Two-stage N-person Prisoner's Dilemma with Social Preferences

Extended Abstract

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ABSTRACT

We examine two-stage games where all players choose the parameters of social preferences at the first stage and play the n-person prisoner's dilemma at the second stage with perfect and imperfect information. This model expresses situations where players can choose how much they depend on the other players' payoffs. In this model, we get the following results. If the game has perfect information, cooperation among all players can be attained in an equilibrium by punishing a deviating player. If each player plays the n-person prisoner's dilemma without knowing the choices of the other players at the first stage, cooperation among a constant number of players can be attained in an equilibrium. In addition, we study two-stage games where all players choose how much they are concerned with the social welfare at the first stage and play the n-person prisoner's dilemma at the second stage. We show that when the players are more concerned with the minimum payoff, the number of players who cooperate at the second stage in an equilibrium weakly decreases.

KEYWORDS

Non-cooperative game; prisoner's dilemma; social preference

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1 INTRODUCTION

The prisoner's dilemma or the n-person prisoner's dilemma (NPPD) is a good model for expressing situations such that agents fail to cooperate even though the cooperation among all agents is efficient. In this model, agents can choose whether to cooperate or to defect. Even if they get a Pareto-efficient allocation when all agents cooperate, defection is a strictly dominant strategy for all the agents. Namely, there is a unique equilibrium such that all agents defect. For many years, researchers have been driven to study this simple model, and many previous studies focus on how agents attain cooperation by modifying the prisoner's dilemma or NPPD, e.g., Okada [7], Kalai [5], Varian [8], and Nishihara [6].

In these works, an implicit assumption, which is common to many works that analyze the emergence of cooperation in such scenarios, is that an agent's utility equals her payoff. Player's preferences depend only on her payoff and not on the payoffs of others. However, in many real-world scenarios, this is not the case. For example, consider a firm that holds the stocks of another firm or when a firm produces a complementary good of the good supplied by another firm. In these cases, the firm's payoff clearly depends on the payoff of the other firm. This dependence of utility on the payoff of others might be based on psychological grounds. Suppose some of a group's members are in a situation like NPPD. If the people in this group have close relationships, they are concerned with the payoffs of others. If people behave as a part of a group, they care about what the group values. Many papers have experimentally identified cases where players care about the payoffs of others, e.g., Andreoni and Miller [1] and Falk et al. [3].

Preference relations, which depend not only on one's own payoff but also the payoffs of others, are called social preferences. Fehr and Schmidt [4] investigate a cooperation game that is almost the same as NPPD under social preferences and conclude that cooperation can be attained in an equilibrium when all the players depend enough on the payoffs of the other players. In their paper, the parameters which express how much the players depend on the payoffs of the other players are given. In contrast, we examine NPPD and possibilities for cooperation under a one-shot game with social preferences whose parameters are determined endogenously.

We argue that in many cases, it is unrealistic to assume that the parameters of social preferences (degree of dependence) are exogenously defined, and to the best of our knowledge, this is the first work that analyzes the emergence of cooperation when players can choose these parameters. Consider the following example. Multiple firms decide how much money to spend on supplying goods and how much to invest in other firms. If a firm invests in another firm, then its utility obviously depends on the payoff of the other firm. If the parameters of social preferences are interpreted as how much money they invest in other firms, the parameters can be controlled by the firms.

We examine two-stage games where all players choose how much they depend on the payoffs of the other players at the first stage and play NPPD at the second stage. Our main results are as follows. If the players do not know the actions of each player at the first stage, a constant number of players, which is uniquely determined by construction, can cooperate in an equilibrium. If the players know the actions of all players at the first stage, two or more players can cooperate in an equilibrium. In particular, all players can cooperate. This difference comes from punishment. If the game has perfect information, players choose an action at the second stage knowing the actions of other players at the first stage. The actions at the first stage convey information to the second stage; if a player tries to defect at the second stage, the other players can punish him when the game has perfect information.

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2 PRELIMINARIES

Let $N = \{1, \ldots, n\}$ be a set of $n \geq 3$ players. The n-person prisoner's dilemma (NPPD) is given by $(N, \{0,1\}, \{x_i\}_{i \in N})$. Every player i chooses $\{0,1\}$ and gets a payoff $x_i : \{0,1\}^n \to \mathbb{R}$:

$$x_i(g_1, \ldots, g_n) = A + a \sum_{j=1}^{n} g_j - g_i,$$

where $A \in \mathbb{R}$, $a \in (\frac{1}{n-1}, 1)$, and $g_j \in \{0,1\}$ for all $j \in N$.

We analyze two types of social preferences: one of the social preferences, which is analyzed in Section 3, is:

$$u_i(x_1, \ldots, x_n) = (1 - \beta_i)x_i + \sum_{j \neq i} \frac{\beta_{i,j}}{n-1} x_j$$

where $\beta_i = \sum_{j \neq i} \frac{\beta_{i,j}}{n-1}$ and $\beta_{i,j} \in [0,1]$. We call these preferences complementary preferences. This utility function is a convex combination of each player's payoff. Each $\beta_{i,j}$ means how much player i depends on x_j.

The other social preferences, which are analyzed in Section 4 (the modified Charness-Rabin preferences), are as follows. The utility of $x = (x_1, \ldots, x_n)$ for the players is

$$u_i^{mCR}(x) = \gamma_i x_i + (1 - \gamma_i) \left\{ \delta \min_{j \in N} x_j + (1 - \delta) \frac{\sum_{j \in N} x_j}{n} \right\},$$

where $\gamma_i \in [0,1]$. To unify the scale of each term, we modify the social preferences proposed by [2]. The difference between the modified Charness-Rabin preferences and the complementary preferences is the item for the minimum payoff when $\beta_{i,j}$ is the same for all $j \in N \setminus \{i\}$.

We assume $n \geq 3$ and $1 > a > \frac{1}{n-1}$. $1 > a$ is a general assumption of NPPD. We assume $a > \frac{1}{n-1}$ because if $a \leq \frac{1}{n-1}$, there is the case where cooperation among all the players is not Pareto-optimal. The concepts of equilibria which we will study are a pure Nash equilibrium and a pure subgame-perfect equilibrium (pure SGPE).

3 EQUILIBRIA OF TWO-STAGE GAMES WITH COMPLEMENTARY PREFERENCES

In this section, we will study the following two-stage game. At the first stage, the players choose how much they depend on the other players' payoff (x_j). For any $i = 1, \ldots, n$, player i chooses $\beta_{i,j} \in [0,1]$ for any $j = 1, \ldots, n$, and the utility function u_i is the complementary preferences.

The following is the process of the game they play:

(1) all players simultaneously choose $\beta_{i,j}$, and
(2) all simultaneously choose g_i; and get $u_i(x_1, \ldots, x_n)$.

We, first, assume that $\beta_{i,j}$ is common knowledge at the second stage.

We can show the next theorem which gives the equilibria of NPPD with complementary preferences.

Theorem 1. $(g_1, \ldots, g_n)$ is a pure Nash equilibrium if and only if

$$\begin{cases} g_i = 1 \text{ if } 1 - a < \beta_i, \\ g_i = 0 \text{ if } 1 - a > \beta_i \end{cases}$$

holds for all $i \in N$.

By this theorem, if a increases, more players cooperate. We can confirm the intuitive result that no players cooperate when $\beta_{i,j} = 0$ for all distinct i, j, and all players cooperate when $\beta_{i,j} = 1$ for all distinct i, j.

Next, we will examine pure subgame-perfect equilibria of the two-stage game. The next theorem states that m players can cooperate at the second stage in a pure SGPE if and only if $m \geq 2$.

Theorem 2. There exist a pure SGPE of the game such that m players choose 1 at the second stage if and only if $m \geq 2$.

This theorem shows that m players can cooperate if and only if $m \geq 2$, and players who will punish a deviating player are needed for cooperation among m players in an equilibrium. The players who mete out punishment must choose $\beta_i = 1 - a$ at the first stage because they should be indifferent between choosing 0 at the second stage and choosing 1 at the second stage. The proof of this theorem implies that two or more players who will punish a deviating player are needed because if one of the players deviates and abandons the role of punisher, the deviating player must be punished. In this theorem, all the players who will cooperate mete out punishment. We call these players punishers.

Next, we will study the two-stage game such that all players choose an action at the second stage without knowing the actions of the other players at the first stage. We examine the Nash equilibria of the game because the players cannot know the actions at the first stage and the best response at the second stage depends only on their own actions at the first stage.

The next theorem characterizes pure Nash equilibria.

Theorem 3. For any pure Nash equilibrium, the number of players who cooperate is either m^* or $m^* - 1$, and in particular, if

$$\frac{n - m^*}{n - 1} > 1 - a,$$

the number is only m^*. Moreover, any player i who cooperates chooses $\beta_{i,j} = 1$ for any j who defects, and any player i who defects chooses $\beta_{i,j} = 0$ for any j who cooperates.

By this theorem, it turns out that there exist only equilibria such that m^* or $m^* - 1$ players cooperate. In the case of the two-stage game with imperfect information, no player can mete out punishment, and hence, there exist only such equilibria.

4 EQUILIBRIUM OF TWO-STAGE GAMES WITH MODIFIED CHARNESS-RABIN PREFERENCES

Consider that the players choose one of the parameters of modified Charness-Rabin preferences at the first stage. In this case, we can obtain some similar results: when the parameter the players choose at the first stage is common knowledge at the first stage, all players' cooperation can be attained in an equilibrium. When the players cannot know the choices of the other players at the first stage, there is a constant number such that cooperation among the number of players can be attained in an equilibrium. In addition, we show that if the players get more concerned with the players who have the minimum payoff, the number of players who cooperate in an equilibrium decreases.

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