Novel Hedonic Games and Lottery Systems

Doctoral Consortium

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ABSTRACT

We present here work on two types of matching problems, namely Hedonic Games, also known as Coalition Formation Games, and on quota-based lottery systems such as the one used in Singapore to allocate public housing. We introduce two Hedonic Games, and investigate the computational complexity of finding optimal partitions of agents into coalitions, or finding — or determining the existence of — stable coalition structures. We propose a new stability notion for hedonic games and prove its distinctness from existing notions of stability. We present results from investigations into the fairness of lotteries with diversity quotas inspired by the public housing allocation system in Singapore.

KEYWORDS

Coalition formation (non-strategic); Cooperative games: computation; Cooperative games:theory & analysis

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1 INTRODUCTION

We study two distinct areas of multiagent systems: coalition formation games and lottery-based distribution systems. Coalition formation games are a class of cooperative games focused on partitioning agents into coalitions. Lottery-based distribution systems provide a means by which a supply of goods can be distributed to a pool of agents.

Though general case coalition formation games can model a multitude of interesting situations, much research in this area focuses on sub-classes of coalition formation games. Our focus is on hedonic games, a sub-class of coalition formation games that assumes an agent's utility is wholly derived from the coalition they are in [1, 4]. We introduce two new classes of hedonic games and present results for one of them. Additionally, we propose a new stability notion to better characterize the behavior of agents under certain realistic assumptions.

Our work on lottery-based distribution systems extends previous work by Benabbou et al. which was inspired by Singapore's public housing allocation system [3]. More than 70% of Singapore's residential real estate is operated by the Housing & Development Board (HDB). The Singaporean government uses this position as a majority operator of residential real estate to promote ethnic diversity in neighborhoods; this is done through diversity quotas which limit the percentage of real estate an ethnic group can own in a given neighborhood. We present analytical results about a simplified version of Singapore's public housing allocation system.

2 HEDONIC GAMES

Hedonic coalition formation games are a powerful tool to model agent behavior in a variety of circumstances. A classic example is the stable marriage problem which seeks to pair off men and women from two equally-sized groups. In the stable marriage problem, a pair is said to be stable if there is no man-woman pair who both want to leave their assigned partners to form a new pair.

2.1 Super Altruistic Hedonic Games

Consider the process of choosing where to live. Previous research on hedonic games has investigated the process of choosing the right roommates or housing units to ensure a stable configuration. We look beyond the individual housing unit to consider the choice of *neighbors*, perhaps in a setting where students are choosing dormitories/hostels. We envision the partitioning of students into living units (floors, buildings, etc.) as a hedonic game. It is clear that we value our friends' happiness with the living situation, as we will hear about it from them; our enemies' happiness could be assumed to also affect how they treat us. (If we stopped there, we would be modeling evaluation as an *Altruistic Hedonic Game (AHG)* [7].) More generally we can also argue that our friends' friends' happiness will affect our friends', and thus indirectly, our own, and that this continues out friendship chains, with decreasing (or at least, non-increasing) effect as we increase the social distance from ourselves.

If we were building intranets, a node could evaluate the quality of the local network in terms of the bandwidth to reachable nodes. However, it would also need to take into account the quality of more distant connections, if it hopes to have its packets relayed. There are many other applications in which agents care not only about immediate connections, but also those farther away. We introduce a family of hedonic games that model such broad evaluations of coalitions: the Super Altruistic Hedonic Games.

Definition 2.1. **Super Altruistic Hedonic Games (SAHGs)** extend the core principal of AHGs[1] so agents consider the preferences of all agents in their coalition. Agents weigh their consideration of each other's preferences according to some polynomially computable value.

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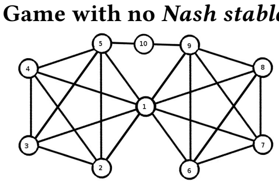
[1]See the full text of the Altruistic Hedonic Games paper by Nguyen et al. for more information [7].

Let parameters (a, g, M, L) be non-negative weights where a and g represent the weights associated with friends and enemies, respectively, while M and L represent the weights associated with personal preference and the average of friends' preferences. Next, let $D(i, j)$ be a polynomial-time computable function that is non-increasing with the graph distance between i and j. Let the number of other agents in coalition C_i be $h_i = |C_i \setminus \{i\}|$. For each agent $i \in N$, let that agent's base preference be $b_i = a|C_i \cap F_i| - g|C_i \cap E_i|$, and let their utility be $u_i = Mb_i + L \sum_{j \in C_i \setminus \{i\}} \frac{D(i,j) \cdot b_j}{h_i}$. If $C_i = \{i\}$ then the sum is set to 0. The default definition of D is the inverse graph distance function: for any pair of agents $i, j \in N : i \neq j$, let d_{ij} be the shortest path distance between them, then let $D(i, j) = 1/d_{ij}$. The **total utility** of a partition π is given by $U_T = \sum_{i \in N} u_i$.

Theorem 2.2. *Not all SAHGs have Nash stable partitions.*

Theorem 2.2 is proved by contradiction using a game based on Figure 1 with weight parameters $(a, g, M, L) = (1, 1, 1, 3)$. The full proof is omitted for space reasons.

Figure 1: Game with no *Nash stable* partition



SAHGs generalize friend and enemy-oriented hedonic games by utilizing weight parameters of $(a, g, M, L) = (n, 1, 1, 0)$ and $(1, n, 1, 0)$ respectively. Thus, complexity results for friend and enemy-oriented hedonic games are inherited as lower complexity bounds for SAHGs.

2.2 Anchored Team Formation Games

Consider tabletop role playing games (TRPG) such as Dungeons and Dragons. More specifically, consider a university club focused on such games. One of the key challenges such a club must face is how to divide up the club members so everyone can participate in a game. In order to play a TRPG, a group must have both players and a game master, or GM. We propose Anchored Team Formation Games (ATFGs) as a model for such situations. A key focus of our work on ATFGs is the impact of unknowns on coalition stability.

Recent work by Barrot, Ota, Sakurai, and Yokoo has investigated the impact of unknowns on FHGs [2]. In their work, an unknown agent is assigned either an epsilon positive or negative utility value depending on whether the agent assessing the unknown is extroverted or introverted. Our interpretation of unknowns is based on work by Konczak and Lang in voting theory where unknowns are assumed to have a true value that is only known ex post facto [5].

2.3 Proximal Stability

This work was inspired by Taywade, et al.'s experimental work on decentralized, multi-agent coalition formation [6]. In that work, agents have additively separable preferences, and explore a grid world, learning about their preferences for other agents as they encounter them, and ultimately forming coalitions. As other agents join a coalition, subgroups may "bud off" from the group to form their own, new coalition. In that work, the budding agents must increase their utility by forming the new coalition, while the remaining agents' utilities are not taken into consideration.

We describe a stability notion defined by coalitions that do not fracture into smaller coalitions. This contrasts with core stability wherein a deviating coalition can comprise agents from up to as many coalitions as there are agents in the new coalition. It is reasonable to think of this as a scenario where agents have limited information about agents from coalitions other than their own.

Our work thus far has focused on distinctions separating proximal stability from core and Nash stability. We have proved that proximal stability neither generalizes or is generalized by Nash stability. We prove that proximal stability generalizes core stability.

3 LOTTERY-BASED ALLOCATION SYSTEMS

We are interested in modeling allocations of goods over diverse populations similar to HDB housing allocation in Singapore and school selection in several US cities.

We construct a lottery-based allocation model that utilizes a set of players N, which is partitioned into k types $\{N_1, ..., N_k\}$, and a set of goods G, partitioned into l blocks $\{G_1, ..., G_l\}$. We do not always define a fixed size for N and sometimes define N simply as a random distribution vector. Each agent $i \in N$ has a set of approved goods $G_i \subseteq G$. The utility some agent $i \in N$ derives from some good $j \in G$ is 1 for approved goods and 0 otherwise.

Using this allocation model, we investigate whether we can ensure fairness with agent type maximum quotas. Additionally, we characterize the probability that an agent i of type h will not receive a good because of their type as a function of their position in the allocation queue. We investigate one variation on this allocation problem, wherein the set of players N is partitioned into groups of fixed size such that each group's type composition closely replicates the type composition of N.

We utilize a series of simulations to better understand how certain conditions impact the outcomes of such allocation systems. We craft a mathematical model of agents' expected utility based on their distribution queue position and their type. We use this expected utility model to evaluate the performance of Singapore's HDB allocation system with 2018 population data and found that members of one of the three ethnic category have substantially worse expected outcomes than the other two.

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