Inverse Kinematics and Sensitivity Minimization of an n-Stack Stewart Platform

Extended Abstract

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ABSTRACT

An autonomous system is presented to solve the problem of in space assembly, which can be used to further the NASA goal of deep space exploration. A prototype of an autonomous manipulator called "Assemblers" was fabricated from an aggregation of Stewart Platform robots for the purpose of researching autonomous in space assembly capabilities. Selecting inverse kinematic poses, defined by a set of translations and rotations, for the Assembler requires coordination between each Stewart Platform and is an underconstrained non-linear optimization problem. For assembly tasks, it is ideal that the pose selected has the least sensitivity to disturbances possible. A method of sensitivity reduction is proposed by minimizing the Frobenius Norm (FN) of the Jacobian of the forward kinematics. The effectiveness of the FN method will be demonstrated through a Monte Carlo simulation to model random motion internal to the structure.

CCS CONCEPTS

• **Computing methodologies** → *Motion path planning*;

KEYWORDS

kinematics; Stewart Platform; in-space assembly; robotics; autonomy

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1 INTRODUCTION

NASA is tasked with developing technologies for deep space exploration and habitation [5][6]. To further that goal, NASA is developing a robotic assembly process of deep space structures [2][7][4]. A recent robotics concept introduces the use of coordinating Stewart platforms [1] arranged in a stack, called Assemblers. The contributions of this paper include the formulation of a solver which can efficiently find inverse kinematic solutions for an Assembler, and the demonstration of optimal pose selection via Frobenius Norm (FN) minimization [3] with numerical results.

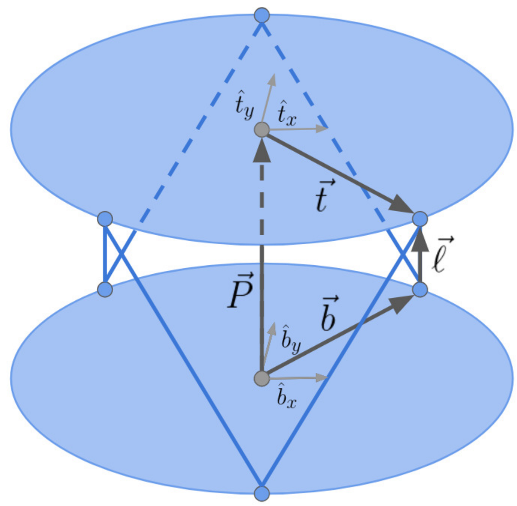

Figure 1: Diagram of Stewart Platform Coordinates

2 FORMULATION

Stewart platforms, or parallel plate manipulators, consist of two plates adjoined by six linear actuators. Fig. 1 shows a simplified diagram of a Stewart Platform with the coordinates we use. Given this coordinate system, the following constraints are placed on the ℓ vector:

$$\ell_{min}^2 \le \vec{\ell}^{\,T} \vec{\ell} \le \ell_{max}^2 \tag{1}$$

$$\vec{\ell}^{\,T} \hat{n} \ge ||\vec{\ell}|| \sin(\theta_{min}) \tag{2}$$

$$\vec{\ell}^{\,T} \mathbf{R} \hat{n} \ge ||\vec{\ell}|| \sin(\theta_{min}) \tag{3}$$

Where $\hat{n} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ is the vector normal to the top plate in its own reference frame, θ_{min} is the minimum angle allowed by the ball joint, and ℓ_{max}, ℓ_{min} are the maximum and minimum possible

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Perturbation Data	Optimal Pose	Non-Opt 1	Non-Opt 2
$\vec{P}_{ee} = [600\ 1000]^T$ $\theta = -\frac{\pi}{2}$ rad	4.67 [0.9, 11.2]	5.47 [1.0, 14.7]	5.31 [1.0, 15.1]
$\vec{P}_{ee} = [145\ 1500]^T$ $\theta = -0.207$ rad	5.10 [0.84, 15.3]	5.40 [0.87, 16.5]	5.56 [0.94, 16.8]
$\vec{P}_{ee} = [-319\ 1532]^T$ $\theta = 0.332$ rad	5.32 [0.89, 16.4]	5.58 [0.93, 17.4]	5.69 [0.92, 17.8]

Table 1: Median distance in mm moved by end effector at different noise levels with 95% confidence interval in brackets; $\vec{P}_{ee}$ given in mm

actuator lengths respectively. The forward kinematics of the end effector position and orientation are given by:

$$\vec{P}_{ee} = \vec{P}_1 + \sum_{i=2}^{n} \prod_{j=1}^{i-1} (\mathbf{R}_j)\vec{P}_i \tag{4}$$

$$\mathbf{R}_{ee} = \prod_{i=n}^{1} \mathbf{R}_i \tag{5}$$

The inverse kinematic problem for Assemblers is to choose all $\mathbf{R}_i$ and $\vec{P}_i$ such that the desired end effector pose is reached, without violating the geometric constraints, and while giving the structure as little sensitivity to internal movement as possible. We formulate the sensitivity as the FN of the forward kinematic Jacobian $||\mathbf{J_{ee}}||_F^2 = Tr(\mathbf{J_{ee}^T J_{ee}})$.

We work in cylindrical polar coordinates with the azimuthal angle ϕ held constant to simplify the problem. We set $\vec{P}_i = [\rho_i, z_i]^T$, and the following equations translate from cylindrical coordinates to Cartesian:

$$\hat{s} = [sin(\phi) \quad -cos(\phi) \quad 0]^T \tag{6}$$

$$\mathbf{R} = \cos(\theta)\mathbf{I} + \sin(\theta)[\hat{s}]_\times + (1 - \cos(\theta))\hat{s}\hat{s}^T \tag{7}$$

$$\vec{P} = \begin{bmatrix} \rho\cos(\phi) & \rho\sin(\phi) & z \end{bmatrix}^T \tag{8}$$

Where θ is the rotation about $\hat{s}$, $\mathbf{R}$ is the rotation matrix about $\hat{s}$ by θ, and $\vec{P}$ is the translation in Cartesian Coordinates.

3 NUMERICAL RESULTS

We use Monte Carlo methods to simulate perturbations in the structure and test the resulting end effector movement. We solve the inverse kinematics with an interior point solver with the constraints from Eq. 1 and the cost function set to minimize $||\mathbf{J_{ee}}||_F^2$. Suboptimal poses are found by running the same interior point solver, but without any minimization requirement.

A sample set of 10,000 data points of random perturbations was generated for each pose of a 4 platform Assembler with three different end effector conditions. Table 1 summarizes the results.

Fig. 2 shows a plot of all found solutions colored by the FN ratio between an optimized pose and a non-optimized pose. Low values tend to be concentrated in the center of the distribution because the DOFs are less constrained and the optimizer has more feasible poses to choose from.

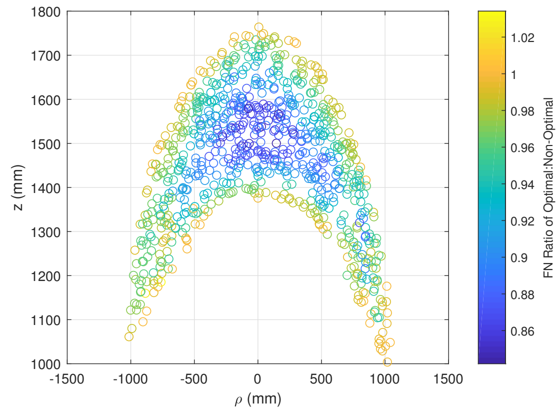

Figure 2: Plot of Found Solutions colored by FN ratio between optimal and non-optimal poses

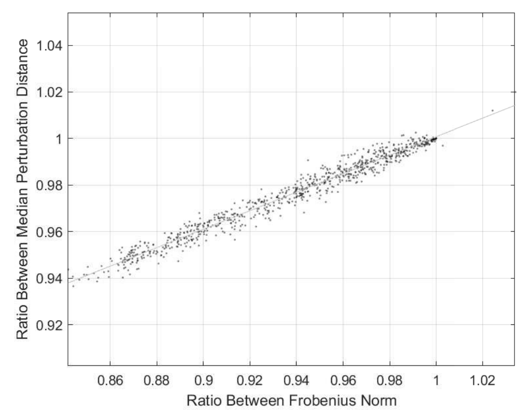

Figure 3: Plot of ratio between optimized and non-optimized poses of FN value and Median Perturbation distance

Fig. 3 empirically shows the relationship between the FN and the perturbation distance. A simple linear regression shows an r^2 value of 0.97, which demonstrates a strong linear correlation with reductions in the FN by 1% causing a reduction in perturbation distance by 0.4%.

4 CONCLUDING REMARKS

The Frobenius Norm methodology was used to optimize the pose selection of over-actuated structures with many degrees of freedom and non-linear forward kinematics. This method was evaluated with a Monte Carlo simulation of Assemblers. Poses with optimized Frobenius Norms were shown to consistently outperform the non-optimized poses. This method can be applied to any over-actuated structure with differentiable forward kinematics. Demonstration of Frobenius Norm minimization on other geometries with hardware validation is proposed for future efforts.

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