Efficient City-Scale Patrolling Using Decomposition and Grafting

Extended Abstract

Wanyuan Wang[1], Zichen Dong[1], Bo An[2], Yichuan Jiang[1]
[1]School of Computer Science and Engineering, Southeast University, China
[2]School of Computer Science and Engineering, Nanyang Technological University, Singapore
[1]{wywang, zcdong, yjiang}@seu.edu.cn, [2]boan@ntu.edu.sg

ABSTRACT

This paper uses an integer program (IP) to formulate the city-scale patrolling (CSP) problem, with the objective of maximizing the police visibility rate (PVR) and the constraint of incident response time guarantee. We decompose the original CSP into two subproblems: minimizing police problem (MinP) and maximizing PVR (MaxP) problem. A polynomial time approximation algorithm is proposed for MinP, and a polynomial time optimal algorithm is proposed for MaxP. We conduct experiments to demonstrate the efficiency of the proposed algorithm.

KEYWORDS

Police Patrolling; multi-objective; Approximation; Decomposition

ACM Reference Format:
Wanyuan Wang, Zichen Dong, Bo An, Yichuan Jiang. 2019. Efficient City-Scale Patrolling Using Decomposition and Grafting. In Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), Montreal, Canada, May 13–17, 2019, IFAAMAS, 3 pages.

1 INTRODUCTION

Existing empirical studies have shown that the presence of police can significantly improve people's feelings of safety (FoS) [5]. Figure 1 shows the association between various types of police presence and people's FoS. From Figure 1, we can find that the presence of police, *no matter how many*, can improve people's FoS significantly [1, 5, 14, 15]. On the other hand, the emergency incidents such as criminal and traffic accidents are time sensitive, and their response time (defined as the interval from the time when the incident occurs to the arrival of a police officer) should be within time guarantees [7, 12]. In Figure 2a, the police always patrol several specific important checkpoints (labeled by ★). However, the incidents (labelled by •) did not spread evenly across regions and could not be fully covered by these fixed checkpoints. Moreover, the volume flow of people (VFoP) varies with time, e.g., schools are more crowded in the morning and afternoon, while shopping malls are more popular during the evening. Figure 2b shows the different VFoP distributions at 16:00 and 20:00. While the topic has been heavily studied, we provide some new insights, such as upon arrival of the incidents, there should be police nearby that can respond within a threshold time, and when there are no incidents, they should patrol around the city to improve PVR [1, 14, 15], rather than returning to the static stations/hot-spots studied in [7, 16]. This bi-objective optimization problem makes 1) existing exact algorithms [2, 4, 9, 13] cannot apply to city-scale instances with fine-grained periods, hundreds of regions and hundreds of police teams, and 2) existing scalable algorithms [3, 8, 10, 11, 17] cannot guarantee PVR.

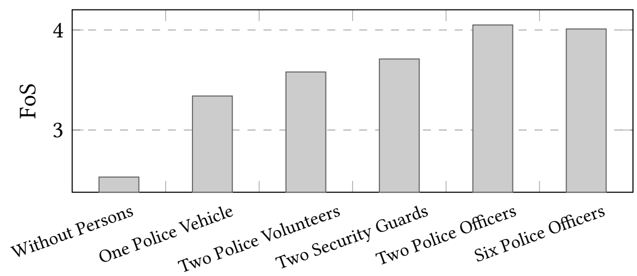
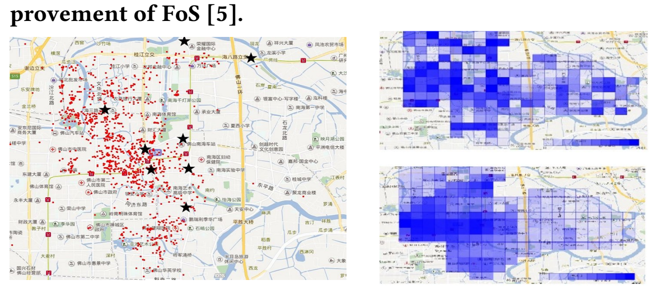


Figure 1: Empirical evidence of police presence on the improvement of FoS [5].



(a) Static Patrolling Policy **(b) Dynamics of VFoP**

Figure 2: Real World Data: (a) ★ indicates static checkpoints and • indicates incident distributions; (b) the top indicates the VFoP at 16:00 and the bottom indicates the VFoP at 20:00.

2 MODEL

Daily Patrolling. There are n police teams $A = \{a_1, a_2, \ldots, a_n\}$. Each day, there are 3 patrolling shifts, each for 8 hours. Each shift is discretized into periods, each period includes $\delta \in [0, 8]$ hour, and the daily patrolling includes periods $\{1, \ldots, T = \frac{24}{\delta}\}$. Each police only serves one shift.

City Network. Let $G = \langle V, D \rangle$ denote the city network where V denotes m regions $\{v_1, v_2, \ldots, v_m\}$, and $D = \{d_{ij} \in \mathbb{Z}^{\geq 0}\}_{v_i, v_j \in V}$ denotes the distance between regions v_i to v_j.

Data-Driven Sampling. We adopt a data-driven sampling model (introduced by [16]) to generate the set of IRs $R = \{r_1, r_2, \ldots, r_l\}$ and the daily volume of flow of people (VFoP) $Q_i = \{q_i^1, q_i^2, \ldots, q_i^T\}$ of each region v_i. Let $\mathcal{P}(i, k, t)$ denote the distribution of the next arriving IR with type k occurring at region v_i during period t and

Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), N. Agmon, M. E. Taylor, E. Elkind, M. Veloso (eds.), May 13–17, 2019, Montreal, Canada.

$\mathcal{Q}(i,t)$ denote the distribution of VFoP of v_i at period t. The parameters of $\mathcal{P}(i,k,t)$ and $\mathcal{Q}(i,t)$ are estimated from historical data.

Objectives. There are two objectives: 1) **IRs Response Time Guarantee.** Let $R = \{r_1, r_2, \ldots, r_l\}$ denote IRs, each $r_k \in R$ is a tuple $\langle v_{r_k}, \tau_{r_k}, \theta_{r_k} \rangle$, where $v_{r_k} \in V$ is the region of occurrence, τ_{r_k} is the time, θ_{r_k} is the threshold response time. Let $N_{r_k} = \{v_j | d_{r_k j} \leq \theta_{r_k}\}$ denote the available regions where the police can respond to r_k with time guarantee. 2) **PVR Maximization.** Let q_j^t denote the VFoP of the region v_j at period t. Let $z_j^t \in \{0,1\}$ denote whether there is police patrolling v_j at period t (= 1) or not (= 0). We can define the PVR of v_j at period t as $\phi_j^t = z_j^t \cdot q_j^t$.

Problem Formulation. The decision variables are defined as follows: $x_i^\beta \in \{0,1\}$: set to 1 if a_i serves the $\beta(\in \{0,1,2\})$th shift; $y_{ij}^t \in \{0,1\}$: set to 1 if a_i patrols the region v_j at period t; $z_j^t \in \{0,1\}$: set to 1 if there is police patrolling v_j at period t. We use an Integer Programming (IP) to formulate the **CSP** problem.

$$\max \textstyle\sum_{v_j \in V} \sum_{1 \leq t \leq T} z_j^t \cdot q_j^t \tag{1}$$

$$\textbf{s.t.} \sum_{\beta=0,1,2} x_i^\beta = 1, \forall a_i, \tag{2}$$

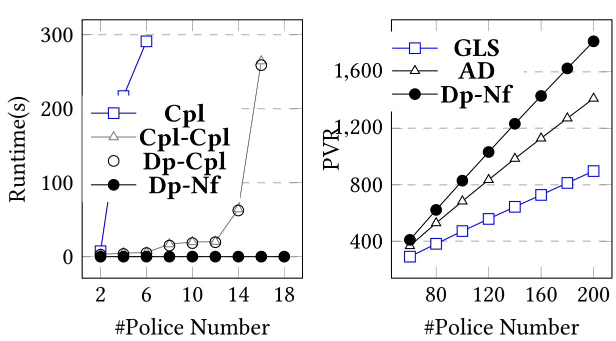
$$\begin{cases} y_{ij}^t + x_i^\beta - 1 < \frac{t - \frac{8}{\delta} x_i^\beta \cdot \beta}{M} + 1, \forall a_i, v_j, t, \beta, \\ y_{ij}^t + x_i^\beta - 1 \leq \frac{\frac{8}{\delta}(x_i^\beta \cdot \beta + 1) - t}{M} + 1, \forall a_i, v_j, t, \beta, \\ \sum_{v_j \in V} y_{ij}^t \leq 1, \forall a_i, t, \end{cases} \tag{3}$$

$$t + d_{jj'} + 1 - t' \leq M(2 - y_{ij}^t - y_{ij'}^{t'}), \forall a_i, v_j, v_{j'}, t < t', \tag{4}$$

$$\sum_{a_i \in A} \sum_{v_j \in N_{r_k}} y_{ij}^{\tau_{r_k}} \geq 1, \forall r_k, \tag{5}$$

$$z_j^t \leq \sum_{a_i \in A} y_{ij}^t, \forall v_j, t. \tag{6}$$

Constraint (2) ensures that each police officer only serves one shift. $M \in \mathbb{R}_{>0}$ is large enough to guarantee the constraint (3). Constraint (4) ensures patrolling consecutiveness. Constraint (5) ensures that each request must be covered.



(a) Runtime (b) Soluation Quality

Figure 3: (a) Runtime; (b) Solution Quality. Each record is statistically significant at 95% confidence level.

3 AN EFFICIENT ALGORITHM

In this section, we present an approximation algorithm to solve such an NP-hard CSP problem. The key idea behind the proposed algorithm is that we decompose the original CSP problem into two weakly-coupled subproblems: 1) *minimum police sub-problem (MinP)*: how to determine the minimum number of police needed to cover all the IRs; and 2) *maximum PVR problem (MaxP)*: how to schedule the remaining police resources to maximize PVR.

Minimum Police Subproblem. Assume that a police a_i is dispatched to the βth shift, let $\tilde{R}^\beta$ denote the remaining IRs after the former police have finished patrolling. To optimize a_i's patrolling plan to cover as many IRs in $\tilde{R}^\beta$ as possible, we design a dynamic programming (DP)-based patrolling plan for a_i. Let $\Omega_i^\beta(t, v_j, \tilde{R}^\beta)$ denote the maximum number of IRs covered by a_i if a_i patrols the region v_j at period t. $\tilde{R}_i^\beta(t, v_j) = \{r_o \in \tilde{R}^\beta | v_{r_o} \in N_{v_j}, \tau_{r_o} = t\}$ denote the IRs near the region v_j that a_i can serve at the period t.

Maximum PVR Subproblem. We provide the interpretation of MaxP problem in terms of a flow network, which can be denoted by a weighted directed graph $G_f = (V_f, E_f)$. Each period-region pair (t, v_i) is denoted by a vertex in $v_{ti} \in V_f$, such that a feasible movement from one vertex v_{ti} to another vertex $v_{t'j}$ (i.e., $t' - t - 1 \geq d_{ij}$) is indicated by a direct edge in E from v_{ti} to $v_{t'j}$. Given two connected vertexes v_{ti} and $v_{t'j}$, we model the 'cost' between them by the negation of the VFoP of the latter vertex $v_{t'j}$. A minimum cost flow is a maximum flow, such that the sum of its edges' weights is the minimum, thereby maximizing its opposite, i.e., the PVR.

Assignment of Police to Shifts. We need to optimally split the police resources A_{MaxP} among the three shifts, so that the total PVR is maximized. We propose an iterative polynomial algorithm to compute the optimal assignment of police to shifts. The main idea behind the optimal police assignment to shifts is that the police resources first are randomly allocated among shifts, and the process continues until there is no beneficial reassignment by moving one police resource from one shift to another shift.

4 EXPERIMENTAL EVALUATION

1) **Scalability Analysis.** We compare the proposed **Dp-Nf** with three benchmarks: **Cpl**, which solves the CSP by the optimal solver CPLEX (version 12.6); **Cpl-Cpl**, which decomposes the CSP into MinP and MaxP, and solves both of them by the optimal solver CPLEX [7]; **Dp-Cpl**, which decomposes the CSP into MinP and MaxP, and solves the former by the dynamic programming and the latter by the optimal solver CPLEX. Figure 3(a) compares the scalability of algorithms with varying numbers of police. The Cpl cannot scale-up to 8 police, and Cpl-Cpl and Dp-Cpl can not scale-up to 18 police with runtime cap of 500 seconds, while Dp-Nf can return the solution within in one second. 2) **Solution Quality Analysis.** We compare the PVR **Dp-Nf** with: *i*) Greedy Local Search (**GLS**), which solves the MinP by requiring each police to patrol the regions greedily and solves the MaxP by requiring each police to search the local accessible regions with the maximum VFoP [10, 16], and *ii*) Abstraction and Division (**AD**), which divides the city into sub-regions [6, 18]. The police is allocated proportionally to each region according to the number of IRs.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (61806053, 61807008 and 61472079), the Natural Science Foundation of Jiangsu Province of China (BK20171363, BK20180356, and BK20180369).

REFERENCES

[1] Anthony A. Braga, Andrew V. Papachristos, and David M. Hureau. 2014. The Effects of Hot Spots Policing on Crime: An Updated Systematic Review and Meta-Analysis. *Justice Quarterly* 31, 4 (2014), 633–663.

[2] Jonathan Chase, Jiali Du, Na Fu, Truc Viet Le, and Hoong Chuin Lau. 2017. Law enforcement resource optimization with response time guarantees. In *2017 IEEE Symposium Series on Computational Intelligence, SSCI 2017, Honolulu, HI, USA, November 27 - Dec. 1, 2017*. 1–7.

[3] Cen Chen, Zachary B. Rubinstein, Stephen F. Smith, and Hoong Chuin Lau. 2017. Tackling Large-Scale Home Health Care Delivery Problem with Uncertainty. In *Proceedings of the Twenty-Seventh International Conference on Automated Planning and Scheduling(ICAPS'17), Pittsburgh, Pennsylvania, USA, June 18-23*. 358–366.

[4] Kevin M. Curtin, Karen Hayslett-McCall, and Fang Qiu. 2010. Determining Optimal Police Patrol Areas with Maximal Covering and Backup Covering Location Models. *Networks and Spatial Economics* 10, 1 (2010), 125–145.

[5] Maria Doyle, Louise Frogner, Henrik Andershed, and Anna-Karin Andershed. 2016. Feelings of Safety In The Presence Of the Police, Security Guards, and Police Volunteers. *European Journal on Criminal Policy and Research* 22, 1 (2016), 19–40.

[6] Yehuda Elmaliach, Noa Agmon, and Gal A. Kaminka. 2009. Multi-robot area patrol under frequency constraints. *Annals of Mathematics and Artificial Intelligence* 57, 3 (01 Dec 2009), 293–320.

[7] Supriyo Ghosh and Pradeep Varakantham. 2018. Dispatch Guided Allocation Optimization for Effective Emergency Response. In *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, New Orleans, Louisiana, USA, February 2-7*.

[8] Burcu B. Keskin, Shirley (Rong) Li, Dana Steil, and Sarah Spiller. 2012. Analysis of an integrated maximum covering and patrol routing problem. *Transportation Research Part E: Logistics and Transportation Review* 48, 1 (2012), 215 – 232.

[9] Hoong Chuin Lau, Zhi Yuan, and Aldy Gunawan. 2016. Patrol scheduling in urban rail network. *Annals of Operations Research* 239, 1 (2016), 317–342.

[10] Ayan Mukhopadhyay, Yevgeniy Vorobeychik, Abhishek Dubey, and Gautam Biswas. 2017. Prioritized Allocation of Emergency Responders Based on a Continuous-Time Incident Prediction Model. In *AAMAS'17*. 168–177.

[11] David Portugal and Rui P. Rocha. 2013. Distributed multi-robot patrol: A scalable and fault-tolerant framework. *Robotics and Autonomous Systems* 61, 12 (2013), 1572 – 1587.

[12] Sandhya Saisubramanian, Pradeep Varakantham, and Hoong Chuin Lau. 2015. Risk Based Optimization for Improving Emergency Medical Systems. In *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence, January 25-30, 2015, Austin, Texas, USA*. 702–708.

[13] Jácint Szabó, Sebastien Blandin, and Charles Brett. 2017. Data-Driven Simulation and Optimization for Incident Response in Urban Railway Networks. In *Proceedings of the 16th Conference on Autonomous Agents and MultiAgent Systems, AAMAS 2017, São Paulo, Brazil, May 8-12, 2017*. 819–827.

[14] Jordi Blanes Vidal and Tom Kirchmaier. 2017. The Effect of Police Response Time on Crime Clearance Rates. *The Review of Economic Studies* 111, 45 (2017), 1–54.

[15] David Weisburd and John E. Eck. 2004. What Can Police Do to Reduce Crime, Disorder, and Fear? *The ANNALS of the American Academy of Political and Social Science* 593, 1 (2004), 42–65.

[16] Yisong Yue, Lavanya Marla, and Ramayya Krishnan. 2012. An Efficient Simulation-Based Approach to Ambulance Fleet Allocation and Dynamic Redeployment. In *Proceedings of the Twenty-Sixth AAAI Conference on Artificial Intelligence, July 22-26, 2012, Toronto, Ontario, Canada.*

[17] Yifeng Zeng, Xuefeng Chen, Xin Cao, Shengchao Qin, Marc Cavazza, and Yanping Xiang. 2015. Optimal Route Search with the Coverage of Users' Preferences. In *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015, Buenos Aires, Argentina, July 25-31, 2015*. 2118–2124.

[18] Chao Zhang, Victor Bucarey, Ayan Mukhopadhyay, Arunesh Sinha, Yundi Qian, Yevgeniy Vorobeychik, and Milind Tambe. 2016. Using Abstractions to Solve Opportunistic Crime Security Games at Scale. In *AAMAS '16*. 196–204.