.463-approximate, unless $\mathbf{P} = \mathbf{NP}$ [11, 24]. Jain *et al.* [12] give a 1.61-approxime algorithm (JMS algorithm). Combining it with cost scaling, the algorithm of Mahdian *et al.* [16] achieves the currently best approximation ratio 1.52 in the deterministic sense. The UFL falls into the *k-facility location problem* (k-UFL) if at most k facilities can be opened. For the k-UFL, Zhang [27] gives the best-known approximation ratio of $2 + \sqrt{3} + \epsilon$.

Facility Location Game. Mechanism design for the facility location game was first studied by Procaccia and Tennenholtz [20]. After that, the model in which strategic customers report their locations have been widely studied, see, e.g., [3, 7, 8, 14, 15, 26]. Archer and Tardos [1] study the model with publicly known service costs and strategic facilities who report their private opening costs, where the differences to our model are (i) customers are not the players of the game; (ii) each winning facility only pays his real opening cost and each losing facility pays nothing. They prove that any optimal algorithm for the UFL admits a truthful mechanism for the social-cost objective (involving both facilities and customers).

Single Parameter Problem. A mechanism design problem is *single parameter*, if each agent holds only one private value. Myerson [17] first gives a characterization of truthful mechanisms in reverse-auction setting where each agent has a private scalar value for "winning", with "losing" having value of 0. Archer and Tardos [1] extend the result to a more general setting, where the cost of agent i is his private data times the amount of load assigned to him. Similar characterizations in somewhat different settings can be found in [18, 19, 21]. Our model of dual-role facility location games falls into the single-parameter framework in a totally different way.

VCG Mechanisms. One of the most celebrated results in mechanism design is the Vickrey-Clarke-Groves (VCG) mechanism [4, 10, 25], which is truthful for the social objective functions. It has been shown that VCG-like mechanisms are the only truthful ones that maximize the social welfare [9, 13, 19]. A main difficulty in applying VCG mechanism is its computational complexity, as finding an optimal solution of the corresponding optimization problem is often NP-hard.

Budget on Payments. A mechanism is *budget feasible* if the total payment provided by the mechanism does not exceed a given budget. Singer [23] initiates the budget-feasible mechanism design problem, and provides constant-approximate algorithms for maximizing monotone submodular objective functions. Later, Chen *et al.* [2] improve the approximation ratio by a greedy scheme.

Our Results and Organizations

This paper extends the family of facility location games to the area with payment, while most of the previous works mainly concern the case without money. We characterize the truthful mechanisms, and design optimal and approximate truthful mechanisms both for the social-cost and the bottleneck-cost objectives. When there is a budget on the payments, we show a lower bound of $\Omega(n)$ for truthful budget feasible mechanisms under the social-cost objective, in contrast of the constant approximation results for maximizing a supermodular function. Our work contributes in two aspects: in terms of facility location games, we provide the first nontrivial results with payment to dual-role agents, and it can lead to the studies with respect to strategic customers or facilities; in terms of single parameter problems, we initiate the study for the class where an agent incurs some cost even it loses (*i.e.*, he is not selected as a winner by the mechanism), and we believe this can be extended to many other topics such as reverse auction, especially combined with budget. The details are as follows.

In Section 2 we formally present our model of dual-role facility location game. It is the first time to merge strategic customers with strategic facilities, where the government only allows facilities to be opened at the places of the customers (for example, other lands in the city are urban green space), and they are combined together to act as an agent. This model offers a new sight on extending the classic facility location games, and possibly avoids the criticism on the common assumption that the government having no information about the locations of customers.

Section 3 provides the characterization of normalized truthful mechanisms. For the facility location game with only strategic facilities, it is known [1] that a mechanism is truthful if and only if the selection function is monotone and the payment to each opened facility is its *threshold bid* at which the facility would no longer be opened. In our setting, because of the extra cost for the "losing" agents, the characterization for the truthfulness is no longer the same as that in [1]. We show that a truthful mechanism for our game is additionally required to keep the service cost of any losing agent unchanged no matter how he increases his bid, and the payment to a winner should be his threshold bid minus the invariant service cost in any outcome where he is a loser. This conclusion can be extended to more general settings.

In Section 4 and 5, we study truthful mechanisms for the social-cost and the bottleneck-cost objectives, respectively. For the former, we give an optimal truthful mechanism, regardless of the time complexity. Although this is a VCG mechanism, it remains a challenge to assure the normality and individual rationality. In special cases, such as the tree space and k-UFL game with constant k, this mechanism is polynomially computable. Further, we provide a polynomial-time 1.61-approximation mechanism, based on JMS algorithm for the UFL [12], and thus the gap between the best-known approximation ratios of polynomial-time algorithms and polynomial-time truthful mechanisms is 0.09. For the bottleneck-cost objective, which minimizes the maximum agent cost, we provide an optimal truthful mechanism that runs in polynomial time.

Section 6 introduces the budget constraint on payments to the dual-role facility location game. Given a budget B, the total payment of the mechanism for the game cannot exceed B. We give a lower bound $\Omega(n)$ (resp. 1.5) on the approximation ratio of any truthful and budget feasible mechanisms w.r.t. the social-cost objective (resp. bottleneck-cost objective), where n is the number of agents in the game. Furthermore, we show that there is an optimal mechanism when only one facility is allowed to open.

2 THE MODEL

Let $N = \{1, 2, \ldots, n\}$ be a set of agents, where each agent takes a dual role of both a customer and a facility. We will use "agent" and "facility" interchangeably. The agents are located in a metric space (Ω, d), where $d : \Omega \times \Omega \to \mathbb{R}_+$ is the metric. Each agent $i \in N$ is located at $l_i \in \Omega$ and has a (facility) opening cost f_i. Let $d(i, j) := d(l_i, l_j)$ denote the distance between any two agents $i, j \in N$. The profile of locations and that of opening costs are written as $\mathbf{l} = (l_1, l_2, \ldots, l_n)$ and $\mathbf{f} = (f_1, f_2, \ldots, f_n)$, respectively.

While the opening cost f_i is agent i's private information for each $i \in N$, the location profile and the distances are publicly known. Each agent i strategically reports his own opening cost as his *bid* b_i (which is not necessarily equal to f_i). Once a mechanism receives all bids $b_1, b_2, \ldots, b_n$ from the agents, it outputs a subset $W \subseteq N$ of agents (referred to as *winners*) for opening facilities, and a payment p_i to each $i \in N$. Formally, a mechanism $\mathcal{M} = (s, p)$ consists of a selection function $s : \mathbb{R}^n_+ \to 2^N$ and a payment function $p : \mathbb{R}^n_+ \to \mathbb{R}^n_+$, which map each bid vector $\mathbf{b} = (b_1, \ldots, b_n)$ to a winner set $s(\mathbf{b}) = W$ and a payment vector $p(\mathbf{b}) = (p_1, p_2, \ldots, p_n)$, respectively. If both functions can be computed in polynomial time, $\mathcal{M}$ is said to be a polynomial-time mechanism.

Given a winner set $W \subseteq N$, each agent $i \in N$ bears a *cost* $c_i(W) = I_W(i) \cdot f_i + d(i, W)$, where $I_W(i)$ equals 1 if $i \in W$ and 0 otherwise, and $d(i, W) = \min_{j \in W} d(i, j)$ is the distance between agent i and W (moreover, $d(i, \emptyset) := Q$ where Q is a big constant).[1] Each agent i wishes to maximize his *utility*: $p_i - c_i(W)$, where p_i is the payment he receives. We call a mechanism $\mathcal{M}$ *truthful*, if bidding the true opening cost is the best strategy for every agent . That is, for every $i \in N$ with bid b_i, and every set of bids $\mathbf{b}_{-i}$ by $N \backslash \{i\}$, it holds that $p_i - c_i(W) \geq p'_i - c_i(W')$, where $(W, \mathbf{p})$ and $(W', \mathbf{p}')$ are the outputs of $\mathcal{M}$ for the input bid vectors $(f_i, \mathbf{b}_{-i})$ and $(b_i, \mathbf{b}_{-i})$, respectively.

We consider some system objective function $C : 2^N \to \mathbb{R}_+$, that depends on the opening cost profile $\mathbf{f}$, and distances between agents (determined by (Ω, d) and location profile $\mathbf{l}$). The mechanism tries to minimize $C(W)$, while it does not know $\mathbf{f}$, and just makes decisions according to public information and the bids reported. Subject to certain constraints for facility opening (possibly none), we say that a mechanism has *approximation ratio* $\alpha(\geq 1)$, if for any facility game instance $(\Omega, d, \mathbf{l}, \mathbf{f})$ the mechanism outputs a winner set W such that $C(W) \leq \alpha \cdot \min_S C(S)$, where the minimum is taken over all S that could be chosen for opening facilities. Our goal is to design polynomial-time truthful mechanisms that exactly or approximately minimize the objective function C. As usual, we assume that the mechanism is *normalized* (i.e., $p_i = 0$ if $i \notin W$) and *individual rational* (i.e., no agent will lose any of his utility by participating this game).

In this paper, we consider two objective functions C and C_B, respectively. For any $W(\subseteq N)$, which is considered as a solution of the facility location game, we define its social cost and bottleneck cost as follows.

- **Social cost:**

$$C(W) = \sum_{i \in N} c_i(W) = \sum_{i \in N} d(i, W) + \sum_{i \in W} f_i.$$

- **Bottleneck cost:**

$$C_B(W) = \max_{i \in N} c_i(W) = \max_{i \in N} (I_W(i) \cdot f_i + d(i, W)).$$

We can tackle the mechanism design when some constraints are imposed on the selection or payment function: (i) given constant integer k, at most k facilities can be open; (ii) given a budget $B \in \mathbb{R}_+$, the total payment does not exceed the budget: $\sum_i p_i \leq B$. A mechanism satisfying (ii) is said to be *budget feasible*.

Remark 2.1. While the property *no positive transfer* (i.e., $p_i \geq 0$) is common in other game settings, it is not needed here. If the potential service cost of an agent in some solution is larger than his opening cost, he might conversely pay an amount of "money" to the government (mechanism), asking

[1] Note that each winner $i \in W$ bears his associated facility opening cost f_i, while each loser $j \notin W$ bears the service cost $d(j, W)$. The words "winner" and "loser" are used to distinguish agents whose locations are selected for opening facilities with those whose locations are not selected. Being a winner does not mean a smaller cost than being a loser .

for a facility to be opened at his location. So a negative payment is also reasonable.

3 CHARACTERIZATION OF TRUTHFULNESS

In this section, we study the properties of normalized truthful mechanisms for the dual-role facility game with payments. This game is a single-parameter mechanism design problem, as the single parameter f_i directly determines the cost function c_i. Single-parameter problems are widely studied in the reverse-auction setting where each agent i has a private scalar data $\bar{f}_i$ which is the cost incurred to him if he wins (losing incurs him 0 cost). In a more general setting, the cost of agent i equals his private data $\bar{f}_i$ times the amount of load assigned to him by the assignment function.

For the reverse-auction setting, Myerson [17] gives a well-known characterization: a normalized mechanism is truthful if and only if (i) the selection function s is monotone (i.e., a winner keeps winning if he unilaterally decreases his bid); and (ii) the payment to each winner is his threshold bid to win. For the general setting, Archer and Tardos [1] proves that an assignment function admits a truthful mechanism (via suitable payments) if and only if it is monotone. Our setting is, however, different from them, as a loser always bears a positive cost which equals the distance to the nearest winner. Before stating the characterization for the truthful mechanisms of our facility game, we give some essential definitions.

Definition 3.1. A selection function s is called *monotone*, if for any player $i \in N$ and bid vector $(b_i, \mathbf{b}_{-i})$ with $i \in s(b_i, \mathbf{b}_{-i})$, we have $i \in s(b'_i, \mathbf{b}_{-i})$ for all $b'_i < b_i$. That is, if agent i wins with bid b_i, then he also wins by bidding any $b'_i < b_i$.

Definition 3.2. For a monotone selection function s, given others' bids $\mathbf{b}_{-i}$, the *threshold value* of agent i is $r_i(\mathbf{b}_{-i}) = \inf_{i \notin s(b_i, \mathbf{b}_{-i})} b_i$, i.e., the "smallest bid" under which agent i loses (facility i is not opened). The threshold value at $\mathbf{b}_{-i}$ is undefined if $\{b_i \mid i \notin s(b_i, \mathbf{b}_{-i})\}$ is empty.

In this paper, we always assume without loss of the generality that the infimum (if exists) in the definition of the threshold belongs to the bid set, i.e., $\inf_{i \notin s(b_i, \mathbf{b}_{-i})} b_i = \min_{i \notin s(b_i, \mathbf{b}_{-i})} b_i$. The following claim about the payment is useful.

LEMMA 3.3. *For any agent i and any fixed bids $\mathbf{b}_{-i}$ of other agents, if agent i wins, then a truthful mechanism must pay the same to i.*

PROOF. Consider payments p_i and p'_i to winner i under bids $(b_i, \mathbf{b}_{-i})$ and $(b'_i, \mathbf{b}_{-i})$, respectively. For the instance with $f_i = b_i$ (resp. $f_i = b'_i$), the truthfulness guarantees $p_i - b_i \geq p'_i - b_i$ (resp. $p'_i - b'_i \geq p_i - b'_i$). It follows that $p_i = p'_i$. □

Denote as $\mathcal{S}_i(\mathbf{b}_{-i}) = \{s(b_i, \mathbf{b}_{-i}) \mid b_i \geq r_i(\mathbf{b}_{-i})\}$ the collection of all possible winner sets when agent i bids at least his threshold value. Now we are ready to state the characterization:

THEOREM 3.4. *A normalized mechanism $\mathcal{M} = (s, p)$ is truthful if and only if the following hold:*

(i) s is monotone;

(ii) For any agent $i \in N$ and winner sets $S, S' \in \mathcal{S}_i(\mathbf{b}_{-i})$, there holds $d(i, S) = d(i, S')$;

(iii) Every winner is paid his threshold value minus the distance to any winner set when he bids at least the threshold value. Precisely, for every agent $i \in N$ and others' bids $\mathbf{b}_{-i}$, if i's threshold value is undefined, then i is paid by a constant independent of his bid; otherwise, for every bid b_i with $i \in s(b_i, \mathbf{b}_{-i})$, i's payment is

$$p_i(b_i, \mathbf{b}_{-i}) = r_i(\mathbf{b}_{-i}) - d(i, S), \tag{1}$$

where $d(i, S)$ is an invariant for all $S \in \mathcal{S}_i(\mathbf{b}_{-i})$.

PROOF. The *"if"* part. Given bids $\mathbf{b}_{-i}$, if i's threshold value is undefined, then i always wins with a constant utility; else by the definition of threshold value, we have $i \notin s(f_i, \mathbf{b}_{-i})$ if $f_i \geq r_i(\mathbf{b}_{-i})$, and $i \in s(f_i, \mathbf{b}_{-i})$ otherwise. To prove the truthfulness, we show that in any case, for i, bidding f_i is no worse than bidding any b_i. Let W and W' denote the winner sets $s(f_i, \mathbf{b}_{-i})$ and $s(b_i, \mathbf{b}_{-i})$, respectively. We may assume $r_i(\mathbf{b}_{-i})$ is defined.

In the case of $f_i \geq r_i(\mathbf{b}_{-i})$, we have $W \in \mathcal{S}_i(\mathbf{b}_{-i})$, and i's utility when telling the truth is $0 - c(W) = -d(i, W)$. Agent i could change this utility only by bidding $b_i < r_i(\mathbf{b}_{-i})$, at which he wins with a utility $p_i(b_i, \mathbf{b}_{-i}) - c(W') = (r_i(\mathbf{b}_{-i}) - d(i, W)) - f_i \leq -d(i, W)$.

In the case of $f_i < r_i(\mathbf{b}_{-i})$, agent i's utility when telling the truth is $p_i(f_i, \mathbf{b}_{-i}) - c_i(W) = r_i(\mathbf{b}_{-i}) - d(i, S) - f_i > -d(i, S)$ for some $S \in \mathcal{S}_i(\mathbf{b}_{-i})$. The only possibly way that agent i could change this utility is bidding $b_i \geq r_i(\mathbf{b}_{-i})$, at which $W' \in \mathcal{S}_i(\mathbf{b}_{-i})$, and i's utility becomes $0 - c_i(W') = -d(i, W') = -d(i, S)$ by (ii).

The *"only if "* part. Condition (i): Suppose for a contradiction that s is not monotone. Then there exist $i \in N$ and $b_i, b'_i, \mathbf{b}_{-i}$ with $b'_i < b_i$ such that $i \in s(b_i, \mathbf{b}_{-i})$ wins with payment p_i when bidding b_i, while $i \notin W' := s(b'_i, \mathbf{b}_{-i})$ loses with payment 0 when biding b'_i. For the instance with $f_i = b_i$, the truthfulness of $\mathcal{M}$ implies $p_i - b_i \geq -d(i, W')$, while for the instance with $f_i = b'_i$, we have $-d(i, W') \geq p_i - b'_i$. A contradiction to $b'_i < b_i$ follows.

Condition (ii): For any winner sets $S = s(b_i, \mathbf{b}_{-i})$ and $S' = s(b'_i, \mathbf{b}_{-i})$ in $\mathcal{S}_i(\mathbf{b}_{-i})$, considering the instances with f_i equal to b_i and b'_i respectively, $\mathcal{M}$'s truthfulness gives $d(i, S) = d(i, S')$.

Condition (iii): Suppose that $i \in s(b_i, \mathbf{b}_{-i})$. If i's threshold value w.r.t. $\mathbf{b}_{-i}$ is undefined, then it is instant from $\mathcal{M}$'s truthfulness that $p_i(b'_i, \mathbf{b}_{-i})$ is the same constant for all b'_i. So we may assume that $r_i(\mathbf{b}_{-i})$ is defined, and $b_i < r_i(\mathbf{b}_{-i})$. Suppose for a contradiction that $p_i(b_i, \mathbf{b}_{-i}) \neq r_i(\mathbf{b}_{-i}) - d(i, S)$ for some $S \in \mathcal{S}_i(\mathbf{b}_{-i})$. On the one hand, if $p_i(b_i, \mathbf{b}_{-i}) > r_i(\mathbf{b}_{-i}) - d(i, S)$, then for the instance whose f_i satisfies $p_i(b_i, \mathbf{b}_{-i}) > f_i - d(i, S) > r_i(\mathbf{b}_{-i}) - d(i, S)$, since $f_i > r_i(\mathbf{b}_{-i})$ and condition (ii) holds, i's utility when telling the truth is $-d(i, S)$, which is lower than his utility $p_i(b_i, \mathbf{b}_{-i}) - f_i$ when he bids b_i, a contradiction to $\mathcal{M}$'s truthfulness. On the other hand, if $p_i(b_i, \mathbf{b}_{-i}) < r_i(\mathbf{b}_{-i}) - d(i,S)$, then for the instance whose f_i satisfies $p_i(b_i, \mathbf{b}_{-i}) < f_i - d(i,S) < r_i(\mathbf{b}_{-i}) - d(i,S)$, by $f_i < r_i(\mathbf{b}_{-i})$, condition (i) and Lemma 3.3, we have $p_i(f_i, \mathbf{b}_{-i}) = p_i(b_i, \mathbf{b}_{-i})$. It follows that i's utility when telling the truth is $p_i(b_i, \mathbf{b}_{-i}) - f_i$. However, i's utility when bidding at least $r_i(\mathbf{b}_{-i})$ is $-d(i,S) > p_i(b_i, \mathbf{b}_{-i}) - f_i$, a contradiction to $\mathcal{M}$'s truthfulness. □

The main difference between the above characterization (Theorem 3.4) and Myerson's is the existence of the extra term $-d(i,S)$ in (1). Similar to the generalization of Myerson's characterization for selection functions [17] to that for assignment functions [1], Theorem 3.4 for cost functions $c_i(W) = I_W(i) \cdot f_i + d(i,W)$ admits a generalization for cost function $\tilde{c}_i(W) = A_W(i) \cdot f_i + \tilde{d}_i(W)$, where $A_W(i)$ is the amount of load assigned to agent i corresponding to winner set W, and $\tilde{d}_i(W)$ is the counterpart to $d(i,W)$.

4 SOCIAL COST

By the characterization (Theorem 3.4), a selection function (algorithm) can be extended to a truthful mechanism if and only if it is monotone and a loser always bears the same (service) cost whenever increasing his bid; and the unique way for the extension is paying agents according to condition (iii). In this section, we design truthful mechanisms to minimize the social cost. The challenge is to find eligible algorithms (satisfying conditions (i) and (ii)) for the UFL with good approximation ratios. In the remainder of the paper, we use algorithm and selection function indiscriminately.

4.1 Optimal Truthful Mechanism

We are first concerned with the existence of an *optimal* truthful mechanism which exactly minimizes the social cost, regardless of the time complexity of the algorithm. Every optimal algorithm is clearly monotone, but it does not necessarily satisfy condition (ii) in Theorem 3.4. Suppose a UFL instance has two different optimal solutions W and W' neither of which contains agent i, and i's distances from them $d(i,W)$ and $d(i,W')$ are different. However, W and W' could be the winner sets output by an optimal algorithm in responses to different bids of i. The algorithm violates condition (ii). To avoid this event, we can adopt a trivial optimal approach that traverses and indexes all feasible solutions, and then outputs the optimal one with lowest index, where the sets are indexed in the same way for any input of reported opening costs (bids). It is easy to see that the trivial optimal algorithm, denoted as s, satisfies condition (ii) in a way that

$$|\{\mathcal{S}_i(\mathbf{b}_{-i})\}| = 1 \text{ for all } i \text{ and } \mathbf{b}_{-i}. \tag{2}$$

More generally, let s denote an optimal algorithm for the UFL which satisfies condition (ii) in a way that $\mathcal{S}_i(\mathbf{b}_{-i})$ consists of a unique set S_i for every i and $\mathbf{b}_{-i}$. Note that $i \notin S_i$. Consider the following mechanism $\mathcal{M} = (s,p)$.

Mechanism 1. $\mathcal{M} = (s,p)$ is defined as: Given bid vector $\mathbf{b} = (b_1, b_2, \ldots, b_n)$,

(i) The winner set is $W = s(\mathbf{b})$.

(ii) For each agent i, his payment $p_i(\mathbf{b})$ is

$$\left(\sum_{j\in N\setminus\{i\}} d(j,S_i) + \sum_{j\in S_i} b_j\right) - \left(\sum_{j\in N\setminus\{i\}} d(j,W) + \sum_{j\in W\setminus\{i\}} b_j\right).$$

Theorem 4.1. *Mechanism 1 for the dual-role facility game is normalized, truthful, individual rational, and optimal for the social-cost objective.*

Proof. *Normality.* For $i \notin W = s(\mathbf{b})$, note that $W = S_i$. It is straightforward from Mechanism 1(ii) that $p_i(\mathbf{b}) = 0$.

Truthfulness. By Theorem 3.4, it suffices to prove (1) holds. Indeed, for every $i \in W = s(\mathbf{b})$, we have $d(i,W) = 0$ and $r_i(\mathbf{b}_{-i}) - d(i,S_i) = (\sum_{j\in N} d(j,S_i) + \sum_{j\in S_i} b_j) - (\sum_{j\in N} d(j,W) + \sum_{j\in W\setminus\{i\}} b_j) - d(i,S_i) = p_i(\mathbf{b})$.

Individual rationality. The individual rationality of losers is obvious. For each winner $i \in s(\mathbf{b})$, if facility i is open, then his "utility" is $p_i(\mathbf{b}) - b_i = r_i(\mathbf{b}_{-i}) - d(i,S_i) - b_i \geq -d(i,S_i)$, where $r_i(\mathbf{b}_{-i}) \geq b_i$ is implied by the monotonicity of s. Otherwise, i has to bear a service cost $d(i,S_i)$, and his "utility" is $-d(i,S_i)$, no more than that when he participates the game. □

VCG mechanism

Recall that, given cost functions $\tilde{c}_i$ of agents $i \in N$, a VCG mechanism (not necessarily normalized) consists of a selection function $\tilde{s}(\tilde{c}_1, \ldots, \tilde{c}_n)$ minimizing the social cost $\sum_{i\in N} \tilde{c}_i(\tilde{s}(\tilde{c}_1, \ldots, \tilde{c}_n))$, and payment functions $\tilde{p}_i(\tilde{c}_1, \ldots, \tilde{c}_n) = h_i(\tilde{c}_1, \ldots, \tilde{c}_{i-1}, \tilde{c}_{i+1}, \ldots, \tilde{c}_n) - \sum_{j\neq i} \tilde{c}_j(\tilde{s}(\tilde{c}_1, \ldots, \tilde{c}_n))$, where h_i is a real function independent of $\tilde{c}_i$. Concerning Mechanism 1 of our facility location game, let us consider $\tilde{c}_i(W) := I_W(i) \cdot b_i + d(i,W)$ for all $i \in N$ and $\tilde{s}(\tilde{c}_1, \ldots, \tilde{c}_n) = s(\mathbf{b})$ as a special case. For any agent i, notice that S_i defined above Mechanism 1 does not contain i, and it does not depend on b_i, nor on $\tilde{c}_i$. It is valid to take $h_i(\tilde{c}_1, \ldots, \tilde{c}_{i-1}, \tilde{c}_{i+1}, \ldots, \tilde{c}_n) := \sum_{j\in N\setminus\{i\}} \tilde{c}_j(S_i) = \sum_{j\in N\setminus\{i\}} d(j,S_i) + \sum_{j\in S_i} b_j$. It is clear that $\sum_{j\in N\setminus\{i\}} \tilde{c}_j(W) = \sum_{j\in N\setminus\{i\}} d(j,W) + \sum_{j\in W\setminus\{i\}} b_j$, and therefore $h_i(\tilde{c}_1, \ldots, \tilde{c}_{i-1}, \tilde{c}_{i+1}, \ldots, \tilde{c}_n) - \sum_{j\in N\setminus\{i\}} \tilde{c}_j(W) = p_i(\mathbf{b})$, from which we see that the payment defined in Mechanism 1(ii) falls within the general VCG framework. Therefore, Mechanism 1 is actually in the family of VCG mechanisms.

Although VCG mechanisms are various, finding a normalized (and individual rational) one needs some tricks, especially when it is additionally required that the optimal algorithm satisfy condition (ii). The main challenge lies in the selection of h_i to yield a normalized mechanism. A most-used choice of h_i in a VCG mechanism is Clarke Pivot Rule (see [19]): $h_i(\tilde{c}_1, \ldots, \tilde{c}_{i-1}, \tilde{c}_{i+1}, \ldots, \tilde{c}_n) = \min_{S\subseteq N\setminus\{i\}} \sum_{j\in N\setminus\{i\}} \tilde{c}_j(S)$. The payment to i under this rule and that in Mechanism 1 can both be regarded as i's marginal contribution to the social cost of other agents. The key difference lies on that under Clarke's rule, $h_i(\tilde{c}_1, \ldots, \tilde{c}_{i-1}, \tilde{c}_{i+1}, \ldots, \tilde{c}_n)$ is the minimum social cost of others provided i is absolutely absent from the problem instance, while in Mechanism 1 it is the total cost of others in a best facility solution that does not contain i (Note that here "best" corresponds to the social cost with all players).

Time complexity

The time complexity of computing payments entirely relies on the selection function. The brute-force search mentioned at the beginning of this subsection takes an exponentially long period of time, and is intolerable in practice. In fact we should not expect to find an optimal solution in polynomial time, as the UFL in a general metric space is a well-known NP-hard problem.

On the other hand, the UFL admits positive results in some special cases. For example, the line is a widely studied metric space for facility location games, because it models a street in reality. When all agents lie on a tree and the distances are given by the path lengths, Shah and Farach-Colton [22] provides an $O(n^2)$-time dynamic programming algorithm for finding an optimal solution of the UFL. As the algorithm (when denoted as s) satisfies (2), we can apply it to Mechanism 1.

Corollary 4.2. *For the dual-role facility location game in a tree space, there is a truthful mechanism that is optimal for the social-cost objective, and runs (i.e., finds the winner set and payments) in polynomial time.*

Alternatively, we can impose restrictions on the cardinality of the winner sets in a way that at most k facilities can be open. Then, the selection function required by Mechanism 1 becomes an algorithm for the k-facility location problem instead of the UFL. When k is a constant, we can implement an exhaustive search, and let the selection function output the optimal solution of the k-facility location problem with the lowest index.

Corollary 4.3. *Let k be a positive constant. For the dual-role facility location game with the constraint that every winner set can contain at most k agents, there is a truthful mechanism that is optimal for the social-cost objective, and runs in polynomial time.*

4.2 Approximate Truthful Mechanism

In a general metric space, the exponential running time of an optimal mechanism is unacceptable, and we turn our attention to truthful mechanisms that approximately optimize the objective function in polynomial time. Recall from Theorem 3.4 that an algorithm admits a truthful mechanism if (and only if) it satisfies conditions (i) and (ii) in the theorem.

We are interested in the price of being truthful, that is, how much is the gap between the best approximation factors of algorithms and truthful mechanisms, both running in polynomial time. Dobzinski [6] provides an extended multi-auction setting, in which deterministic polynomial-time truthful mechanisms cannot guarantee any bounded approximation ratio, but a non-truthful FPTAS exists. On the other hand, Dhangwatnotai *et al.* [5] prove that for a minimum makespan scheduling problem, there are a randomized monotone PTAS and a deterministic monotone quasi-PTAS, where monotonicity can guarantee the truthfulness. These related works show that the gap in our concern may be arbitrarily large or small, depending on different problem settings.

As far as the UFL is concerned, Jain *et al.* [12] give a 1.61-approximate algorithm, referred to as *JMS Algorithm*. Combing it with cost scaling, the algorithm of Mahdian *et al.* [16] achieves the best-known deterministic approximate-ratio 1.52. Next, we state JMS Algorithm, and show that it admits a truthful mechanism; then we remark that why the 1.52-approximation algorithm [16] cannot be extended to a truthful mechanism.

JMS Algorithm [12]

Phase 1. At the beginning, all agents (facilities) are *unconnected* (*unopened*). Set $\alpha_j = 0$ for every agent j. At every moment, each agent j offers some money to each unopened facility i. The amount of this offer is equal to $\max\{\alpha_j - d(i,j), 0\}$ if j is unconnected, or $\max\{d(i',j) - d(i,j), 0\}$ if j has already been connected to some other facility i' (that has been opened).

Phase 2. While there is an unconnected agent, increase the parameter α_j of each unconnected agent j at the same rate, until one of the following two events occurs:

- For some unopened facility i, the total offer he receives is equal to his opening cost f_i. In this case, we open facility i, and connect j to i for every agent j (connected or unconnected) which has a non-zero offer to i.
- For some unconnected agent j, and some facility i that has already been opened, $\alpha_j = d(i,j)$. In this case, we connect j to i.

JMS algorithm runs in time $O(n^3)$, and the corresponding threshold values are polynomially computable.

Theorem 4.4. *JMS algorithm can induce a* 1.61*-approximate efficiently-computable truthful mechanism.*

Proof. Take JMS algorithm as the selection function s. It is easy to see that s is monotone, satisfying condition (i) of Theorem 3.4. To prove the truthfulness, we verify that s satisfies (2).

Suppose that agent i bids $b_i \geq r_i(\mathbf{b}_{-i})$. Then facility i will not be unopened. In the algorithm, the increasing process of all parameters α_j will stop at a time point t before the total offer that agent i receives reaches $r_i(\mathbf{b}_{-i})$, and all the connections of agents and opening of facilities have been determined at this time point t. Note that t has the same value for all $b_i \geq r_i(\mathbf{b}_{-i})$, which implies $|\mathcal{S}_i(\mathbf{b}_{-i})| = 1$. □

The best-known 1.52-approximation deterministic algorithm, however, does not satisfy condition (ii). It utilizes a cost scaling technique, and the solution is sensitive to the scaled-up or scaled-down (reported) opening costs. So the current gap between the approximation ratios of polynomial-time algorithms and truthful mechanisms is $1.61 - 1.52 = 0.09$. This is a quite small value, indicating that for the dual-role facility location game the requirement of truthfulness produces little influences on the approximation ratios.

5 BOTTLENECK COST

In this section, we study the bottleneck-cost objective function $C_B(W) = \max_{i\in N} c_i(W)$, instead of the social cost. This objective partly means that neither a large opening cost nor a large service cost is welcome. Most of the conclusions in the previous sections adapt to this bottleneck objective. First, the characterization of truthfulness in Theorem 3.4 does not depend on the objective function. So we can design algorithms that satisfy conditions (i) and (ii), which implies truthfulness. The difference is that we can exactly solve this minimum bottleneck-cost problem in polynomial time, which stands in contrast to the NP-hardness of the UFL problem. The following mechanism consists of the exact algorithm and its corresponding payments.

Mechanism 2. Sort the bids in nondecreasing order: $b_1 \le b_2 \le \cdots \le b_n$, renaming if necessary. Set $i := 1, W := \{1\}$.

(i) **while** $b_{i+1} \le \max\{\max_{j\in N} d(j, W), b_i\}$ **do**
$W := W \cup \{i + 1\}$; $i := i + 1$;
The winner set $s(\mathbf{b})$ is $W^* := W$.

(ii) For each $i \in W$, let r_i^* be the optimal value of the program: minimize r, subject to $\max_{j\in N} d(j, S_r) \le r$, where $S_r = \{j \mid b_j \le r, j \in N \setminus \{i\}\}$; the payment to i is $p_i(\mathbf{b}) := r_i^* - d(i, S_{r_i^*})$. For each $i \notin W$, $p_i(\mathbf{b}) = 0$.

Lemma 5.1. *For any bid vector* $\mathbf{b}$*, the selection algorithm s in Mechanism 2 finds an optimal winner set* $W^* \in \arg\min_{W\subseteq N} \max_{i\in N}\{I_W(i) \cdot b_i + d(i, W)\}$.

Proof. In view of the nondecreasing ordering of bids, we may suppose that the output winner set $W^* = \{1, \ldots, i\}$. For every $S \subseteq N$, define $\tilde{C}_B(S) := \max_{i\in N}\{I_S(i)\cdot b_i+d(i, S)\}$. By contradiction, suppose that $S(\subseteq N)$ has a lower bottleneck-cost $\tilde{C}_B(S) < \tilde{C}_B(W^*)$. Note that $b_i \le \tilde{C}_B(W^*) < b_{i+1}$, enforcing $S \subseteq W^*$. It follows from $\tilde{C}_B(W^*) > \tilde{C}_B(S) \ge \max_{j\in N} d(j, S) \ge \max_{j\in N} d(j, W^*)$ that $\tilde{C}_B(W^*) = b_i > \max_{j\in N} d(j, \{1, \ldots, i\})$.

At each iteration of the algorithm we add one agent to W, maintaining $\tilde{C}_B(W)$ and $\max_{j\in N} d(j, W)$ nonincreasing. If $b_1 > \max_{j\in N} d(j, \{1\})$, then $\tilde{C}_B(W^*) = b_1 \ge \tilde{C}_B(S)$ gives a contradiction. So we take the largest $h(\le i)$ such that $\max_{j\in N} d(j, \{1, \ldots, h\}) \ge b_h$, which combined with $b_i > \max_{j\in N} d(j, \{1, \ldots, i\})$ implies $b_{h+1} > \max_{j\in N} d(j, \{1, \ldots, h+1\})$ and $b_h \le b_{h+1} = \cdots = b_i \le \max_{j\in N} d(j, \{1, \ldots, h\})$ since the iteration stops at step i. As $\tilde{C}_B(S) < b_i$, it follows that $S \subseteq \{1, \ldots, h\}$ and $\tilde{C}_B(S) \ge \max_{j\in N} d(j, \{1, \ldots, h\}) \ge b_i$, a contradiction. □

Lemma 5.2. *The threshold value of winner* $i \in s(\mathbf{b})$ *is* r_i^*.

Proof. We discuss the outcome when agent i bids r below or above r_i^*. When $r < r_i^*$, it cannot be a feasible solution of the programm, that is, $r < \max_{j\in N} d(j, S_r)$. According to the selection algorithm s, i must be added to the winner set $s(r, \mathbf{b}_{-i})$. When $r > r_i^*$, the optimal objective value r^* of the program is actually the minimum bottleneck cost of a solution that does not contain i . It follows from the optimality of the selection algorithm (Lemma 5.1) that $i \notin s(r, \mathbf{b}_{-i})$. □

Theorem 5.3. *Mechanism 2 for the dual-role facility location game is normalized, truthful, individual rational, optimal for the bottleneck-cost objective, and runs in polynomial time.*

Proof. The normality and polynomial-time efficiency is clear from the context. The optimality has been verified in Lemma 5.1. To see the truthfulness, we show that conditions (i) – (iii) in Theorem 3.4 are satisfied. By the process of the selection algorithm s, s is clearly monotone, giving condition (i), and no single loser can change the winner set by unilaterally increasing his bid, yielding condition (ii). Combining this with Lemma 5.2 (which implies condition (iii)), we deduce from Theorem 3.4 that Mechanism 2 is truthful. As in the proof of Theorem 4.1, individual rationality follows from the fact that every winner's threshold value is at least his bid. □

6 BUDGETED GAMES

In this section, we study the game with a budget constraint. Given a budget $B \in \mathbb{R}_+$ (suppose $B \ge \min_{i\in N}\{f_i\}$), a truthful mechanism is *budget feasible* if its total payment to agents does not exceed B. Payments could be considered as "compensations" for agents to open their facilities. To measure the performance of a budget-feasible mechanism for the social-cost objective, we compare it to the optimum of the UFL under the budget constraint of the total opening cost being at most B. Formally, a budget-feasible mechanism $\mathcal{M}$ is *pseudo-optimal* (resp. *pseudo-α-approximate*) if for any instance, the social cost of the outcome by $\mathcal{M}$ equals (resp. is no more than α times) the optimal objective value of the programm: $\min_{W\subseteq N} C(W)$ subject to $\sum_{i\in W} f_i \le B$. The counterparts for the bottleneck-cost objective are defined analogously, with $C_B(\cdot)$ in place of $C(\cdot)$.

Social-cost objective

We show that, however, the additional requirement of budget feasibility almost excludes possibility of acceptable pseudo-approximation ratios. The high-level idea is: Consider the instance where the optimal solution opens at least two facilities with opening costs far less than the budget, while all suboptimal solutions have unacceptable social costs. Then the threshold value of each winner is large, which may come up to an exceeded total payment.

Theorem 6.1. *Every truthful and budget-feasible mechanism has a pseudo-approximation ratio of* $\Omega(n)$.

Proof. Consider an instance with an even number of agents represented by nodes in the tree network depicted in Figure. 1, where agents (nodes) 1 and 2 are neighbors, both having degree $\frac{n}{2}$, all other $n - 2$ agents (nodes) are leaves; the edge between agents 1 and 2 has length $L = \frac{B}{8}$, and all other $n - 2$ (pendent) edges each having a very small length ϵ. The distances between agents are defined as the lengths of the shortest paths between them. We show that for the opening cost profile $\mathbf{f} = (\delta, \delta, 2B, \ldots, 2B)$ with a very small δ, any truthful mechanism $\mathcal{M}$ with pseudo-approximation ratio α less than $\Theta(n)$ cannot be budget feasible.

First, with any bids, a truthful mechanism which selects a facility with opening cost $2B$ must violate the budget, since its threshold is at least $2B$ (at $2B$, he still wins), and the total payment is at least $2B - 3L > B$ (agent 1 and 2 may have payment $-L$). Second, under $\mathbf{f}$, the optimal solution is $W = \{1, 2\}$ with social cost approximately 0, and in turn, $\alpha < \Theta(n)$ enforces that $\mathcal{M}$ must output W as the winner set.

Now we study the threshold, and consider the bid vector $\mathbf{b} = (\frac{3B}{4}, \mathbf{f}_{-1})$. We have $\sum_{i\in N} d(i, W)+\sum_{i\in W} b_i \approx \frac{3B}{4}$, while the other two budget-feasible solutions $\{1\}$ and $\{2\}$ of the UFL have social costs at least $\frac{nL}{2} = \frac{nB}{16}$. Thus under $\mathbf{b}$, $\alpha < \Theta(n)$ dictates again the selection W. Therefore agent 1 wins under $\mathbf{b}$ and his threshold value w.r.t. $\mathbf{f}_{-1}$ is at least $\frac{3B}{4}$. Similarly, agent 2 has a threshold at least $\frac{3B}{4}$ w.r.t. $\mathbf{f}_{-2}$. Then by payment formula (1), $\mathcal{M}$ should pay at least $\frac{3B}{4} - L = \frac{5B}{8}$ to each agent 1 and 2. At this case, the total payment exceeds the budget B. It is a contradiction. □

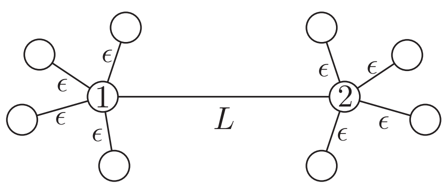

Figure 1: A tree network for the lower bound.

From the above we may find that, even if the government plans to open at most two facilities and within the budget, a corresponding truthful and budget-feasible mechanism still has a pseudo-approximation ratio at least $\Theta(n)$. Next, we show that if only one facility is allowed to open, there is an optimal truthful and budget feasible mechanism.

Mechanism 3. Given input: bid vector $\mathbf{b} = (b_1, \ldots, b_n)$ and budget B, for each agent $i \in N$, define $\tilde{C}(\{i\}) = \sum_{j\in N} d(j, i) + b_i$. Sort all m agents with bids no more than B as $\tilde{C}(\{1\}) \leq \tilde{C}(\{2\}) \leq \cdots \leq \tilde{C}(\{m\})$, breaking the tie arbitrarily and renaming if necessary. Then

- If $m = 0$, the selection function gives $s(\mathbf{b}) = \emptyset$, and there is no payment. Else, $s(\mathbf{b}) = \{1\}$.
- If $m = 1$, the payment to agent 1 is $p_1(\mathbf{b}) = B - Q$. If $m \geq 2$, $p_1 = \min\{\tilde{C}(\{2\}) - \tilde{C}(\{1\}) + b_1, B\} - d(1, 2)$. For each other agent, the payment is 0.

Theorem 6.2. *If at most one facility is allowed to open, Mechanism 3 is an optimal truthful and budget feasible mechanism, running in polynomial time.*

Proof. To show the truthfulness, it suffices to verify the payment is defined as (1), since the selection rule satisfies both condition (i) and (ii) in Theorem 3.4. When $m = 1$, we have $\mathcal{S}_1(b_{-1}) = \{\emptyset\}$, and the payment should be $r_1(\mathbf{b}_{-1})-d(1, \emptyset) = B - Q$. When $m \geq 2$, agent 1 is selected if and only if his bid does not exceed B and $\tilde{C}(\{2\}) + b_1 - \tilde{C}(\{1\})$, otherwise the mechanism will select agent 2. So the threshold value is $\min\{\tilde{C}(\{2\}) + b_1 - \tilde{C}(\{1\}), B\}$.

The optimality, budget feasibility and polynomial computability are all obvious. It remains to show the individual rationality. The case $m \leq 1$ is trivial. Suppose $m \geq 2$ and agent 1 rejects to participate the game, then his utility changes from $p_1 - b_1$ to $-d(1, 2)$. As $B \geq b_1$ and $\tilde{C}(\{2\}) \geq \tilde{C}(\{1\})$, we have $p_1 - b_1 \geq -d(1, 2)$, giving the individual rationality. □

Bottleneck-cost objective

Generally, with the budget constraint, we can find a lower bound 1.5 for the approximation ratio of any truthful and budget feasible mechanisms for the dual-role facility location game with bottleneck-cost objective, by a similar instance in Theorem 6.1. To find an upper bound needs more techniques, for which we have no idea so far. When we are allowed to open only one facility, there is an optimal and budget-feasible mechanism: Define $\tilde{C}_B(\{i\}) = \max\{\max_{j\in N} d(j, i), b_i\}, i \in N$, and modify Mechanism 3 by replacing $\tilde{C}(\cdot)$ with $\tilde{C}_B(\cdot)$. The optimality, truthfulness and budget feasibility can be easily checked, similar to the proof of Theorem 6.2.

7 CONCLUSIONS

Chen *et al.* [2] study designing budget-feasible mechanisms for maximizing monotone submodular function. They provide a 7.91-approximate randomized algorithm and a 8.34-approximate deterministic algorithm by a greedy scheme, according to marginal contributions relative to cost. In this paper, the objective function $C(\cdot)$ is supermodular, and it seems natural to ask what would happen if we turn our objective into maximizing an appropriate constant minus $C(\cdot)$, which is a submodular set function. This function, however, is not monotone, as generally the number of facilities opened should be moderate. In view of this, the greedy scheme is no longer applicable here.

This paper initiates the study of single parameter problems where an agent incurs some cost even it loses, and we believe this can be extended to many other interesting topics such as reverse auction, especially combined with budget. In terms of facility location games, different from previous works which mainly consider non-money case, our study provides results with payment, and it can lead to various studies with respect to strategic customers or facilities.

ACKNOWLEDGMENTS

Xujin Chen is supported in part by NNSF of China under Grant No. 11531014 and 11571258. Minming Li is supported by NNSF of China under Grant No. 11771365, and sponsored by Project No. CityU 11200518 from Research Grants Council of HKSAR. Changjun Wang is supported by NNSF of China under Grant No.11601022 and by YESS Program of CAST under Grant No. 2018QNRC001.

REFERENCES

[1] Aaron Archer and Éva Tardos. 2001. Truthful mechanisms for one-parameter agents. In *Foundations of Computer Science, 2001. Proceedings. 42nd IEEE Symposium on.* IEEE, 482–491.

[2] Ning Chen, Nick Gravin, and Pinyan Lu. 2011. On the approximability of budget feasible mechanisms. In *Proceedings of the twenty-second annual ACM-SIAM symposium on Discrete Algorithms.* Society for Industrial and Applied Mathematics, 685–699.

[3] Xujin Chen, Xiaodong Hu, Xiaohua Jia, Minming Li, Zhongzheng Tang, and Chenhao Wang. 2018. Mechanism Design for Two-Opposite-Facility Location Games with Penalties on Distance. In *Algorithmic Game Theory - 11th International Symposium, SAGT 2018, Proceedings.* 256–260.

[4] Edward H Clarke. 1971. Multipart pricing of public goods. *Public choice* 11, 1 (1971), 17–33.

[5] Peerapong Dhangwatnotai, Shahar Dobzinski, Shaddin Dughmi, and Tim Roughgarden. 2011. Truthful approximation schemes for single-parameter agents. *SIAM J. Comput.* 40, 3 (2011), 915–933.

[6] Shahar Dobzinski. 2009. A note on the power of truthful approximation mechanisms. *arXiv preprint arXiv:0907.5219* (2009).

[7] Itai Feigenbaum and Jay Sethuraman. 2015. Strategyproof Mechanisms for One-Dimensional Hybrid and Obnoxious Facility Location Models.. In *AAAI Workshop: Incentive and Trust in E-Communities.*

[8] Michal Feldman and Yoav Wilf. 2013. Strategyproof facility location and the least squares objective. In *Proceedings of the fourteenth ACM conference on Electronic commerce.* ACM, 873–890.

[9] Jerry Green and Jean-Jacques Laffont. 1977. Characterization of satisfactory mechanisms for the revelation of preferences for public goods. *Econometrica: Journal of the Econometric Society* (1977), 427–438.

[10] Theodore Groves. 1973. Incentives in teams. *Econometrica: Journal of the Econometric Society* (1973), 617–631.

[11] S. Guha and S. Khuller. 1998. Greedy strikes back: improved facility location algorithms. 649–657.

[12] M. Evangelos A. Saberi K. Jain, M. Mahdian and V. Vazirani. 2003. Greedy facility location algorithms analyzed using dual fitting with factor-revealing LP. *Journal of the Acm* 50, 6 (2003), 795–824.

[13] Jean-Jacques Laffont and Eric Maskin. 1980. A differential approach to dominant strategy mechanisms. *Econometrica: Journal of the Econometric Society* (1980), 1507–1520.

[14] Pinyan Lu, Xiaorui Sun, Yajun Wang, and Zeyuan Allen Zhu. 2010. Asymptotically optimal strategy-proof mechanisms for two-facility games. In *Proceedings of the 11th ACM conference on Electronic commerce.* ACM, 315–324.

[15] Pinyan Lu, Yajun Wang, and Yuan Zhou. 2009. Tighter bounds for facility games. In *International Workshop on Internet and Network Economics.* Springer, 137–148.

[16] M. Mahdian, Y. Ye, and J. Zhang. 2002. Improved Approximation Algorithms for Metric Facility Location Problems. In *International Workshop on Approximation Algorithms for Combinatorial Optimization.* 229–242.

[17] Roger B Myerson. 1981. Optimal auction design. *Mathematics of operations research* 6, 1 (1981), 58–73.

[18] Noam Nisan and Amir Ronen. 2001. Algorithmic mechanism design. *Games and Economic behavior* 35, 1-2 (2001), 166–196.

[19] Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay V Vazirani. 2007. *Algorithmic game theory.* Cambridge University Press.

[20] Ariel D Procaccia and Moshe Tennenholtz. 2009. Approximate mechanism design without money. In *Proceedings of the 10th ACM conference on Electronic commerce.* ACM, 177–186.

[21] John G Riley and William F Samuelson. 1981. Optimal auctions. *The American Economic Review* 71, 3 (1981), 381–392.

[22] R. Shah and M. Farach-Colton. 2002. Undiscretized dynamic programming:faster algorithms for facility location and related problems on trees. In *Thirteenth Acm-Siam Symposium on Discrete Algorithms.* 108–115.

[23] Yaron Singer. 2010. Budget feasible mechanisms. In *Foundations of Computer Science (FOCS), 2010 51st Annual IEEE Symposium on.* IEEE, 765–774.

[24] Maxim Sviridenko. 2002. An Improved Approximation Algorithm for the Metric Uncapacitated Facility Location Problem. In *International IPCO Conference on Integer Programming and Combinatorial Optimization.* 240–257.

[25] William Vickrey. 1961. Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance* 16, 1 (1961), 8–37.

[26] Hongning Yuan, Kai Wang, Ken CK Fong, Yong Zhang, Minming Li, et al. 2016. Facility location games with optional preference. (2016).

[27] Peng Zhang. 2007. A new approximation algorithm for the k-facility location problem. *Theoretical Computer Science* 384, 1 (2007), 126–135.