A Fully Rational Argumentation System for Preordered Defeasible Rules

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ABSTRACT

Structured argumentation is a family of formal approaches for the handling of defeasible, potentially inconsistent information. Many models for structured argumentation distinguish between strict and defeasible inference rules. Defeasible rules often come with varying degrees of strength which is formally represented by a preorder over the defeasible rules. Various lifting principles have been presented in the literature to determine the relative strength of an argument by considering the strength of the defeasible rules used in its construction. The strength of arguments then comes into play when determining whether an attack (a purely syntactic relationship between arguments) results in a defeat (i.e. a successful attack). In [5, 22], several rationality postulates were proposed that serve as a measure to assess the normative rationality of structured argumentation formalisms. In [14], the first formalism satisfying all rationality postulates for structured argumentation when taking into account totally ordered defeasible rules was proposed. In many settings, assuming a total order greatly limits the realistic modelling capabilities of a formal system, e.g. when agents do not know the actual preferences of each rule or since different agents have different preferences over defeasible rules. Our paper shows that in the more general setting of preorders, violations of several rationality postulates can occur. We show how for a wide class of lifting principles, these violations can be avoided, resulting in the first Dung-based system that satisfies all four rationality postulates for preordered defeasible rule bases.

KEYWORDS

Non-monotonic Reasoning; Preferences; Formal Argumentation; Structured Argumentation

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1 INTRODUCTION

Structured argumentation is an approach to the formalization of defeasible reasoning with many fruitful applications in multi-agent systems [15, 20, 25]. It is often useful to distinguish between strict and defeasible inference rules. Defeasible rules guarantee the truth of their conclusion only provisionally: from the antecedents of the rules we can infer their conclusion unless and until we encounter feasible counterarguments. Strict rules, in contrast, are outside doubt: the truth of the antecedents is carried over to the conclusion. In structured argumentation, the concept of argumentative attack is used to give a formal explication of the fact that two arguments express conflicting information. When constructing arguments with defeasible rules, it seems sensible that whenever an argument a concludes the contrary of a defeasible argument b, a should be allowed to attack b. The attack form known as rebuttal does exactly this. One can either allow for unrestricted rebuttal or restrict the reach of a rebuttal. In a framework allowing for unrestricted rebuttals, such as ASPIC$^-$ [7], any defeasible argument can be rebutted. This contrasts sharply with more restricted notions of rebuttal to arguments only the last link of which is defeasible, as found e.g. in the ASPIC$^+$-framework [18]. In [7], it has been argued that, at least in a dialectical context, unrestricted rebut is more intuitive than restricted rebut. Recent empirical research [26] supports this claim.

Defeasible information often comes in varying degrees of strength. This feature of defeasible reasoning is represented formally by a preorder over the defeasible rules. Various lifting principles have been presented in the literature to determine the relative strength of an argument by looking at the strength of the defeasible rules used in the construction of the argument. The strength of arguments then comes into play when determining whether an attack (a relationship between arguments based solely on logical form) results in a defeat (i.e. a successful attack). It is common to require that an argument a can only defeat an argument b if a is not weaker then b.

To facilitate the study of such structured argumentation systems, [5] proposed several postulates the output of any sensible argumentation system should satisfy. For example, it seems reasonable to require that the output of an argumentation system is consistent. Likewise, the output of an argumentation system should be closed under strict rules.

Strict rules can be based on some kind of deductive system, like classical logic. When using sufficiently strong deductive systems such as classical logic, however, one needs to be wary of problems that are caused by rules such as *ex falso quodlibet*: this may cause two syntactically disjoint argumentation systems to interact in undesirable ways. The absence of such problems has been labelled Crash Resistance in [22]. A violation of crash resistance can render an argumentation system ineffective since given conflicting defeasible rules, the conflict can spread to unrelated, innocent bystanders and thus contaminate the whole output. This seems to defeat the purpose of structured argumentation frameworks, since it is meant to give us a sensible output *especially* in the case of conflicting but defeasible information.

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ASPIC$^-$, a system allowing for unrestricted rebuts, was shown to violate crash-resistance in [14]. This shortcoming was remedied by generalizing the attack rule of unrestricted rebut. The resulting system ASPIC$^\ominus$ was shown to satisfy all the usual rationality postulates for totally ordered prioritized rule bases while retaining the intuitiveness of unrestricted rebuttal. In fact, to the best of our knowledge, ASPIC$^\ominus$ is the only system for which all rationality postulates were proven to hold when taking into account preferences and allowing for defeasible rules. In [14], the rationality postualtes were only proven for total orders. In the paper, we will generalize this result for any preorder over the defeasible rules.

2 THE ASPIC-FAMILY

A well-known, general and popular family of frameworks for structured argumentation is the ASPIC-family. In ASPIC arguments are constructed using an argumentation system.[1]

Definition 2.1. An *Argumentation System* AS is a tuple $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ consisting of:

(1) a formal language $\mathcal{L}$ based on a set of atoms $\mathcal{A}$
(2) a set of strict rules $\mathcal{S} \subseteq 2^{\mathcal{L}} \times \mathcal{L}$ of the form $A_1, \ldots, A_n \to B$
(3) a set of defeasible rules $\mathcal{D} \subseteq 2^{\mathcal{L}} \times \mathcal{L}$ of the form $A_1, \ldots, A_n \Rightarrow B$.
(4) a contrariness function $\overline{} : \wp_{\text{fin}}(\mathcal{L}) \to \mathcal{L}$ mapping finite subsets of the language $\wp_{\text{fin}}(\mathcal{L})$ to $\mathcal{L}$.[2]
(5) an $\mathcal{S}$-consistent[3] set of strict premises $\mathcal{K} \subseteq \mathcal{L}$.
(6) a preorder $\leq$ over $\mathcal{D}$.

$A_1, \ldots, A_n$ are called the antecedents and B is called the consequent of $A_1, \ldots, A_n \to B$ resp. $A_1, \ldots, A_n \Rightarrow B$.

Definition 2.2. Let $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ be an argumentation system. An argument a is one of the following:

(1) $a = \langle A \rangle$ where $A \in \mathcal{K}$
we let $\mathsf{conc}(a) = A$, $\mathsf{Sub}(a) = \{a\}$, $\mathsf{DefR}(a) = \emptyset$
(2) $a = \langle a_1, \ldots, a_n \to B \rangle$ where $a_1, \ldots a_n$ (with $n \geqslant 0$) are arguments such that $\mathsf{conc}(a_1), \ldots \mathsf{conc}(a_n) \to B \in \mathcal{S}$
we let $\mathsf{conc}(a) = B$, $\mathsf{Sub}(a) = \{a\} \cup \bigcup_{i=1}^n \mathsf{Sub}(a_i)$, $\mathsf{DefR}(a) = \bigcup_{i=1}^n \mathsf{DefR}(a_i)$.
(3) $a = \langle a_1, \ldots, a_n \Rightarrow B \rangle$ where $a_1, \ldots a_n$ (with $n \geqslant 0$) are arguments such that $\mathsf{conc}(a_1), \ldots \mathsf{conc}(a_n) \Rightarrow B \in \mathcal{D}$
we let $\mathsf{conc}(a) = B$, $\mathsf{Sub}(a) = \{a\} \cup \bigcup_{i=1}^n \mathsf{Sub}(a_i)$, $\mathsf{DefR}(a) = \bigcup_{i=1}^n \mathsf{DefR}(a_i) \cup \{\mathsf{conc}(a_1), \ldots \mathsf{conc}(a_n) \Rightarrow B\}$.

By Arg(AS) we denote the set of arguments that can be built from AS. An argument a will be called *defeasible* if $\mathsf{DefR}(a) \neq \emptyset$ and *strict* otherwise. We lift DefR to sets of arguments as usual: $\mathsf{DefR}(\{a_1, \ldots, a_n\}) = \bigcup_{i=1}^n \mathsf{DefR}(a_i)$. We furthermore define $\mathsf{C}(a) = \{\mathsf{conc}(b) \mid b \in \mathsf{Sub}(a)\}$ to be the set of the conclusions of subarguments of a. Finally, an argument is $\mathcal{S}$*-consistent* iff $\mathsf{C}(a)$ is $\mathcal{S}$-consistent. If an argument is not $\mathcal{S}$-consistent, it is $\mathcal{S}$*-inconsistent.*

[1]In this chapter we will, due to spatial restrictions, omit several features of the original ASPIC$^+$ framework of [18], such as defeasible premises, issues, undercutting and undermining attacks.

[2]In the context of ASPIC$^+$ usually $\overline{}$ associates formulas with a set of contrary formulas. To simplify the presentation we opt here for the simpler variant where each set of formulas is associated with a unique contrary formula. We will motivate the shift on the left hand side from formulas to finite sets of formulas below.

[3]A set of formulas Γ is $\mathcal{S}$-inconsistent iff $\overline{\Gamma'}$ is derivable via the strict rules $\mathcal{S}$ from no assumptions for some $\Gamma' \subseteq \Gamma$. If Γ is not $\mathcal{S}$-inconsistent it is $\mathcal{S}$*-consistent.*

Example 2.3. The paradigmatic example for generating a set of strict rules $\mathcal{S}_{\text{CL}}$ by an underlying logic is to use classical logic **CL**:

$$A_1, \ldots, A_n \to A \in \mathcal{S}_{\text{CL}} \text{ iff } \{A_1, \ldots, A_n\} \vdash_{\text{CL}} A$$

Contrariness is defined by $\overline{A} = \neg A$. We will use this system as a guiding example throughout this chapter.

We now give an example of a specific argumentation system using the strict rule base $\mathcal{S}_{\text{CL}}$. Let $AS_1 = (\mathcal{L}, \mathcal{S}_{\text{CL}}, \mathcal{D}_1 = \{\top \Rightarrow_2 \neg p \vee \neg q, \top \Rightarrow_1 p, p \Rightarrow_1 q\}, \emptyset, \overline{}, \leq)$. In this and the following examples, the subscripts of $\Rightarrow$ are used to express the priority ordering over $\mathcal{D}$, i.e. $(A_1, \ldots, A_n \Rightarrow_i B) \leq (A'_1, \ldots, A'_m \Rightarrow_j B')$ iff $i \leq j$. Here are some arguments in $\text{Arg}(AS_1)$:

a_1: $\top \Rightarrow_2 \neg p \vee \neg q$ a_2: $\top \Rightarrow_1 p$ a_3: $a_2 \Rightarrow_1 q$
a_4: $a_1, a_2 \to \neg q$ a_5: $a_2, a_3 \to p \wedge q$ a_6: $a_1, a_3 \to \neg p$

2.1 Attacks and Defeats

In structured argumentation, the concept of argumentative attack is used to give a formal explication of the fact that two arguments express conflicting information. When constructing arguments with defeasible rules, it seems sensible that whenever an argument a concludes the contrary of a defeasible argument b, we would like a to attack b. The attack form known as rebuttal does exactly this. There are at least two roads one can take when giving formal substance to this attack form: to allow for *unrestricted* rebut or to allow only for *restricted* rebut. In a framework allowing for unrestricted rebuttals *any* defeasible argument can be rebutted. This contrasts sharply with more restricted notions of rebuttal to arguments only the last link of which is defeasible, as found e.g. in the ASPIC$^+$-framework [16, 17]. In [7], it has been argued that, at least in a dialectical context, unrestricted rebut is more intuitive than restricted rebut. We do not aim to have the final word on this matter, but shall assume that at least in some context, unrestricted rebut is an intuitive attack form. In this paper, following [14], we use a generalized form of unrestricted rebut called *generalized rebut*, which allows for an argument to attack another one if its conclusion claims that a subset of the commitments of the attacked argument are not tenable together. This attack form is the reason that in Definition 2.1, the contrariness operator $\overline{} : \wp_{\text{fin}}(\mathcal{L}) \setminus \emptyset \to \mathcal{L}$ was defined with finite sets of formulas as a domain. This can be contrasted with most other formalisms for structured argumentation where the contrary is function that maps single formulas to either other formulas or sets of formulas. This difference can be easily explained since in most formalisms for structured argumentation, rebut is *pointed* in the sense that one has to construct an argument concluding the contrary of a *single* assumption.

Example 2.4. In the context of classical logic, one may express $\overline{\{A_1, \ldots, A_n\}}$ by means of disjunction $\bigvee_{i=1}^n \overline{A_i}$ or by means of conjunction $\overline{\bigwedge_{i=1}^n A_i}$.

Below, it will prove convenient to define $\mathsf{C}(a) =_{\text{df}} \{\mathsf{conc}(b) \mid b \in \mathsf{Sub}(a)\}$ to be the set of the conclusions of subarguments of a.

Definition 2.5. Where $a, b \in \text{Arg(AS)}$ and $\Delta \subseteq \mathsf{C}(b)$:
a *gen-rebuts* b ($a\mathsf{GeRe}b$) iff b is defeasible and $\mathsf{conc}(a) = \overline{\Delta}$.

When two arguments conflict, one of the arguments may defeat the other due to its higher priority. To account for defeat via priorities the *weakest link* [16] lifting was considered in [14]:

Definition 2.6. Given $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ and $a, b \in \mathsf{Arg(AS)}$, $a \leq_{wl} b$ iff $\mathsf{DefR}(b) = \emptyset$ or there is an $\alpha \in \mathsf{DefR}(a)$ such that for every $\beta \in \mathsf{DefR}(b)$: $\alpha \leq \beta$.

Remark 1. *Given a relation $\leq$ over* $\mathsf{Arg(AS)}$, *we define $a \prec b$ iff $a \leq b$ and $b \not\leq a$. Furthermore, $a \not\prec b$ if it is not the case that $a \prec b$.*

Definition 2.7. Where $a, b \in \mathsf{Arg(AS)}$ and $\prec \subseteq \mathsf{Arg(AS)} \times \mathsf{Arg(AS)}$: a *defeats* b iff there is a $c \in \mathsf{Sub}(b)$ such that $a\mathsf{GeRe}c$ and $c \not\prec a$. We write $(a, b) \in \mathsf{GeRe}_{\preceq}(\mathsf{Arg(AS)})$.[4]

Where $\mathsf{Arg(AS)}$ is clear from the context, we will often just write $\mathsf{GeRe}_{\preceq}$ instead of $\mathsf{GeRe}_{\preceq}(\mathsf{Arg(AS)})$.

2.2 Grounded Semantics

Definition 2.8. An *argumentation framework* (AF) for an argumentation system AS is the pair $(\mathsf{Arg(AS)}, \mathsf{GeRe}_{\preceq}(\mathsf{Arg(AS)}))$.

Given an AF, we can apply Dung's acceptability semantics [9] for evaluating arguments.

Definition 2.9. Let $AF = (\mathsf{Arg(AS)}, \mathsf{GeRe}_{\preceq})$, $\mathcal{A} \subseteq \mathsf{Arg(AS)}$ and $a \in \mathsf{Arg(AS)}$. a is *acceptable w.r.t.* $\mathcal{A}$ (or, $\mathcal{A}$ defends a) iff for all b such that $(b, a) \in \mathsf{GeRe}_{\preceq}$ there is a $c \in \mathcal{A}$ such that $(c, b) \in \mathsf{GeRe}_{\preceq}$. $\mathsf{Acc}(\mathcal{A})$ denotes the set of all acceptable arguments w.r.t. $\mathcal{A}$. $\mathcal{A}$ is *conflict-free* iff for all $a, b \in \mathcal{A}$, $(a, b) \notin \mathsf{GeRe}_{\preceq}$. $\mathcal{A}$ is a *complete extension* iff it is conflict-free and $\mathsf{Acc}(\mathcal{A}) = \mathcal{A}$. The minimal complete extension is the *grounded extension*, written $\mathcal{G}(AF)$.

Remark 2. *In [9] it was shown that the grounded extension can alternatively be defined as the least fixed point of* Acc.

We define a consequence relation based on the grounded extension[5] for AFs as follows.

Definition 2.10. Where $AF = (\mathsf{Arg(AS)}, \mathsf{GeRe}_{\preceq})$ is an AF for AS. $AS \mid\!\sim^{\mathsf{GeRe}}_{\preceq} A$ iff there is an argument $a \in \mathcal{G}(AF)$ with $\mathsf{conc}(a) = A$.

3 RATIONALITY POSTULATES

In [5, 6] desirable properties for argumentation-based consequence relations $\mid\!\sim$ are defined:

Postulate 1. $\mid\!\sim$ *satisfies* Direct Consistency *for an argumentation system* AS *if there is no $A \in \mathcal{L}$ for which $AS \mid\!\sim A$ and $AS \mid\!\sim \overline{A}$.*

Postulate 2. $\mid\!\sim$ *satisfies* Closure *for an argumentation system* AS, *if whenever $AS \mid\!\sim A_i$ for $1 \leq i \leq n$ and B follows via the strict rules of AS from $\{A_1, \ldots, A_n\}$, then also $AS \mid\!\sim B$.*

Postulate 3. $\mid\!\sim$ *satisfies* Indirect Consistency *for an argumentation system* AS *with strict rules $\mathcal{S}$, if for all $A_1, \ldots, A_n$ such that $AS \mid\!\sim A_i$ for $1 \leq i \leq n$, $\{A_1, \ldots, A_n\}$ is $\mathcal{S}$-consistent.*

Where $\Delta \subseteq \mathcal{L}$, let $\mathsf{Atoms}(\Delta)$ be the set of all atoms occurring in Δ. Furthermore, where $\Delta \cup \Delta' \subseteq \mathcal{L}$, Δ *is irrelevant to* Δ' iff $\mathsf{Atoms}(\Delta) \cap \mathsf{Atoms}(\Delta') = \emptyset$. A set of defeasible rules and formulas Δ is irrelevant to a set of rules and formulas Δ' iff $\{A_1, \ldots, A_n, B \mid A_1, \ldots, A_n \Rightarrow B \in \Delta\} \cup \{A \in \mathcal{L} \mid A \in \Delta\}$ is irrelevant to $\{A_1, \ldots, A_n, B \mid A_1, \ldots, A_n \Rightarrow B \in \Delta'\} \cup \{A \in \mathcal{L} \mid A \in \Delta'\}$.

Postulate 4. $\mid\!\sim$ *satisfies* Non-Interference *(for a class of argumentation systems Ξ) if for any two argumentation systems $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ and $AS' = (\mathcal{L}, \mathcal{S}, \mathcal{D}', \mathcal{K}', \overline{}, \leq')$ (in Ξ), where $\mathcal{K} \cup \mathcal{K}'$ is $\mathcal{S}$-consistent, $\mathcal{D} \cup \mathcal{K}$ is irrelevant $\mathcal{D}' \cup \mathcal{K}'$ and there is a $\leq^+$ such that $\leq$ [$\leq'$] is the restriction of $\leq^+$ to $\mathcal{D} \times \mathcal{D}$ [$\mathcal{D}' \times \mathcal{D}'$], we have: $AS' \mid\!\sim A$ iff $AS^+ \mid\!\sim A$ where $AS^+ = (\mathcal{L}, \mathcal{S}, \mathcal{D} \cup \mathcal{D}', \mathcal{K} \cup \mathcal{K}', \overline{}, \leq^+)$ and* $\mathsf{Atoms}(A) \subseteq \mathsf{Atoms}(AS')$.[6]

4 SATISFACTION FOR RATIONALITY POSTULATES FOR TOTAL ORDERS

When proving the rationality postulates for ASPIC$^{\ominus}$, [14] assumed that the set of strict rules $\mathcal{S}$ of a given AS satisfies Transposition (**T**), Resolution (**R**) and Cut (**C**) (where $\Delta' \subseteq \Delta \in \wp_{\mathsf{fin}}(\mathcal{L})$ and $\Theta' \subseteq \Theta \in \wp_{\mathsf{fin}}(\mathcal{L})$ are sets of formulas):

T: $(\Delta \setminus \Delta') \cup \Theta' \to \overline{(\Theta \setminus \Theta') \cup \Delta'}$ if $\Delta \to \overline{\Theta}$.
R: $\Delta \cup \overline{\Delta \cup \Theta} \to \overline{\Theta}$.[7]
C: $\Delta \cup \Theta \to A$ if $\Delta \cup \{D\} \to A$ and $\Theta \to D$.

[14] supposed furthermore that there is a conjunction symbol in the language that works in the usual way: e.g., $\Delta \to A$ iff $\bigwedge \Delta \to A$; $\Delta \to \bigwedge \Delta$ and $\bigwedge \Delta \to A$ where $A \in \Delta$ are available rules.

The generality of these requirements ensure that the framework of [14] can be instantiated by a broad class of rule bases. E.g., a wide variety of Tarski consequence relation, such as **CL**, intuitionistic logic and many modal logics, can be used to generate a set of strict rules. Likewise, closing a set of domain specific rules under the three above defined properties generates such a strict rule base. Note that Transposition was already required in e.g. [18] and [7].

Example 4.1. For the instantiation $\mathcal{S}_{\mathbf{CL}}$ in terms of **CL** proposed in Ex. 2.3 the requirements read:
T: If $A_1, \ldots, A_n \to \neg B_1 \vee \ldots \vee \neg B_m \in \mathcal{S}_{\mathbf{CL}}$ then
$A_1, \ldots, A_l, B_k, \ldots, B_m \to \neg A_{l+1} \vee \ldots \vee \neg A_n \vee \neg B_1 \vee \ldots \vee \neg B_{k-1} \in \mathcal{S}_{\mathbf{CL}}$.
R: $A_1, \ldots, A_n, \neg A_1 \vee \ldots \vee \neg A_n \vee \neg B_1 \vee \ldots \vee \neg B_m \to \neg B_1 \vee \ldots \vee \neg B_m \in \mathcal{S}_{\mathbf{CL}}$.
C: If $A_1, \ldots, A_n \to A \in \mathcal{S}_{\mathbf{CL}}$ and $B_1, \ldots, B_m \to A_i \in \mathcal{S}_{\mathbf{CL}}$ then $A_1, .., A_{i-1}, A_{i+1}, .., A_n, B_1, .., B_m \to A \in \mathcal{S}_{\mathbf{CL}}$.

For non-interference it has to be assumed that $\mathcal{S}$ is *uniform*:

Definition 4.2. We say that $\mathcal{S}$ is *uniform* iff for any $\Gamma, \Gamma' \in \wp_{\mathsf{fin}}(\mathcal{L})$ and $A \in \mathcal{L}$ such that $\Gamma \cup \{A\}$ is irrelevant to the $\mathcal{S}$-consistent Γ' we have: $\Gamma \to A$ if $\Gamma \cup \Gamma' \to A$.

For a rule base $\mathcal{S}$ closed under **T**, **R** and **C** and given a total preorder $\leq$ over the defeasible rules, [14] showed that $\mid\!\sim^{\mathsf{GeRe}}_{\prec_{wl}}$ satisfies all the four rationality postulates:

Theorem 4.3. $\mid\!\sim^{\mathsf{GeRe}}_{\prec_{wl}}$ *satisfies Direct Consistency, Closure, Indirect Consistency and Non-Interference for the class of argumentation systems $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ whose set of strict rules $\mathcal{S}$ is uniform and closed under* **T**, **R** *and* **C** *and where $\leq$ is a total preorder.*

[4]Since in [14] attention was restricted to total orders, the definition of defeat there required that $a \leq c$ instead of $a \not\prec c$. For total orders, these two conditions are equivalent, but for preorders that are not necessarily totally ordered, the former is clearly not adequate.
[5]Semantics other than grounded (such as preferred or stable) may lead to violations of rationality postulates in the presence of unrestricted rebuttal, see e.g. [4], which is why we restrict our study to the grounded semantics.
[6]A related postulate is *Crash Resistance*, which follows from Non-Interference under some very weak criteria on the strict rules (cf. [6]).
[7]**R** follows from **T** whenever $\to$ is reflexive.

In particular, this means that for any argumentation system based on classical logic (see Example 2.3) using a total preorder, all rationality postulates are satisfied.

Corollary 4.4. $\mid\!\sim^{\mathsf{GeRe}}_{<_{wl}}$ *satisfies Direct Consistency, Closure, Indirect Consistency and Non-Interference for the class of argumentation systems* $AS = (\mathcal{L}, \mathcal{S}_{\mathrm{CL}}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ *for which* $\leq$ *is a total preorder.*

5 AN INDUCTIVE CHARACTERIZATION OF THE GROUNDED EXTENSION

In the following we will only consider lifting principles for the given priorities on the defeasible rules which only depend on the defeasible rules used in the respective arguments. Formally, given an argumentation system AS and any arguments $a, b, c \in \mathrm{Arg(AS)}$ for which $\mathrm{DefR}(b) = \mathrm{DefR}(c)$, we have $a \prec b$ iff $a \prec c$, and, $b \prec a$ iff $c \prec a$. For instance, weakest link falls into this category, while the last link principle [16] does not since it only depends on the last defeasible link and thus on the internal structure of arguments.

This requirement immediately warrants the following fact:

Fact 1. *Where* $a_1, \ldots, a_n, b_1, \ldots, b_m \in \mathrm{Arg(AS)}$ *are strict arguments,* $a, b \in \mathrm{Arg(AS)}$, $\mathrm{conc}(a), \mathrm{conc}(a_1), \ldots, \mathrm{conc}(a_n) \to A \in \mathcal{S}$ *and* $\mathrm{conc}(b), \mathrm{conc}(b_1), \ldots, \mathrm{conc}(b_m) \to B \in \mathcal{S}$*: if* $a \not\prec b$ *then* $\langle a, a_1, \ldots, a_n \to A\rangle \not\prec b$ *and* $a \not\prec \langle b, b_1, \ldots, b_m \to B\rangle$.

Subsequently we also assume that the underlying set of defeasible rules $\mathcal{D}$ of a given argumentation system AS is finite. This allows us to characterize the grounded extension inductively:[8]

Theorem 5.1. *Given an argumentation system* $\mathrm{AS} = \langle \mathcal{L}, \mathcal{S}_{\mathrm{CL}}, \mathcal{D}, \mathcal{K}, \overline{}, \leq\rangle$ *where* $\mathcal{D}$ *is finite and where* $AF = (\mathrm{Arg(AS)}, \mathsf{GeRe}_{\leq})$, *we have:* $\mathcal{G}(AF) = \bigcup_{i\geq 0} \mathcal{G}_i(AF)$ *where* $\mathcal{G}_0(AF) = \mathrm{Acc}(\emptyset)$ *is the set of all arguments in* Arg(AS) *that have no defeater and* $\mathcal{G}_{i+1}(AF) = \mathrm{Acc}(\mathcal{G}_i(AF))$ *is the set of all arguments that are defended by* $\mathcal{G}_i(AF)$.

The theorem follows with the help of the following three lemmas. The following fact follows immediately from **T** and **C**.

Fact 2. *If* $A_1, \ldots, A_n \to B_1 \in \mathcal{S}$ *and* $C_1, \ldots, C_m \to \overline{B_1, \ldots, B_k} \in \mathcal{S}$ *then* $C_1, \ldots, C_m \to \overline{A_1, \ldots, A_n, B_2, \ldots, B_k} \in \mathcal{S}$.

Lemma 5.2. *If* $a, b \in \mathrm{Arg(AS)}$, a *is* $\mathcal{S}$*-consistent, and* a *defeats* b *then there is an* $a' = a, a_1, \ldots, a_n \to \overline{b_1, \ldots, b_n} \in \mathrm{Arg(AS)}$ *that defeats* b *in some* $b_1, \ldots, b_n$ *where each* b_i *is of the form* $b'_1, \ldots, b'_m \Rightarrow B_i$ *and each* a_i *is of the form* $\langle A_i\rangle$ *where* $A_i \in \mathcal{K}$.

Sketch of the proof. Suppose a defeats b in $c_1, \ldots, c_k$. We recursively (over the structure of b) transform a into a' by means of the following recipe. If some c_i is of the form $\langle C_i\rangle$ then $a, c_i \to \overline{c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_k} \in \mathrm{Arg(AS)}$ (obtained via **T**) defeats b in view of Fact 1. For each $c_i = \langle d_1, \ldots, d_l \to C_i\rangle$, also $a \to \overline{c_1, \ldots, c_{i-1}, d_1, \ldots, d_l, c_{i+1}, \ldots, c_k} \in \mathrm{Arg(AS)}$ defeats b (by Facts 1 and 2). Note that since a is $\mathcal{S}$-consistent it is easy to verify that at least one c_i is defeasible. So by the termination of our inductive procedure we will be left with only defeasible conclusions in which the defeat takes place. □

[8] As demonstrated in [2], for infinite argumentation frameworks the grounded extension has a transfinite inductive characterization, but an ω-induction (as used above in Theorem 5.1) is in general not sufficient.

Lemma 5.3. *If* $a \in \mathcal{G}_k(AF)$ *for some* $k \geq 0$, $a_i = \langle A_i\rangle$ *for* $i = 1, \ldots, n$ *and* $A_i \in \mathcal{K}$, *and* $\mathrm{conc}(a), A_1, \ldots, A_n \to A \in \mathcal{S}$ *then also* $a' = \langle a, a_1, \ldots, a_n \to A\rangle \in \mathcal{G}_k(AF)$.

Proof. The proof is by induction on k. Let $\Lambda = a_1, \ldots, a_n$.[9] For the base case suppose $a \in \mathcal{G}_0(\mathrm{AS})$ and that b defeats a'. By Lemma 5.2, there is a b' that defeats a' only in defeasible conclusions and for which $\mathrm{DefR}(b') = \mathrm{DefR}(b)$. Since $\mathrm{DefR}(a') = \mathrm{DefR}(a)$, b' also defeats a which is a contradiction to $a' \in \mathcal{G}_0(\mathrm{AS})$.For the inductive step ($k \Rightarrow k+1$), suppose $a \in \mathcal{G}_{k+1}(AF)$ and that b defeats a'. Again, by Lemma 5.2, there is a b' that defeats a' only in defeasible conclusions and for which $\mathrm{DefR}(b') = \mathrm{DefR}(b)$. Since $\mathrm{DefR}(a') = \mathrm{DefR}(a)$, b' also defeats a. Thus, there is a $c \in \mathcal{G}_k(AF)$ that defeats b'. By Lemma 5.2, there is a $c' = \langle c, c_1, \ldots, c_m \to \overline{\Lambda}\rangle$ for which each c_i is of the form $\langle C_i\rangle$ where $C_i \in \mathcal{K}$ that defeats b' only in defeasible conclusions. Since $\mathrm{DefR}(c') = \mathrm{DefR}(c)$ and $\mathrm{DefR}(b) = \mathrm{DefR}(b')$, c' also defeats b. By the inductive hypothesis $c' \in \mathcal{G}_k(AF)$. So a' is defended by $\mathcal{G}_k(AF)$ and hence $a' \in \mathcal{G}_{k+1}(AF)$. □

Lemma 5.4. *For finite* $\mathcal{D}$*:* $\mathrm{Acc}\big(\bigcup_{i\geq 0} \mathcal{G}_i(AF)\big) = \bigcup_{i\geq 0} \mathcal{G}_i(AF)$

Proof. Suppose a is defended by $\bigcup_{i\geq 0} \mathcal{G}_i(AF)$ and let $\{b_j \mid j \in J\}$ be the (possibly infinite) set of all attackers of a. Thus, for each b_j there is a $k_j \geq 0$ and a $c_j \in \mathcal{G}_{k_j}(AF)$ that defeats b_j. Since $\mathcal{D}$ is finite, $\{\mathrm{DefR}(b_j) \mid j \in J\}$ is finite. So, there is a finite subset $\{b_{j_1}, \ldots, b_{j_m}\}$ of $\{b_j \mid j \in J\}$ for which $\{\mathrm{DefR}(b_j) \mid j \in J\} = \{\mathrm{DefR}(b_{j_1}), \ldots, \mathrm{DefR}(b_{j_m})\}$. For every $k = 1, \ldots, m$ there is a c_{j_k} that defeats b_{j_k}. By Lemma 5.2, there is also a $c'_{j_k} = \langle c_{j_k}, d_1, \ldots, d_l \to \overline{\Delta}\rangle$ that defeats b_{j_k} only in defeasible conclusions Δ and for which each d_i (where $i = 1, \ldots, l$) is of the form $\langle D_i\rangle$. By Lemma 5.3, $c'_{j_k} \in \mathcal{G}_{k_j}(AF)$. Note that for every b_j ($j \in J$) there is a $k \in \{1, \ldots, m\}$ such that $\mathrm{DefR}(b_{j_k}) = \mathrm{DefR}(b_j)$. Thus, b_j is also defeated by c'_{j_k} since $\mathrm{DefR}(c'_{j_k}) = \mathrm{DefR}(c_{j_k})$. Hence, a is defended by $\mathcal{G}_k(AF)$ where $k = \max(\{k_{j_1}, \ldots, k_{j_m}\})$. Thus, $a \in \mathcal{G}_{k+1}(AF)$. So $\mathrm{Acc}\big(\bigcup_{i\geq 0} \mathcal{G}_i(AF)\big) = \bigcup_{i\geq 0} \mathcal{G}_i(AF)$. □

Theorem 5.1 follows now with Lemma 5.4 and Remark 2 since $\bigcup_{i\geq 0} \mathcal{G}_i(AF)$ was shown to be the least fixed point of Acc. (Note that in specific case the fixed point may be reached after finitely many iterations.)

6 FROM TOTAL ORDERS TO PREORDERS

The rationality postulates proven in [14] (as stated in Theorem 4.3) hold only for *total* orders. In many settings, it has been argued that a move from total orders to orders allowing for incomparable elements can greatly increase the realistic modelling capabilities of a formal system. In an epistemic setting, this move may be motivated in terms of *epistemic incomparability*: it may be the case that the user who supplies the defeasible knowledge base does not know the actual preferences of each element in the base; enforcing a preference in such cases might lead to unwanted consequences [23]. Another motivation for giving up the totality assumption is *deontic incomparability*. When reasoning with norms or conditional obligations (as in [3]), for instance, different sources of such norms might

[9] To avoid clutter we will abuse notation and use lists and sets interchangeably in this and some of the following proofs.

be incomparable. Suppose we have a Christian soldier is given (possibly conditional) commands by a captain, a general and a priest. Any of the captain's commands is less preferred than those from the general, but both types are incomparable with any command from the priest. A similar reason for incomparability is given by the values with respect to which we compare elements. Suppose for example Mary and Diana are looking to stay together in a hotel in Bielefeld. Mary prefers hotels with a gym and Diana prefers hotels close to the railway station. If only two hotels exist in Bielefeld, one with a gym but not close to the station, the other close to the railway station but without gym, the two hotels might be considered incomparable in terms of Mary's and Diana's preferences.

However, a move from a total order to any preorder is not trivial (as has already been observed in other contexts [13, 21, 23, 24]). Indeed, using GeRe based on the weakest link as defined above (Definition 2.6) gives rise to violations of crash-resistance when allowing for incomparable elements, as demonstrated in Ex. 6.1.

Example 6.1. Let $AS_2 = (\mathcal{L}, \mathcal{S}_{\text{CL}}, \mathcal{D}_2, \emptyset, \overline{\ \ }, \leq_2)$ where $\mathcal{D}_2 = \{\top \Rightarrow_1 p, \top \Rightarrow_2 \neg p\}$ and $\mathcal{D}_2' = \{\top \Rightarrow_\alpha s\}$, where α is incomparable with 1 and 2 and $1 < 2$. We have (among others) the following arguments:

$$a_1\colon \top \Rightarrow_2 \neg p \quad a_2\colon \top \Rightarrow_1 p \quad a_3\colon \top \Rightarrow_\alpha s \quad a_4\colon a_2, a_3 \to p$$

Notice that $(a_1, a_2), (a_1, a_4), (a_4, a_1) \in \mathsf{GeRe}_{\prec_{wl}}$ and $(a_2, a_1) \notin \mathsf{GeRe}_{\prec_{wl}}$. One would expect $\neg p$ to be justified in the argumentation framework based on $\mathcal{D}_2 \cup \mathcal{D}_2'$, since it is justified in the argumentation framework based on $\mathcal{D}_2$. However, when using the weakest link lifting $\prec_{wl}$ from Definition 2.6, a_4 is incomparable to a_1, which results in a violation of non-interference.

As a cause of this problem one could point to the fact that the weakest link lifting $\prec_{wl}$ used in [14] gives rise to a too weak notion of incomparability, i.e. it makes too many arguments incomparable. In Example 6.1, the violation can be avoided by having a_4 *comparable to* a_1 (resulting in a_4 being strictly less preferred than a_1 and their being a unilateral defeat from a_1 to a_4 instead of the bilateral defeat between a_1 and a_4). In more detail, we have to avoid that *adding* defeasible rules to an argument a makes the resulting argument b incomparable to arguments to which the original argument a was comparable. More precisely, we have to require that $\prec$ is *anti-monotonic*: adding defeasible rules to an argument can only decrease the relative preference of an argument.

LAMC If $\text{DefR}(a) \subseteq \text{DefR}(b)$ and $a \prec c$ then $b \prec c$.

As an example of a lifting principle that is anti-monotonic, consider the *left-pointed dominance* principle, which requires for an argument a to be at least as strong as an argument b, for every minimal element β of b there is a minimal element α of a that is at least as preferred as β.

Definition 6.2. Given an argumentation system $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{\ \ }, \leq)$ and $a, b \in \text{Arg(AS)}$, $a \preceq_{lpd} b$ iff for every $\beta \in \min(\text{DefR}(b))$ there is an $\alpha \in \min(\text{DefR}(a))$ such that $\alpha \leq \beta$. $a \prec_{lpd} b$ iff $a \preceq_{lpd} b$ and $b \not\preceq_{lpd} a$.

Remark 3. *Note for any strict b and any a, $a \preceq_{lpd} b$.*

Notice that in Example 6.1, $a_4 \prec_{lpd} a_1$ (since $a_4 \preceq_{lpd} a_1$ and $a_1 \not\preceq_{lpd} a_4$), and thus there is no defeat from a_4 to a_1. We can thus say that left-pointed dominance is a lifting principle with a stronger notion of incomparability than $\prec_{wl}$, since e.g. a_1 and a_4 are not incomparable using $\prec_{lpd}$ (since $a_1 \prec_{lpd} a_4$), whereas $\prec_{wl}$ rendered a_4 and a_1 incomparable.

Fact 3. $\preceq_{wl} \subseteq \preceq_{lpd}$

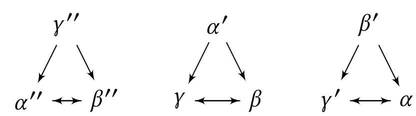
Proof. Suppose that $a, b \in \text{Arg(AS)}$ and $a \preceq_{wl} b$. Thus, either $\text{DefR}(b) = \emptyset$ or $\text{DefR}(b) \neq \emptyset$ and (†) there is some $\alpha \in \text{DefR}(a)$ such that for every $\beta \in \text{DefR}(b)$, $\alpha \leq \beta$. In the first case, clearly $a \preceq_{lpd} b$. In the second case, take any $\beta' \in \min(\text{DefR}(b))$. By †, $\alpha \leq \beta'$. If $\alpha \in \min(\text{DefR}(a))$ we are done. Otherwise, by transitivity, there will be an $\alpha' \in \min(\text{DefR}(a))$ such that $\alpha' \leq \beta'$. □

Having too strong a notion of incomparability might, however, give rise to violations of closure, as demonstrated in Example 6.3.

Example 6.3. Let $AS_3 = (\mathcal{L}, \mathcal{S}_{\text{CL}}, \mathcal{D}_3, \emptyset, \overline{\ \ }, \leq_3)$ with

$$\mathcal{D}_3 = \begin{Bmatrix} \top \Rightarrow_\alpha p; & p \Rightarrow_{\alpha'} p_2; & p_2 \Rightarrow_{\alpha''} p_3; \\ \top \Rightarrow_\beta q; & q \Rightarrow_{\beta'} q_2; & q_2 \Rightarrow_{\beta''} q_3; \\ \top \Rightarrow_\gamma r; & r \Rightarrow_{\gamma'} r_2; & r_2 \Rightarrow_{\gamma''} \neg(p_3 \wedge q_3) \end{Bmatrix}$$

The order of the values is determined by the following graph (where an arrow from x to y means that $y \leq x$):



We have (among others) the following arguments:

a:	$\langle\langle \top \Rightarrow_\alpha p\rangle \Rightarrow_{\alpha'} p_2\rangle \Rightarrow_{\alpha''} p_3$
b:	$\langle\langle \top \Rightarrow_\beta q\rangle \Rightarrow_{\beta'} q_2\rangle \Rightarrow_{\beta''} q_3$
c:	$\langle\langle \top \Rightarrow_\gamma r\rangle r \Rightarrow_{\gamma'} r_2\rangle \Rightarrow_{\gamma''} \neg(p_3 \wedge q_3)$
$a \oplus b$:	$a, b \to p_3 \wedge q_3$
$a \oplus c$:	$a, c \to \neg q_3$
$b \oplus c$:	$b, c \to \neg p_3$

These arguments have the following minimal values:

a:	$\{\alpha, \alpha', \alpha''\}$	$a \oplus b$:	$\{\gamma'', \beta'', \beta', \alpha\}$
b:	$\{\beta, \beta', \beta''\}$	$a \oplus c$:	$\{\alpha'', \gamma, \gamma', \alpha\}$
c:	$\{\gamma, \gamma', \gamma''\}$	$b \oplus c$:	$\{\beta, \beta'', \gamma, \gamma'\}$

Notice that $b \oplus c \preceq_{lpd} a$ yet $a \not\preceq_{lpd} b \oplus c$, i.e. $b \oplus c \prec_{lpd} a$. Similarly, $a \oplus c \prec_{lpd} b$ and $a \oplus b \prec_{lpd} c$. We get the defeat graph in Figure 1. Since a, b and c are in the grounded extension for AS_3, we have $AS_3 \mid\!\sim^{\mathsf{GeRe}}_{\prec_{lpd}} p_3$, $AS_3 \mid\!\sim^{\mathsf{GeRe}}_{\prec_{lpd}} q_3$ and $AS_3 \mid\!\sim^{\mathsf{GeRe}}_{\prec_{lpd}} \neg(p_3 \wedge q_3)$. Hence, indirect consistency is violated. Furthermore, since $AS_3 \not\mid\!\sim^{\mathsf{GeRe}}_{\prec_{lpd}} p_3 \wedge q_3$, closure is violated.

What seems to go wrong here is that the notion of incomparability implicit in the lifting principle $\preceq_{lpd}$ might result in an argument (e.g. a) being incomparable with two other arguments (e.g. b and c) but not with the aggregation of these two arguments (e.g. $b \oplus c$). In that case, ternary conflicts such as that between the conclusions p_3, q_3 and $\neg(p_3 \wedge q_3)$ might not be handled adequately, since for such a conflict to be adequately represented in the argumentation graph, we have to resort to the aggregation of two of the three arguments that constitute the ternary conflict. This is guaranteed by the following anti-monotonicity property:

RAMC If $\text{DefR}(a) \subseteq \text{DefR}(b)$ and $c \prec b$ then $c \prec a$.

A variant of left-pointed dominance that satisfies both anti-monotonicity properties **LAMC** and **RAMC** is *strict left-pointed dominance*:

Definition 6.4. Given $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ and $a, b \in \mathsf{Arg(AS)}$, $a \prec_{slpd} b$ iff $\mathsf{DefR}(a) \neq \emptyset$ and for every $\beta \in \min(\mathsf{DefR}(b))$ there is an $\alpha \in \min(\mathsf{DefR}(a))$ such that $\alpha < \beta$.

Remark 4. *For any strict b and any a,* $b \not\prec_{slpd} a$.

Indeed, using $\prec_{slpd}$ the argumentation system from Example 6.3 does not violate closure or indirect consistency.

Example 6.5. Notice that now $a \not\prec_{slpd} b \oplus c$ and $b \oplus c \not\prec_{slpd} a$, i.e. $\prec_{slpd}$ gives rise to a bilateral defeat relation between a and $b \oplus c$ (and between b and $a \oplus c$ and between c and $a \oplus b$). We get the defeat graph in Figure 1. Note that now, the grounded extension satisfies all the rationality postulates (as shown below).

It is not hard to prove that for any argumentation system AS, if $a \prec_{slpd} b$ then $a \prec_{lpd} b$:

Proposition 6.6. $\prec_{slpd} \subseteq \prec_{lpd}$.

Proof. Suppose that $a \prec_{slpd} b$. Thus for every $\beta \in \min(\mathsf{DefR}(b))$ there is an $\alpha \in \min(\mathsf{DefR}(a))$ such that $\alpha < \beta$. This means that for every $\beta \in \min(\mathsf{DefR}(b))$ there is an $\alpha \in \min(\mathsf{DefR}(a))$ such that $\beta \geqslant \alpha$ and thus $a \preceq_{lpd} b$.

To see that $b \not\preceq_{lpd} a$, we consider two cases: $\mathsf{DefR}(b) = \emptyset$ and $\mathsf{DefR}(b) \neq \emptyset$. Suppose first that $\mathsf{DefR}(b) = \emptyset$. Since $\mathsf{DefR}(a) \neq \emptyset$ (in view of $a \prec_{slpd} b$), we immediately obtain that $b \not\preceq_{lpd} a$. Suppose now $\mathsf{DefR}(b) \neq \emptyset$. In that case, take some $\beta' \in \min(\mathsf{DefR}(b))$. Since $a \prec_{slpd} b$, there is an $\alpha \in \min(\mathsf{DefR}(a))$ such that $\beta' > \alpha$. Suppose now for a contradiction there is some $\beta'' \in \min(\mathsf{DefR}(b))$ such that $\beta'' \leq \alpha$. By transitivity of $>$, this means that $\beta' > \beta''$, which contradicts $\beta' \in \min(\mathsf{DefR}(b))$. □

That the other direction doesn't hold is witnessed by the fact that in Example 6.3, $a \oplus c \prec_{lpd} b$ yet $a \oplus c \not\prec_{slpd} b$.

In order to obtain all the rationality postulates we need yet another constraint on liftings, which makes use of the notion of the aggregation of arguments. Where $a_1, \ldots, a_n \in \mathsf{Arg(AS)}$, we define:

$$a_1 \oplus \ldots \oplus a_n =_{df} a_1, \ldots, a_n \to \textstyle\bigwedge_{i=1}^{n} \mathsf{conc}(a_i).$$

We will also denote $a_1 \oplus \ldots \oplus a_n$ by $\bigoplus_{i=1}^{n} a_i$.

We are now ready to introduce the property of *left shifting*:

LSH If $b \not\prec c \oplus a$ then either $b \oplus c \not\prec a$ or $b \oplus a \not\prec c$.[10]

Fact 4. $\prec_{slpd}$ *satisfies* **RAMC**, **LAMC** *and* **LSH**.

Proof. We give the proof for (**RAMC**). The proofs of (**RAMC**) and (**LSH**) are similar and left to the reader. Suppose $a, b, c \in \mathsf{Arg(AS)}$. We show the fact for $\mathsf{DefR}(b) \neq \emptyset$. The case $\mathsf{DefR}(b) = \emptyset$ is trivial. Suppose furthermore that $\mathsf{DefR}(a) \subseteq \mathsf{DefR}(b)$ and $c \prec_{slpd} b$, i.e. for every $\beta \in \min(\mathsf{DefR}(b))$ there is a $\gamma \in \min(c)$ such that $\gamma < \beta$. Now take some $\alpha \in \min(\mathsf{DefR}(a))$. Either $\alpha \in \min(\mathsf{DefR}(b))$ or there is some $\beta \in \min(\mathsf{DefR}(b))$ such that $\beta < \alpha$. In either case (by transitivity) there is a $\gamma \in \min(\mathsf{DefR}(c))$ such that $\gamma < \alpha$. Hence, $c \prec_{slpd} a$. □

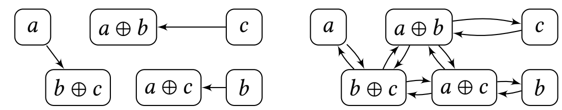


Figure 1: Defeat graphs for Ex. 6.3 (left) and Ex. 6.5 (right)

For a lifting $\prec$ satisfying **RAMC** and **LAMC**, all four rationality postulates hold for *any* preorder:

Theorem 6.7. $\mid\!\sim^{\mathsf{GeRe}}_{\leq}$ *satisfies Direct Consistency, Closure, Indirect Consistency and Non-Interference for for the class of argumentation systems* $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ *whose set of strict rules* $\mathcal{S}$ *is closed under* **T**, **R** *and* **C**, *where* $\leq$ *is a preorder, the lifting* $\prec$ *(over* $\mathsf{Arg(AS)}$*) is transitive and satisfies* **RAMC**, **LAMC** *and* **LSH**.

The theorem follows from Lemmas 6.11 and 6.15 proven below.

Corollary 6.8. $\mid\!\sim^{\mathsf{GeRe}}_{\leq_{slpd}}$ *satisfies Direct Consistency, Closure, Indirect Consistency and Non-Interference for for the class of argumentation systems* $AS = (\mathcal{L}, \mathcal{S}_{\mathrm{CL}}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ *where* $\leq$ *is a preorder.*

The following facts will be used below:

Fact 5. *Where* $a, b, c \in \mathsf{Arg(AS)}$,

(1) *if a defeats b and* $\mathsf{Sub}(b) \subseteq \mathsf{Sub}(c)$ *then a defeats c;*
(2) *if* $a \in \mathcal{G}_i(AF)$ *(for some* $i \geq 0$*) and* $\mathsf{Sub}(b) \subseteq \mathsf{Sub}(a)$ *then also* $b \in \mathcal{G}_i(AF)$.

Proof. Ad 1. Suppose a defeats b in Δ. Since $\mathsf{Sub}(b) \subseteq \mathsf{Sub}(c)$ also $\mathsf{DefR}(b) \subseteq \mathsf{DefR}(c)$ and $\mathbf{C}(b) \subseteq \mathbf{C}(c)$. Since $a \not\prec b$, by **RAMC** also $a \not\prec c$. Thus, a also defeats c in Δ. Ad 2. Suppose some c defeats b. By item 1 also c defeats a. Thus, b is defended by $\mathcal{G}_{i-1}(AF)$ and so $b \in \mathcal{G}_i(\mathrm{AS})$. □

In the following we suppose, unless noted otherwise, an arbitrary but fixed $AS = (\mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq)$ such that $\mathcal{S}$ is closed under **T**, **R** and **C**, and the lifting $\prec$ satisfies **RAMC**, **LAMC**, and **LSH**.

Fact 6. (1) *If* $b = \langle b_1, \ldots, b_n \to B \rangle$ *and* $\mathcal{G}_k(AF)$ *defeats b then* $\mathcal{G}_k(AF)$ *defeats b in some* Δ *s.t.* $b \notin \Delta$.
(2) *If* $a \in \mathcal{G}_k(AF)$, $b = \langle a_1, \ldots, a_n \to A \rangle \in \mathsf{Arg(AS)}$ *where* $a_1, \ldots, a_n \in \mathsf{Sub}(a)$, *then* $b \in \mathcal{G}_k(AF)$.
(3) *If* $a \in \mathcal{G}(AF)$ *and b attacks a then* $\mathcal{G}(AF)$ *defeats b.*

Proof. Ad 1. This follows by Lemmas 5.2 and 5.3. Ad 2. Suppose some c defeats b in Δ. If $b \notin \Delta$ then c also defeats a (note that in view of **RAMC** $c \not\prec a$) and thus $G_{k-1}(AF)$ defeats b. Else by Fact 2 and Fact 1 $c' = \langle c \to \overline{\Delta \cup \{a_1, \ldots, a_n\}) \setminus \{b\}} \rangle$ defeats a. Again, $\mathcal{G}_{k-1}(AF)$ defeats c' and by item 1 it also defeats c.

[10] Our three requirements resemble some properties from epistemology. E.g., Rott in [19, p. 1240] states the *Choice principle* about entrenchments of beliefs as follows: $B \cap C < A$ and $B \cap A < C$ iff $B < C \cap A$, where A, B, C are propositions (sets of possible worlds) and where $A < B$ means that the belief in B is more entrenched than the belief in A. We compare this to the contraposition of **LSH**: $b \oplus c \prec a$ and $b \oplus a \prec c$ implies $b \prec c \oplus a$. When considering the commitment to a $\prec$-stronger argument as being deeper entrenched than the one to a weaker argument, we obtain a similar interpretation. Similarly, **LAMC** and **RAMC** resemble the postulates *Continuing up* and *Continuing down* resp. known from the study of plausibility orderings (e.g., [19, p. 1229]). For instance, the former states that if A is less plausible than B then adding information C to A (resulting in $A \cap C$) also leads to a less plausible proposition than B. Similarly, according to **LAMC**, if a is $\prec$-weaker than b (and so less plausible) then also a_c is $\prec$-weaker than b, where a_c adds more defeasible information to a.

Ad 3. Suppose that $a \in \mathcal{G}(AF)$ is attacked in $a_1, \ldots, a_n$ by b. If $b \not\prec a$ then $\mathcal{G}(AF)$ defeats b since b defeats a. Suppose $b \prec a$. By **T**, $a' = a_1, \ldots, a_n \to \overline{b} \in \text{Arg(AS)}$. Since $b \prec a$ and $\text{DefR}(a') \subseteq \text{DefR}(a)$, by **LAMC**, also $b \prec a'$. Thus, a' defeats b. By Item 2, $a' \in \mathcal{G}(AF)$. □

Lemma 6.9. *$a_1, \ldots, a_n \to A \in \mathcal{G}_i(AF)$ iff $a_1 \oplus \bigoplus_{i=2}^n a_i \to A \in \mathcal{G}_i(AF)$.*

Proof. The ⇒-direction is shown by induction on $i \geq 0$. We show the inductive step. Suppose that $a' = a_1, \ldots, a_n \to A \in \mathcal{G}_i(AF)$. Suppose some b attacks $a = a_1 \oplus \bigoplus_{i=1}^n a_i \to A$. By Lemma 5.2 there is a b' that defeats a only in defeasible conclusions and for which $\text{DefR}(b') = \text{DefR}(b)$. Since $\text{DefR}(a) = \text{Def}(a')$, b' also defeats a'. Thus, some $c \in \mathcal{G}_{i-1}(AF)$ defeats b'. By Lemmas 5.2 and 5.3 there is a $c' \in \mathcal{G}_{i-1}(AF)$ that defeats b' only in defeasible conclusions. Since $\text{DefR}(b') = \text{DefR}(b)$, c' also defeats b. Thus, a is defended by $\mathcal{G}_{i-1}(AF)$ and hence $a \in \mathcal{G}_i(AF)$. The other direction is analogous. □

Lemma 6.10. *If $a, b \in \mathcal{G}(AF)$ then $a \oplus b \in \mathcal{G}(AF)$.*

Proof. Where for any $c \in \mathcal{G}(AF)$, let $\kappa(c)$ be the minimal $k \geq 0$ for $c \in \mathcal{G}_k(AF)$. We prove the lemma by induction on $\kappa(a) + \kappa(b)$.

For the base case suppose that $\kappa(a) = \kappa(b) = 0$. Suppose now for a contradiction that some $c \in \text{Arg(AS)}$ defeats $a \oplus b$ (in $c_1, \ldots, c_n$). Since $c \not\prec a \oplus b$, by **LSH** either $a \oplus c \not\prec b$ or $b \oplus c \not\prec a$. Wlog suppose the former and let $\{c_1, \ldots, c_k\} = \{c_1, \ldots, c_n\} \cap \text{Sub}(a)$. By **T**, $d = c, c_1, \ldots, c_k \to \overline{c_{k+1}, \ldots, c_n} \in \text{Arg(AS)}$. Since $\text{DefR}(d) \subseteq \text{DefR}(a \oplus c)$ and $a \oplus c \not\prec b$, by **LAMC**, $d \not\prec b$. Thus d defeats b, in contradiction to $b \in \mathcal{G}_0(AF)$. For the inductive case suppose that the lemma holds for any $h, h' \in \mathcal{G}$ such that $\kappa(h) + \kappa(h') < n$ and let $\kappa(a) + \kappa(b) = n$. Suppose furthermore that some c attacks $a \oplus b$. Since $c \not\prec a \oplus b$, by **LSH** either $a \oplus c \not\prec b$ or $b \oplus c \not\prec a$. Wlog suppose the former. Let $\{c_1, \ldots, c_k\} = \{c_1, \ldots, c_n\} \cap \text{Sub}(a)$. By **T**, $d = c, c_1, \ldots, c_k \to \overline{c_{k+1}, \ldots, c_n} \in \text{Arg(AS)}$. Since $\text{DefR}(d) \subseteq \text{DefR}(a \oplus c)$, by **LAMC**, $d \not\prec b$. Thus d defeats b. Since $b \in \mathcal{G}_{\kappa(b)}(AF)$, there is a $e \in \mathcal{G}_{\kappa(b)-1}(AF)$ such that e defeats d in $e_1, \ldots, e_l$. Let $\{e_1, \ldots, e_o\} = \{e_1, \ldots, e_l\} \cap \text{Sub}(a)$. Notice that with **T**, $f = e_{o+1}, \ldots, e_l \to \overline{e, e_1, \ldots, e_o} \in \text{Arg(AS)}$. Since $\kappa(a) + \kappa(e) = n - 1$, $a \oplus e \in \mathcal{G}(AF)$ by the inductive hypothesis. Since f attacks $a \oplus e \in \mathcal{G}(AF)$, by Fact 6 (Item 3), some $g \in \mathcal{G}(AF)$ defeats f. By Fact 6 (Item 1) we can suppose that g defeats f in $f_1, \ldots, f_n \in \text{Sub}(c)$. Since $\text{DefR}(f) \subseteq \text{DefR}(c)$, by **RAMC** we get that g also defeats c. □

Lemma 6.11. *If $a_1, \ldots, a_n \in \mathcal{G}(AF)$ and $a_1, \ldots, a_n \to A \in \text{Arg(AS)}$ then $a_1, \ldots, a_n \to A \in \mathcal{G}(AF)$.*

Proof. This follows from Lemmas 5.3, 6.9, and 6.10. □

Fact 7. *1. If $a \in \mathcal{G}(AF)$ then a is $\mathcal{S}$-consistent. 2. If $b \in \text{Arg(AS)}$ is $\mathcal{S}$-inconsistent then it is defeated by $\mathcal{G}_0(AF)$.*

Proof. Ad 2. To see this suppose a is $\mathcal{S}$-inconsistent and hence there is a $\Delta \subseteq \text{C}(a)$ for which $\to \overline{\Delta} \in \mathcal{S}$. But then a is defeated by $b = \langle \to \overline{\Delta} \rangle$. Since b has no defeaters, $b \in \mathcal{G}_0(AF)$. Ad 1. This follows by Item 2 and the fact that $\mathcal{G}(AF)$ is conflict-free. □

Fact 8. *If b is $\mathcal{S}$-consistent in AF and $b' = \langle b \to B \rangle \in \text{Arg(AS)}$, then b' is $\mathcal{S}$-consistent in AF.*

Proof. Suppose b' is $\mathcal{S}$-inconsistent and hence $\to \overline{\Theta}$ for some $\Theta \subseteq \text{C}(b')$. If $\Theta \subseteq \text{C}(b)$ also b is $\mathcal{S}$-inconsistent. Otherwise $B \in \Theta$ and thus $\to \overline{\{\text{Conc}(b)\} \cup (\Theta \setminus \{B\})} \in \mathcal{S}$ (by **C** and **T**) which shows that b is $\mathcal{S}$-inconsistent. □

Remark 5. *Any argument $a \in \text{Arg(AS)}$ can be transformed into an argument $\text{cut}(a)$ such that (1) any argument $b \in \text{Sub}(\text{cut}(a))$ of the form $b = \langle b_1, \ldots, b_n \to B \rangle$ is such that each b_i is either of the form $\langle B_i \rangle$ or of the form $\langle b'_1, \ldots, b'_m \Rightarrow B_i \rangle$, (2) $\text{DefR}(\text{cut}(a)) = \text{DefR}(a)$, (3) $\text{C}(\text{cut}(a)) \subseteq \text{C}(a)$, and (4) $\text{Conc}(a) = \text{Conc}(\text{cut}(a))$. The way to achieve this is to apply Cut "as much as possible". That means for any subargument $b = \langle b_1, \ldots, b_n \to B \rangle$ of a for which some b_i is of the form $\langle b'_1, \ldots, b'_m \to B_i \rangle$ we replace b in a by $b' = \langle b_1, \ldots, b_{i-1}, b'_1, \ldots, b'_m, b_{i+1}, \ldots, b_n \to B \rangle$.*

Fact 9. *If $a \in \mathcal{G}_i(AF)$ then also $\text{cut}(a) \in \mathcal{G}_i(AF)$.*

In the following we let $AS = \langle \mathcal{L}, \mathcal{S}, \mathcal{D}, \mathcal{K}, \overline{}, \leq \rangle$ and $AS' = \langle \mathcal{L}, \mathcal{S}, \mathcal{D}', \mathcal{K}', \overline{}, \leq' \rangle$ where $\mathcal{K} \cup \mathcal{D}$ is irrelevant to $\mathcal{K}' \cup \mathcal{D}'$. We denote $\langle \mathcal{L}, \mathcal{S}, \mathcal{D} \cup \mathcal{D}', \mathcal{K} \cup \mathcal{K}', \overline{}, \leq \cup \leq' \rangle$ by AS^+. We suppose furthermore that $\leq'$ and $\leq$ satisfy **LAMC**, **RAMC**, and **LSH**, and $\mathcal{S}$ is closed under **T**, **R** and **C**. We denote by AF, AF' and AF^+ the corresponding argumentation frameworks.

Lemma 6.12. *If $a \in \text{Arg}(AS^+) \setminus \text{Arg}(AS')$ is $\mathcal{S}$-consistent and $\text{Atoms}(\text{Conc}(a)) \subseteq \text{Atoms}(\mathcal{K}' \cup \mathcal{D}')$ then there is a $c \in \text{Arg}(AS')$ for which $\text{C}(c) \subseteq \text{C}(a)$ and $\text{DefR}(c) = \text{DefR}(a) \cap \mathcal{D}'$.*

Proof. We transform $\text{cut}(a)$ recursively over its tree structure beginning with the top rule. If $\text{cut}(a)$ is of the form $\langle a_1, \ldots, a_n \Rightarrow A \rangle$, we leave it and proceed further with $b = a_1, \ldots, a_n$. Note that in this case $\text{Conc}(a_1), \ldots, \text{Conc}(a_n) \Rightarrow A \in \mathcal{D}'$ (since $\text{Atoms}(A) \subseteq \text{Atoms}(\mathcal{D}' \cup \mathcal{K}')$). If $\text{cut}(a)$ is of the form $\langle a_1, \ldots, a_n \to A \rangle$ we know that each a_i is either of the form $\langle A_i \rangle$ or $\langle b_1, \ldots, b_m \Rightarrow A_i \rangle$. Hence, for each A_i either $\text{Atoms}(A_i) \subseteq \text{Atoms}(\mathcal{D} \cup \mathcal{K})$ or $\text{Atoms}(A_i) \subseteq \text{Atoms}(\mathcal{D}' \cup \mathcal{K}')$. Let $\{A_{j_1}, \ldots, A_{j_l}\}$ be the set of all A_i for which $\text{Atoms}(A_i) \subseteq \text{Atoms}(\mathcal{D}' \cup \mathcal{K}')$. By uniformity $\langle a_{j_1}, \ldots, a_{j_l} \to A \rangle \in \text{Arg}(AS^+)$ (since $\text{Atoms}(A) \subseteq \text{Atoms}(\mathcal{D}' \cup \mathcal{K}')$). We proceed further with $b = a_{j_1}, \ldots, a_{j_l}$. By the construction, the resulting argument c satisfies the requirements of the lemma. □

Lemma 6.13. *If $a = \text{cut}(a)$, $a \in \mathcal{G}(AF')$ implies $a \in \mathcal{G}(AF^+)$.*

Proof. Suppose $a = \text{cut}(a)$. We show by induction that if $a \in \mathcal{G}_i(AF')$ then $a \in \mathcal{G}(AF^+)$.

($i = 0$) Suppose $a \in \mathcal{G}_0(AF')$. By Fact 7, a is $\mathcal{S}$-consistent. Suppose some $b \in \text{Arg}(AF^+)$ defeats a in $a_1, \ldots, a_n$. If b is $\mathcal{S}$-inconsistent it is, by Fact 7, attacked by $\mathcal{G}_0(AF^+)$ and hence a is defended from b. Suppose now b is $\mathcal{S}$-consistent. By Lemma 5.2 there is a b' with $\text{DefR}(b) = \text{DefR}(b')$ that defeats a in defeasible conclusions. By Lemma 6.12, there is a $c \in \text{Arg}(AF')$ for which $\text{DefR}(c) \subseteq \text{DefR}(b')$ and $\text{Conc}(c) = \text{Conc}(b')$. By Fact 1 $b \not\prec a$ implies $b' \not\prec a$, and by **LAMC**, $b' \not\prec a$ implies $c \not\prec a$, and thus c defeats a. This is a contradiction to $a \in \mathcal{G}_0(AF')$.

($i \Rightarrow i + 1$) Suppose $a \in \mathcal{G}_{i+1}(AF')$. By Fact 7, a is $\mathcal{S}$-consistent. Suppose some $b \in \text{Arg}(AS^+)$ defeats a in $a_1, \ldots, a_n$. As in the base we can disregard the case that b is $\mathcal{S}$-inconsistent. Suppose thus that

b is $\mathcal{S}$-consistent. As in the base case we know that there is a $c \in \text{Arg}(AS')$ for which $\text{DefR}(c) \subseteq \text{DefR}(b)$, $\text{Conc}(c) = \text{Conc}(b)$, and c defeats a. Since $a \in \mathcal{G}_{i+1}(AF')$ there is a $d \in \mathcal{G}_i(AF')$ that defeats c in some $d_1, \ldots, d_k$. By Lemma 5.2 there is a $d' = \langle d, e_1, \ldots, e_m \rightarrow \overline{\Delta}\rangle$ where each e_i ($i = 1, \ldots, m$) is of the form $\langle E_i \rangle$ that defeats c in only defeasible conclusions. By Lemma 5.3, $d' \in \mathcal{G}_i(AF')$. Since $\text{DefR}(c) \subseteq \text{DefR}(b)$ and $\text{DefR}(d') = \text{DefR}(d)$, by **RAMC**, d' also defeats b. Since d' defeats b it defends a. Altogether we have shown that a is defended by $\mathcal{G}(AF^+)$ and thus $a \in \mathcal{G}(AF^+)$. □

Lemma 6.14. *Where* $\text{Atoms}(\text{Conc}(a)) \subseteq \text{Atoms}(\mathcal{D}' \cup \mathcal{K}')$*, if* $a \in \mathcal{G}_i(AF^+)$ *then there is an* $a' \in \mathcal{G}_i(AF')$ *with* $\text{Conc}(a') = \text{Conc}(a)$, $\text{C}(a') \subseteq \text{C}(a)$ *and* $\text{DefR}(a') = \text{DefR}(a) \cap \mathcal{D}$.

Proof. Shown by induction on i where $a \in \mathcal{G}_i(AF^+)$. We show the inductive step. Suppose $a \in \mathcal{G}_{i+1}(AF^+)$. By Lemma 6.12 and Fact 7, there is an $a' \in \text{Arg}(AS')$ with $\text{Conc}(a') = \text{Conc}(a)$, $\text{C}(a') \subseteq \text{C}(a)$ and $\text{DefR}(a') = \text{DefR}(a) \cap \mathcal{D}'$. Suppose some $c \in \text{Arg}(AS')$ defeats a'. Hence, c also defeats a (with **RAMC**) and there is a $b \in \mathcal{G}_i(AF^+)$ that defeats c. By the inductive hypothesis, there is a $b' \in \mathcal{G}_i(AF')$ with the same conclusion as b that defeats c (again with **RAMC**) . Thus, $a' \in \mathcal{G}_{i+1}(AF')$. □

Lemma 6.15. $\mid\!\sim$ *satisfies Non-Interference.*

Proof. ($\Rightarrow$) Suppose $AS' \mid\!\sim A$. Hence, there is an $a \in \mathcal{G}(AF')$ such that $A = \text{Conc}(a)$. By Fact 9, $\text{cut}(a) \in \mathcal{G}(AF')$. By Lemma 6.13, $\text{cut}(a) \in \mathcal{G}(AF^+)$. Hence $AS^+ \mid\!\sim A$. ($\Leftarrow$) Suppose $AS^+ \mid\!\sim A$. Hence, there is an $a \in \mathcal{G}(AF^+)$ with $A = \text{Conc}(a)$. By Lemma 6.14, there is a $b \in \mathcal{G}(AF')$ such that $\text{Conc}(b) = \text{Conc}(a)$. Hence, $AS' \mid\!\sim A$. □

7 DISCUSSION

In this paper, we have investigated conditions under which all four rationality postulates from [1, 22] are satisfied for a *preordered* defeasible rule base for ASPIC$^\ominus$, thus generalizing the results from [14]. We discussed where potential pitfalls lay for such a generalization and gave general conditions on the lifting principle that are sufficient for the satisfaction of the rationality postulates. To the best of our knowledge, this is the first system for which all rationality postulates were proven to hold when taking into account any preorder over the defeasible rules. For ASPIC$^+$, [22] shows that when inconsistent arguments are filtered out, the four postulates can be shown to hold when preferences over defeasible rules are not taken into account at all. When preferences are taken into account, violations of closure and indirect consistency can occur (see [22, Ex. 6.7]). In [12], the strategy to avoid interference is to disallow for the chaining of strict rules (we refer to [12] for motivations). Even though they are able to avoid interference *caused by inconsistent arguments*, none of the postulates can be proven to hold for this approach. Thus, it is clear that regardless of which form of rebut is studied, satisfaction of the four rationality postulates in the context of preferences is a challenging task and we hope our results will be helpful when investigating similar problems for ASPIC+ or other nonmonotonic formalisms. Finally, we should mention that there are some structured argumentation systems that allow for rational reasoning with prioritized defeasible premises: [8]. The behaviour of defeasible rules in these systems, however, has not been investigated yet. In future work, we want to investigate whether there are other lifting principles besides $\prec_{slpd}$ that satisfy the conditions we formulated. Furthermore, we want to look at conditions alternative to **T**, **R** and **C** on the strict rule base $\mathcal{S}$, such as the ones suggested by [10, 11]. Finally, we also plan to generalize our results to argumentation systems that allow for defeasible premises and the attack form of undercut.

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