Peer Reviewing in Participatory Guarantee Systems: Modelisation and Algorithmic Aspects

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ABSTRACT

The term Participatory Guarantee Systems (PGS) refers to quality certification systems based on the active participation of stakeholders, i.e., producers, consumers, and experts. Unlike to the more common Third Party Certification system, quality standards are guaranteed by peer review: visits of production sites by producers themselves. A critical issue in PGS is the assignment of the peers carrying each review visit, in a way that incentivizes participation. This paper explores algorithmic aspects of this peer assignment, so as to better address challenges faced by PGS. First, we propose a mathematical model of this task that can express diverse local PGS situations, as well as possible extensions. Then, we show that this model leads to computationally challenging problems and identify restrictions that are easy to handle. Finally, we develop an encoding of the model in Answer Set Programming and use it to solve realistic scenarios of PGS.

KEYWORDS

Peer Review Assignment, Participatory Guarantee System, Graph Factors, Computational Complexity, Algorithms, Answer-Set Programming

ACM Reference Format:


1 INTRODUCTION

The term Participatory Guarantee Systems (PGS) refers to locally focused social organisations providing guarantees on production quality, typically organic standards in farming.1 Contrary to the dominant Third Party Certification (TPC), PGS are grounded on the active participation of stakeholders, predominantly producers but also consumers and experts, in local communities. PGS are generally considered cheaper and more concerned with local socio-technical situations, and thus more suitable to small-scale producers than TPC [28, 29, 32]. Since the formalisation of the concept of PGS by the International Federation of Organic Agriculture (IFOAM) [8], PGS initiatives have received increasing attention and approximately 240 PGS initiatives were operating or developing in more than 60 countries as of 2017.2 However, PGS are facing challenges that the scientific community is only starting to identify and address [16, 21, 22]. Securing and maintaining the level of participation of producers is the main challenge and it is influenced by producers’ workload, the overall credibility of the system or the lack thereof, and by how personal conflicts are mitigated [2, 3, 29]. The PGS activity that weighs most on these questions is the reviewing visits of the production sites by producers, whether for initial certification or regular monitoring. Therefore, a critical issue is the peer review selection process, i.e., how to assign producers to production sites reviewing visits in a way that incentivise their participation. This paper aims at exploring the algorithmic aspects of this selection, so as to better address the challenges faced by PGS.

Diverse requirements are imposed by PGS regulations on the peer review selection process: Each production site has to receive a fixed number of peer reviews, i.e., reviews from producers. Some PGS may also require additional reviews from non-producer stakeholders, i.e., consumers and experts. A minimal number of skilled stakeholders are required at each review of production site. To reduce the possibility of collusion, regulations usually forbid pairs of producers to review each other production site. Finally, some reviews can be considered infeasible because of external constraints or personal conflicts. The outcome of the peer review selection is a multiple assignment where each producer is assigned several stakeholders that review his production site.

1.1 Related Works

To the best of our knowledge, this is the first formal investigation of peer review assignment in PGS. Various assignment problems similar to PGS have been investigated in the literature, but they address different problems with different constraints and objectives. First, the Conference Assignment problem (CA) considers the setting where a set of papers has to be evaluated by a set of reviewers, in order to select the papers that will be published [14]. As the most popular AI conferences involve thousands of papers and reviewers, the assignment process is critical. Some aspects of CA are similar to PGS. A paper has to receive a minimal number of evaluation. Reviewers have a capacity limit over the number of evaluation they provide. Moreover, reviewers can express conflict of interest with specific papers. However, contrary to PGS, papers


2www.ifoam.bio/en/pgs-maps
and reviewers form two disjoint sets that allow to model CA as a matching problem. Moreover, reviewers are allowed to report preferences over the set of papers and the goal of CA is then to find an assignment that reflect as much as possible the preferences [5, 6, 23, 33]. A recent review on how AI tools are used to solve CA has been published in [31].

Next, the assignment of judges to competition considers similar constraints as CA, i.e., capacity constraints and conflict of interests [20]. In addition, an expertise level/requirement is associated to judges/competitions, and judges do not express preferences over the set of competition. However, the set of judges and competition are also disjoints and thus judges to competition differs strongly from PGS. Closely related is the assignment of referees to league match, where there exist periodic constraints due to repetition of matches [1, 24, 25].

Another interesting domain is peer evaluation, which studies situations where agents are evaluated by agents with the same status. In this general setting, the focus is usually on how to grade and not on how to choose peer-reviewers. There exist two main trends in peer evaluation. The first one has educational purposes and studies the benefit of different methods or peer-grading on learning performances [7, 26, 34]. The second one is related to Massive Open Online Courses (MOOCs), where issues are how to reveal the true grade of students given that each student only receives few grades and students are not experts in grading [4, 17–19].

1.2 Contributions and outline

Our contributions in this paper are threefold. First, we propose a mathematical model of the peer review selection in PGS, as well as possible extensions, in Section 3. Our parametric model is rich enough to let one express diverse local PGS situations. Then, we unveil a correspondence between the graph-theoretical problem of $r$-factors and peer review selection in PGS. We investigate the algorithmic properties of the PGS model and while we show that it leads to computationally challenging problems, we also identify tractable restrictions, in Section 4. Our computational results are summarized in Table 1. Finally, we develop an Answer Set Programming (ASP) encoding of our PGS model, together with selected extensions, and demonstrate that solving realistic scenarios of PGS is within reach of modern ASP implementations, in Section 5.

2 MATHEMATICAL PREREQUISITES

2.1 Some Notions on Graphs

A graph is a pair $G = (V, E)$, where $V = \{1, \ldots, n\}$ is a set of vertices and $E$ is a set of unordered (resp. ordered) pairs, called edges (resp. arrows). For two vertices $x, y \in V$, we denote $(x, y)$ if the pair is ordered and $(x, y)$ otherwise. Graph $G$ is called directed if $E$ is a set of ordered pairs, i.e., $E \subseteq \{(x, y) \in V \times V : x \neq y\}$, and undirected if the pairs are unordered, i.e., $E \subseteq \{(x, y) \in V \times V : x \neq y\}$. In an undirected graph $G$, the degree of a node $v \in V$, denoted $\deg_G(v)$, is the number of edges to which $v$ belongs. In a directed graph $G$, the indegree (resp. outdegree) of a node $v$, denoted $\deg^-_G(v)$ (resp. $\deg^+_G(v)$), is the number of arrows whose ending point (resp. starting point) is $v$. A crucial notion for PGS is the notion of $f$-factor (see the survey from Plummer [30]). Given a function $f : V \mapsto \mathbb{N}$, an undirected subgraph $G' = (V, E' \subseteq E)$ of $G$ is called an $f$-factor if for all $v \in V$, $\deg_{G'}(v) = f(v)$. In this paper, we are interested in $f$-factors for constant functions, i.e., for all $v \in V$, $f(v) = r$, for $r \in \mathbb{N}$, which we denote $r$-factor.

2.2 Primer on Complexity

In computational complexity theory, a decision problem is a set of instances that is partitioned into YES-instances and NO-instances. Depending on their inherent complexity, decision problems are categorized into complexity class, among which class $P$ and class $NP$ are the most studied. $Class P$ corresponds to decision problems that can be solved in polynomial time in the size of the instance. Problems in $P$ are generally considered tractable or easy to solve.

$Class NP$ represents the decision problems for which the verification of a YES-instance admits a polynomial time algorithm. The most difficult problems in $NP$ are called $NP$-complete and it is generally assumed that there exists no polynomial algorithm to solve $NP$-complete problems. In our complexity proof, we use the $NP$-complete Set Cover problem [9].

**Set Cover:**
Instance: $X$ a set of elements, $C$ a collection of subsets of $X$, and $t$ a positive integer.
Question: Do $t$ subsets exist in $C$ such that their union covers $X$?

2.3 Primer on Answer Set Programming

Answer set programming (ASP) is a paradigm of declarative programming oriented towards combinatorial search problems [12]. It is based on the stable model semantics of logic programming. The main idea is to reduce a search problem to a logic program, by formulating constraints in terms of rules, such that minimal stable models correspond exactly to problem solutions.

Most ASP systems are composed of a grounder and a solver. The grounder "grounds" the problem file by replacing all variables in

<table>
<thead>
<tr>
<th>Veto</th>
<th>Skills ($\epsilon$)</th>
<th>$k$</th>
<th>$k'$</th>
<th>$z$</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty</td>
<td>Input</td>
<td>Any</td>
<td>Any</td>
<td>$= 0$</td>
<td>$\text{NP-c (Th. 4.15)}$</td>
</tr>
<tr>
<td></td>
<td>Input</td>
<td>Any</td>
<td>Any</td>
<td>$= 2$</td>
<td>$\text{P (Th. 4.9)}$</td>
</tr>
<tr>
<td></td>
<td>Any</td>
<td>$= 3$</td>
<td>$\text{P (Th. 4.8)}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sym.</td>
<td>$= 1$</td>
<td>Any</td>
<td>Any</td>
<td>$\geq 5$</td>
<td>$\text{NP-c (Th. 4.11)}$</td>
</tr>
<tr>
<td></td>
<td>$\geq 2$</td>
<td>Any</td>
<td>Any</td>
<td>$\geq 2$</td>
<td>$\text{NP-c (Th. 4.14)}$</td>
</tr>
<tr>
<td>General</td>
<td>Any</td>
<td>$= 1$</td>
<td>Any</td>
<td>$= 2$</td>
<td>$\text{NP-c (Th. 4.12)}$</td>
</tr>
</tbody>
</table>

Table 1: Summary of the results for PGS($k, k', z, \epsilon$, veto) (see Definition 3.5). Veto can be empty, symmetric between producers, or general. Variable $\epsilon$ represents the number of different skills. Variables $k$ and $k'$ refer to the number of reviews from producers and consumers, respectively. Variable $z$ refers to the minimal size of an cycle in a feasible assignment. Input means that the variable is part of the input. Any means that the result holds for each value of the variable.
As a first approximation, we identify producers and production sites. Let \( P = \{ 1, \ldots, n \} \) be a set of producers and \( C = \{ n+1, \ldots, n+m \} \) be a set of consumers constituting a set of stakeholders \( S = \{ 1, \ldots, n+m \} \). The goal is to find an assignment \( A \subseteq S \times P \), where \((i, j) \in A\) means that stakeholder \( i \) reviews producer \( j \), that satisfies the following requirements.

### Infeasible review
Some reviews can be infeasible because of personal conflicts or external constraints (e.g., distance between production sites). We model infeasible reviews with a binary relation between \( S \) and \( P \), denoted \( V \), where \((i, j) \in V\) means that stakeholder \( i \) cannot evaluate producer \( j \).

#### Definition 3.1 (\( V \)-respecting).
An assignment \( A \) is said to be \( V \)-respecting if \( A \cap V = \emptyset \).

#### Number of reviews
Each production site should receive a review committee comprising \( k \) producers \((k \geq 1)\) and \( k' \) consumers \((k' \geq 0)\). The workload is equally distributed across producers by imposing that each producer participates in the same number of review committees. Consumers’ workload is not as crucial for PGS and thus we do not impose equal distribution of reviews across consumers.

#### Definition 3.2 \((k, k')\)-reviewable.
An assignment \( A \) is said to be \((k, k')\)-reviewable if each producer \( p \in P \) performs \( k \) reviews, i.e., \( |A \cap (\{ p \} \times P)| = k \), receives \( k \) reviews from other producers, i.e., \( |A \cap (P \times \{ p \})| = k \), and receives \( k' \) reviews from consumers, i.e., \( |A \cap (C \times \{ p \})| = k' \).

#### Credibility of the PGS
A possible criticism against PGS is the possibility of collusion between producers. A first step towards reducing collusion opportunities is to forbid situations where two producers review each other. We generalize this idea and forbid reviewing cycles of length smaller than a threshold \( z \). In practice, most PGS are looking for a threshold \( z = 2 \).

#### Definition 3.3 (\( z \)-credible).
An assignment \( A \) is said to be \( z \)-credible if there exists no sequence of producers \((p_1, p_2, \ldots, p_l)\) of length \( l \leq z \) such that \((p_i, p_{i+1}) \in A\) for \( 1 \leq i < l \) and \((p_1, p_l) \in A \).

#### Necessary expertise
Some skills (e.g., knowledge on agroecology, or expertise in reviewing) may be required to handle reviews. Let \( E \) denote the set of possible skills, and, given a field of expertise \( e \in E \), let \( E_e \) denote the set of producers having expertise \( e \). For each skill \( e \) in \( E \), we impose that at least one stakeholder should possess skill \( e \) at each visit.

#### Definition 3.4 (\( E \)-compatible).
An assignment \( A \) is said to be \( E \)-compatible if for all \( p \in P \) and all \( e \in E \), \( A \cap (E_e \times \{ p \}) \neq \emptyset \).

## 3 FORMAL MODEL

### 3.1 Peer Review Selection Model for PGS

#### As a first approximation, we identify producers and production sites.
Let \( P = \{ 1, \ldots, n \} \) be a set of producers and \( C = \{ n+1, \ldots, n+m \} \) be a set of consumers constituting a set of stakeholders \( S = \{ 1, \ldots, n+m \} \). The goal is to find an assignment \( A \subseteq S \times P \), where \((i, j) \in A\) means that stakeholder \( i \) reviews producer \( j \), that satisfies the following requirements.

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## 3.2 Possible Extensions

Several extensions that better reflect the issues faced in PGS can be straightforwardly handle by our model.

#### Diversity of stakeholders.
In many PGS, stakeholders include producers and consumers as well as experts, as a way of building trust in the community and facilitate access to the market. Experts may actively participate in the review process, inducing constraints which are similar to those implied by consumers participation. Hence, we can model experts as consumers by adding appropriate skills in \( E \).

#### Knowledge exchange.
One promoted benefit of PGS is to foster knowledge creation and exchange. Knowledge exchange can be fostered by taking into account past reviews, e.g., by promoting review between producers who have not reviewed each other yet.

A simple way of implementing this idea is adding past reviews into the set of infeasible reviews of the following review selection process, as done in our ASP implementation in Section 5.

## 3.3 Decision Problems

Let us now formally define the decision problems that we investigate in this paper. Our analysis depends on the values of parameter \( k, k', z, \) the number of skills \( e \), and whether \( V \) is empty (vetos are not allowed), includes only symmetric relations between producers, or also includes asymmetric relations between producers. Symmetric vetos arise from external constraints such as prohibiting distance between two production sites, whereas asymmetric vetos represent infeasibility from personal conflicts.

#### Definition 3.5.
For a number of producer reviews \( k \geq 1 \), of consumer reviews \( k' \geq 0 \), a credibility \( z \geq 2 \), a number of skills \( e \geq 0 \), and a parameter \( \text{vetos} \subseteq \{ \text{empty, symmetric, general} \} \), we define PGS\((k, k', z, e, \text{vetos})\) as the following decision problem.

#### PGS\((k, k', z, e, \text{vetos})\):
Instance: A set of producers \( P \), a set of consumers \( C \), a set of vetos \( V \), and, for \( 1 \leq e \leq e \), the subset of stakeholders \( E_e \subseteq P \) having skill \( e \).

Question: Is there a reviewing assignment \( A \) that is \( V \)-respecting, \((k, k')\)-reviewable, \( z \)-credible, and \( E \)-compatible?

Note that each parameterization of \( (k, k', z, e, \text{vetos}) \) leads to an individual decision problem PGS\((k, k', z, e, \text{vetos})\). Furthermore, when presenting our complexity results, we may replace variables of PGS\((k, k', z, e, \text{vetos})\) by the notation \( \text{INPUT} \), which means that the corresponding variable is considered as part of the input.

The PGS decision problem can be defined from a graph-theoretic perspective. Indeed, we can associate to any PGS instance a graph, that we call the potential review graph, where nodes represent stakeholders and edges represent feasible reviews.

#### Definition 3.6 (potential review graph).
Given a PGS instance, the potential review graph is the graph \( G = (S = P \cup C, (S \times P) \setminus V) \).

Similarly, any assignment \( A \) can be seen as a directed graph \( G^* = (S, A) \), and the following characterization is immediate: \( A \) is \( V \)-respecting if \( A \subset (S \times P) \setminus V \), that is, if \( G^* \) is a subgraph of \( G \); \( A \) is \((k, k')\)-reviewable if each vertex in \( P \) has an indegree equal to
Table 2: Summary of Section 4.1 on the equivalence between assignments in symmetric PGS and factors in potential graphs. If a symmetric PGS instance admits a $k$-reviewable, $z$-credible assignment, then its potential graph admits a $2k$-factor with no cycles of length smaller than $z$ (Lemma 4.1). The converse is only guaranteed for some values of $k$ and $z$, as indicated above.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$z$</th>
<th>Invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$z \leq 2$</td>
<td>Equivalent notions according to Lemma 4.2</td>
</tr>
<tr>
<td>2</td>
<td>$z = 3$</td>
<td>Counterexamples exist (Proposition 4.4)</td>
</tr>
</tbody>
</table>

$k + k'$ and an outdegree equals to $k$; and $A$ is $z$-credible if $G'$ has no cycles of length smaller than $z$.

4 ALGORITHMIC ANALYSIS

Let us first mention that while our model is faithful to the diversity of PGS situations (see Table 3), the parameter $k'$ has little computational impact in theory. Hence, unless stated otherwise, we assume in this section that there are no consumers in the review process ($C = 0, k' = 0$) and we use the notations PGS($k, z, e$, veto) and $k$-reviewable.

We start our algorithmic analysis by examining the connections between finding reviewable credible assignments in symmetric PGS instances and the graph-theoretic problem of finding $r$-factors with no short cycles. The correspondence is summarized in Table 2 and will let us transfer some complexity results in graph theory to the symmetric PGS problem.

4.1 Correspondence between PGS peer review selection and $r$-factors

PGS peer review selection is closely related to the graph-theoretic problem of finding $r$-factors, when consumers do not participate in the review process ($C = 0, k' = 0$), when no skills are required ($e = 0$), and when $V$ is symmetric. Lemmas 4.1, 4.2, and 4.3 present the condition under which the two problems are equivalent.

**Lemma 4.1.** For any $k$ and $z$, an instance of PGS($k, z, 0$, symmetric) has a solution if the potential review graph has a $2k$-factor that does not include cycles of length smaller than $z$.

**Proof.** Assume that the potential review graph admits a $2k$-factor, denoted $(P, E_0)$, which does not include cycles of length smaller than $z$. Algorithm 1 computes an assignment $A$ for PGS($k, z, 0$, symmetric). Intuitively, Algorithm 1 turns cycles from $E_0$ into directed circuits and adds them to $A$. Notice that it satisfies the following loop invariants.

- $E_0 = E \cup \{(x, y) \mid (x, y) \in A\}$.
- Each producer $p$ provides as many reviews as it receives in $A$: $|\{x \mid (x, p) \in A\}| = |\{y \mid (p, y) \in A\}|$.
- For each producer $p$, edges are conserved going from $E$ to $A$, that is $2k = \deg_{(P, E)}(p) + |\{x \mid (x, p) \in A\}| + |\{y \mid (p, y) \in A\}|$.

At the end of the loop, since $E$ is empty, we derive from the loop invariants that $A$ is $V$-respecting, $k$-reviewable, and $z$-credible. □

**Algorithm 1: 2k-factor to Assignment ($E_0$)**

1. $E \leftarrow E_0$
2. $A \leftarrow \emptyset$
3. While $E \neq \emptyset$ do
   4. select an arbitrary cycle $C$ from $E$
   5. orientate it to obtain a directed circuit $C'$
   6. $E \leftarrow E \setminus C$
   7. $A \leftarrow A \cup C'$
8. return $A$

At the end of the loop, since $E$ is empty, we derive from the loop invariants that $A$ is $V$-respecting, $k$-reviewable, and $z$-credible. □

**Lemma 4.2.** For any $k$, if an instance of PGS($k, 2, 0$, symmetric) admits a solution then its potential review graph has a $2k$-factor.

**Proof.** Assume that $G$ admits a solution for PGS($k, 2, 0$, symmetric), described as a directed subgraph $G' = (P, A)$. In $G'$, since each producer provides and receives $k$ reviews, the degree of each node is $2k$. Hence, by removing the orientation of the arrows in $A$, graph $G'$ is a $2k$-factor for $G$. □

**Lemma 4.3.** For any $z$, if an instance of PGS($1, z, 0$, symmetric) admits a solution then its potential review graph has a $2z$-factor that does not include cycles of length smaller than $z$.

**Proof.** Assume that $G$ admits a solution for PGS($1, z, 0$, symmetric), described as a directed subgraph $G' = (P, A)$. It implies that $A$ is a partition of $P$ into disjoint oriented cycles that does not contain any cycle of length smaller than $z$. Hence, by removing the orientation of arrows in $A$, graph $G'$ is a $2z$-factor for $G$ that does not contain any cycle of length smaller than $z$. □

Given an instance of PGS($k, z, 0$, symmetric), Lemma 4.1 states that a $2k$-factor in its potential review graph provides a solution for this instance. Conversely, Lemma 4.2 and 4.3 show that, when $k = 1$ or $z = 2$, a solution for a PGS($k, z, 0$, symmetric) instance implies a $2k$-factor without cycle of size lower than $z$ in its potential review graph. Therefore, when either $k = 1$ or $z = 2$, PGS($k, z, 0$, symmetric) is equivalent to finding a $2k$-factor in the potential review graph. However, Proposition 4.4 shows that this relation does not extend to arbitrary $k$ and $z$.

**Proposition 4.4.** For any $k \geq 2$ and $z \geq 3$, there exists a PGS($k, z, 0$, symmetric) instance that admits a $k$-reviewable, z-credible, and $V$-respecting assignment while any $2k$-factors of the potential graph includes cycles of length $\epsilon$ for every $3 \leq \epsilon \leq z$.

**Proof.** Given $k \geq 2$ and $z \geq 3$, we define the potential review graph $G = (P, E)$, with $P = \{p_i^j \mid 0 \leq i \leq z, 0 \leq j < k\}$ and $E = \{(p_i^j, p_i^j) \mid 0 \leq i \leq z, 0 \leq j < k - 1, j < i \leq k\}$

\[\cup \{(p_i^j, p_{i+1}^j) \mid 0 \leq i < z, 0 \leq j < k, 0 \leq j \leq l \leq j\}\]

First note that $G$ is $2k$-regular, and thus $G$ admits a unique $2k$-factor which is $G$ itself. Moreover, given $e$ such that $3 \leq e \leq z$, the set of edges $\{(p_i^0, p_0^j) \mid 0 \leq i \leq \left\lceil \frac{e}{2} \right\rceil - 1\} \cup \{(p_i^1, p_{i+1}^1) \mid 0 \leq i \leq \left\lceil \frac{e}{2} \right\rceil - 1\}$
Now we identify parameterizations of PGS\((k, z, \varepsilon, \text{veto})\) and PGS\((k, k', z, \varepsilon, \text{veto})\) that lead to tractable problems. We start by showing how PGS\((k, k', z, \varepsilon, \text{veto})\) reduces to PGS\((k, z, \varepsilon, \text{veto})\) when no skill is required, i.e., \(\varepsilon = 0\).

**Lemma 4.6.** Deciding PGS\((k, k', z, 0, \text{veto})\) reduces to deciding PGS\((k, z, 0, \text{veto})\) in polynomial time.

**Proof.** Notice that a producer \(p\) can receive \(k'\) reviews from consumers if and only if \(p\) does not veto more than \(|C| - k'\) consumers. Hence, computing an assignment of the consumers such that each producer receives \(k'\) reviews is polynomial. Since \(\varepsilon = 0\), combining this partial assignment with an assignment for PGS\((k, z, 0, \text{veto})\) forms a solution for PGS\((k, k', z, 0, \text{symmetric})\). □

The two first tractability results that we present rely on the equivalence between peer review selection in PGS and \(r\)-factors.

**Theorem 4.7.** For any \(k\), PGS\((k, 2, 0, \text{symmetric})\) is in \(P\).

**Proof.** Given a PGS instance \(P, V\) and a producer review target \(k\), we can compute in polynomial time whether its potential review graph admits a 2-\(r\)-factor [27]. We can then invoke Lemmas 4.1 and 4.2 to conclude.

**Theorem 4.8.** PGS\((1, 3, 0, \text{symmetric})\) is in \(P\).

**Proof.** Given a PGS instance \(P\) and \(V\), we can compute in polynomial time whether its potential review graph admits a 2-\(r\)-factor that does not contain cycles of size smaller than 3 [15]. We can then invoke Lemmas 4.1 and 4.3 to conclude.

Both results extend to the case where consumers do participate in the review process, as the following theorem shows.

**Theorem 4.9.** For any \(k\) and \(k'\), PGS\((k, k', 2, 0, \text{symmetric})\) and PGS\((1, k', 3, 0, \text{symmetric})\) are in \(P\).

**Proof.** Given a PGS instance \(S = (P, C), V\), and review targets \(k\) and \(k'\), we first reduce PGS\((k, k', 2, 0, \text{symmetric})\) and PGS\((1, k', 3, 0, \text{symmetric})\) to PGS\((k, 2, 0, \text{symmetric})\) and PGS\((1, 3, 0, \text{symmetric})\), respectively, in polynomial time by Lemma 4.6. Then, computing an assignment for PGS\((k, 2, 0, \text{symmetric})\) or PGS\((1, 3, 0, \text{symmetric})\) is polynomial by Theorems 4.7 and 4.8.

Theorems 4.7, 4.8, and 4.9 also give rise to a polynomial time algorithms when considering a weighted model of PGS with an utilitarian score (see future works in Section 6). Indeed, Meijer et al. [27]’s algorithm produces a \(r\)-factor by computing a perfect matching in a modified graph. This construction can be adapted to find a maximum weight perfect matching, which is known to be polynomial.

Next result identifies a tractable case for PGS\((k, z, s, \text{symmetric})\) when some skills are required at each reviewing visit, which doesn’t easily extend to the presence of consumers.

**Proposition 4.10.** For any fixed \(s\), PGS\((1, 2, s, \text{symmetric})\) is in \(P\).

**Proof.** When \(k = 1\), each producer has to possess all the skills, otherwise he cannot provide a review. Hence, if any producer misses a skill, i.e., if there exists \(i\in S\) such that \(S_i \subseteq V\), then we have a NO-instance. Otherwise, PGS\((1, 2, s, \text{symmetric})\) is equivalent to PGS\((1, 2, 0, \text{symmetric})\) which is in \(P\) by Theorem 4.7. □
Now, we show that parameterizations of PGS\((k, z, \epsilon, \text{veto})\) lead to computationally hard problems in general. We start with the cases where no skill is required.

**Theorem 4.11.** For any \(z \geq 5\), PGS\((1, z, 0, \text{symmetric})\) is NP-complete.

**Proof.** By Lemmas 4.1 and 4.3, PGS\((1, z, 0, \text{symmetric})\) is equivalent to finding a 2-factor that contains no cycle of length smaller than \(z\), which is an NP-complete problem when \(z \geq 5\). \(\square\)

Notice that the complexity of PGS\((k, 4, 0, \text{symmetric})\) appears difficult to decide since the complexity of finding a 2-factor which does not contain cycles of length smaller than 4 is still under investigation by the graph-theory community.

**Theorem 4.12.** PGS\((1, 2, 0, \text{general})\) is NP-complete.

**Proof.** A solution of an instance of PGS\((1, 2, 0, \text{general})\) partitions the vertices of the potential review graph into cycles. Hence, PGS\((1, 2, 0, \text{general})\) is equivalent to finding a partition of the vertices into hamiltonian subgraphs, which is NP-complete when cycles have to be of size greater than 3. \(\square\)

By Lemma 4.6, both results extend to the presence of consumers in the review process.

**Theorem 4.13.** For any \(k'\) and \(z \geq 5\), PGS\((2, k', z, 0, \text{symmetric})\) and PGS\((1, k', 2, 0, \text{general})\) are NP-complete.

Our main results, Theorem 4.14 and 4.15, show that the most realistic PGS settings lead to computationally hard problems.

**Theorem 4.14.** For any fixed number of reviews \(k \geq 2\), any fixed credibility \(z \geq 2\), and any fixed number of skills \(\epsilon \geq 1\), PGS\((k, z, \epsilon, \text{symmetric})\) is NP-complete.

**Proof.** We give a reduction for the result in the case of \(k = 2, z = 2\), and \(\epsilon = 1\). For other cases, the reduction can be adapted by introducing dummy producers as auxiliary needed.

We reduce PGS\((1, 2, 0, \text{general})\), which is NP-complete by Theorem 4.11, to PGS\((2, 2, 1, \text{symmetric})\) as follows. Let \((V, E)\) be the potential review graph of an instance \(I\) of PGS\((1, 2, 0, \text{general})\). We create an instance \(I'\) of PGS\((2, 2, 1, \text{symmetric})\) with potential review graph \((V', E')\) and skilled producer set \(S_1\), by using gadgets described in Figure 2.

To simplify notations, we assume in the following that producer indices are considered modulo 5, i.e., \(x_{i+5}\) is the same producer \(x_i\).

\[
V' = V \cup \{x_i | x \in V, 0 \leq i < 5\} \\
\cup \{x_i^y \mid (x, y) \in E, 0 \leq i < 5\} \\
S_1 = \{x_1, x_2, x_3 \mid x \in V\} \cup \{x_i^y, x_{i+1}^y, x_{i+2}^y \mid (x, y) \in E\} \\
E' = \{(x, x_0), (x_0, x) \mid x \in V\} \\
\cup \{(x_i, x_{i+1}) \mid x \in V, 0 \leq i < 5\} \\
\cup \{(x_i, x_{i+2}) \mid x \in V, 0 \leq i < 5\} \\
\cup \{(x_i^y, x_{i+1}^y), (x_{i+1}^y, x_{i+2}^y) \mid (x, y) \in E, 0 \leq i < 5\} \\
\cup \{(x_i^y, x_{i+1}^y), (x_{i+1}^y, x_{i+2}^y) \mid (x, y) \in E, 0 \leq i < 5\} \\
\cup \{(x_i^y, x_{i+1}^y), (x_{i+1}^y, x_{i+2}^y) \mid (x, y) \in E, 0 \leq i < 5\} \\
\cup \{(x_i^y, x_{i+1}^y), (x_{i+1}^y, x_{i+2}^y) \mid (x, y) \in E, 0 \leq i < 5\}
\]

![Figure 2: Gadgets. A circle around a producer signifies that they are skilled.](image)

Intuitively, for each vertex \(x \in V\), we create a vertex gadget (see Fig.2 (a)) which ensures that \(x\) provides and receives one review to/from \((x_i)_{0 \leq i < 5}\). In addition, for each directed edge \((x, y)\) in \(E\), we create an edge gadget (see Fig.2 (b)) which ensures that \((x_i^y)_{0 \leq i < 5}\) can only provide a skilled review to vertex \(y\).

Let us first prove that if instance \(I\) admits a solution, then the constructed instance \(I'\) also admits a solution. Let \(A' \subseteq E\) be the solution assignment for \(I\). Then we construct an assignment for \(I'\), by selecting all edges in the vertex gadgets and most edges in the edge gadgets, and orienting them as follows.

\[
A' = \{(x, x_0), (x_1, x) \mid x \in V\} \\
\cup \{(x_i, x_{i+1}) \mid x \in V, 0 \leq i < 4\} \\
\cup \{(x_i, x_{i+2}) \mid x \in V, 0 \leq i < 5\} \\
\cup \{(x_i^y, x_{i+1}^y) \mid (x, y) \in E, 0 \leq i < 4\} \\
\cup \{(x_i^y, x_{i+2}^y) \mid (x, y) \in E, 0 \leq i < 5\} \\
\cup \{(x_i^y, x_{i+1}^y), (x_{i+1}^y, x_{i+2}^y) \mid (x, y) \in A\} \\
\cup \{(x_i^y, x_{i+1}^y) \mid (x, y) \in E \setminus A\}
\]

One can directly check that \(A'\) is indeed a solution since every vertex is assigned two reviewers at least one of whom is skilled.

Let us now prove that if the constructed instance \(I'\) has a solution, then instance \(I\) also has a solution. Let \(A' \subseteq E\) be the solution assignment for \(I'\). Then we construct the assignment for \(I\) by selecting edges based on which skilled reviewer reviews the original vertices in \(V'\).

\[
A = \{(x, y) \mid x, y \in V, (x_i^y, y) \in A'\}
\]

To prove that \(A\) is a satisfying assignment, let us first show that each producer \(y \in V\) receives at least one review. Producer \(y\) is also a producer in \(I'\) and thus receives at least one skilled review in \(A'\). Since the only skilled producers adjacent to \(y\) in \(A'\) are of the form \(x_{i+4}^y\) for some \(x \in V\), there exists an \(x\) such that \((x_{i+4}^y, y) \in A'\). Thus there is \(x\) such that \((x, y) \in A\) and so \(y\) receives at least one review in \(A\).

We now prove that no producer \(x \in V\) participate in more than one review in \(A\). Consider the producers \(x_0, \ldots, x_4\). There exist only 9 edges linking them in \(E'\), which is not enough for all of them to receive two reviews. Therefore, at least one review among \((x_0, x)\) and \((x, x_4)\) belongs to \(A'\). As a result, for any \(x \in V\), there cannot be more than one \(y \in V\) such that \((x, x_i^y) \in A'\). \(\square\)
Theorem 4.15. For any fixed $k \geq 1$ and $z \geq 0$, $\text{PGS(INPUT, z, INPUT, empty)}$ is NP-complete.

Proof. We give an explicit reduction for the case $\text{PGS(INPUT, 0, INPUT, empty)}$. The reduction easily adapts to other values of $z$.

We reduce from $\text{Set Cover}$, a classic NP-complete problem defined in Section 2.2.

Let $X, C, t$ be a $\text{Set Cover}$ instance. Without loss of generality, we may assume that the subsets in the collection are numbered: $C = \{ c_0, \ldots, c_m \}$. We construct a PGS instance as follows. The set of producers is

$$P = \{ v_i, f_i \mid 0 \leq i < m \}$$

$$\cup \{ e^j_i \mid 0 \leq i < m, 0 \leq j < t - 2 \}$$

$$\cup \{ e \}$$

where $v_i$ are called the subset producers, $f_i$ are the full producers, and $e^j_i$ and $e$ are the empty producers. We create one skill per element of $X$ and define the skill sets such that all full producers have all skills, no empty producer has any skill, and a subset producer has the skills corresponding to its set. That is, for $x \in X$, we have $S_x = \{ v_i \mid x \in c_i \} \cup \{ f_i \mid 0 \leq i < m \}$.

We will prove that $(P, (S_x)_{x \in X})$ admits a $(t, 0)$-reviewable and E-compatible assignment if and only if $X, C$ admits of cover of size $t$.

To simplify notations, we will assume that the producer indices are cyclical so that $f_{m+i}$ is the same producer as $f_i$, and $e^j_{m+i}$ is the same as $e^j_i$. Let $D \subseteq C$ be a subcollection of subsets, we can create an assignment as follows

$$A_D = \{(f_i, f_{i+1}) \mid 0 \leq i < m \}$$

$$\cup \{(f_i, v_i), (v_i, f_{i+1}) \mid 0 \leq i < m \}$$

$$\cup \{(f_i, e^j_i), (e^j_i, f_{i+1}) \mid 0 \leq i < m, 0 \leq j < t - 2 \}$$

$$\cup \{(v_i, e^j_i), (e^j_i, v_{i+1}) \mid 0 \leq i < m, 0 \leq j < t - 2 \}$$

$$\cup \{(e^j_i, e^{j+k}_{i+1}) \mid 0 \leq i < m, 0 \leq j < k \leq t - 2 \}$$

$$\cup \{(v_i, e), (e, v_{i+1}) \mid 0 \leq i < m, c_i \notin D \}$$

$$\cup \{(v_i, v_{i+1}) \mid 0 \leq i < m, c_i \notin D \}$$

It is straightforward to observe that if $D$ has size $t$ and covers $X$ then $A_D$ is a $(t, 0)$-reviewable and E-compatible assignment. Thus, if the $\text{Set Cover}$ instance admits a solution, then so does the PGS instance.

For the other direction, observe first that there are $tm + 1$ producers and $m$ full producers. In any $(t, 0)$-reviewable assignment $A$, each full producer reviews $t$ other people, so at the very least one producer $p$ is not reviewed by a full producer. Choose one such producer $p$ arbitrarily and define $D_A = \{ c_i \mid (v_i, p) \in A \}$ to be the subsets corresponding to the subset producers reviewing $p$ in $A$. Since $A$ is $(t, 0)$-reviewable, we have $|D_A| \leq t$. It is easy to see that if $A$ is E-compatible, then $D_A$ is cover of $C$. Thus, if the PGS instance admits a solution, then so does the $\text{Set Cover}$ instance.

Notice that Theorem 4.14 and Theorem 4.15 can be extended to the presence of consumers by adding dummy consumers. Furthermore, with a similar reduction as the one for Theorem 4.15, we can show that $\text{PGS(k, INPUT, z, INPUT, empty)}$ is also NP-complete.

Theorem 4.16. For any fixed $k \geq 1$ and $k' \geq 0$, any fixed $z \geq 2$, and any fixed $\varepsilon \geq 1$, $\text{PGS(k, k', z, \varepsilon, symmetric)}$ is NP-complete. For any fixed $k \geq 1$ and $k' \geq 0$, and $z \geq 0$, $\text{PGS(INPUT, k', z, INPUT, empty)}$ and $\text{PGS(k, INPUT, z, INPUT, empty)}$ are NP-complete.

5 ANSWER SET PROGRAMMING MODELLISATION

While some parameterizations of the PGS decision problem are tractable, they require restrictions that are not desirable for end-users. However, the membership in the class NP (Proposition 4.5) means that we can encode the problem in existing solving formalisms (such as SAT, Integer Programming, or Answer Set Programming) and invoke high-performance off-the-shelf software to solve our PGS instances.

We chose to develop an Answer Set Programming approach to solve PGS scenarios which will be made available to the PGS community. In our approach, and in accordance with practice in ASP, we separate the encoding of the problem from the encoding of the instances. Specifically, we have a 3-layered approach: in the first layer, a single file (spg.constraints.lp) contains the logic and constraints relevant to PGS in a generic way; in the second layer, a file contains the parameterization corresponding to the rules of a specific PGS organization or country (e.g., spg.config.india.1p); the last layer contains the data corresponding to an instance we want to solve, i.e., the name of the stakeholders and which skills they possess as well as the vetoes between them, if any (for instance the file spg.data.india.2019.1p).

In Section 5.1, we describe three distinct parameterizations of our PGS model that correspond to real-world PGS organizations. The specific data we used in our experimentation is presented in Section 5.2.

We experimented with six scenarios altogether, and all our simulations can be solved within 1 second on a standard laptop machine with the Clingo Answer Set Programming solver [10]. This demonstrates that although PGS is intractable in theory, real-world instances are small enough to be addressed automatically.

5.1 Modeling real-world PGS organizations

The initial motivation for this model comes from real-life PGS stakeholders in three countries: Morocco, France and India. The model, which is built to easily adapt to the diversity of PGS, was tested on several instances of these three cases. In all cases, only a minimal credibility rule is applied: $z = 2$. We describe these cases in this section. See Table 3 for a summary of parameterizations.

Reviewers’ assignments differ from one year to the following, both to promote knowledge exchange and to increase the credibility of the system (by decreasing the risk of collusion). Nevertheless, some groups prefer to keep an identical reviewer from one year to the following (e.g. a producer or a consumer in Morocco, or the

\footnote{The ASP encoding and the data files can be found at https://bitbucket.org/Abdallah/participatory-guarantee-systems/. Our experiments can be reproduced using the ASP solver Clingo [10].}
In 2018, SPG Agroécologie had 16 producers and 10 consumers, of which 15 skilled in review) and 8 consumers (all skilled in review).

In 2019, including newcomers, the PGS had 27 producers and 19 consumers. In total, taking into account new skills acquired by initial producer members plus consumers. All groups organize reviewing in the same fashion. Each production site is reviewed by one producer and one consumer: \( k = 2 \) and \( k' = 1 \). The PGS takes into account two skills: \( E = \{ \text{review}, \text{agroecology} \} \). Valid reviews require that each skill should be possessed by at least one reviewer. All producers undertake to participate to at least two farm reviews.

Nature & Progrès (France) was created in 1964 and keeps evolving and expanding.\(^5\) It is organized as a network of 23 relatively independent groups. Local groups account from 20 to 60 members producer members plus consumers. All groups organize reviewing in the same fashion. Each production site is reviewed by one producer and one consumer: \( k = 1 \) and \( k' = 1 \). All producers undertake to participate to at least one farm review. For now, a single skill is taken into account: \( E = \{ \text{review} \} \). Each producer needs to be reviewed by at least one person that is skilled in reviewing.

PGS India Organic is a governmental system, which was created in 2011.\(^6\) In 2019, the PGS comprised 18 179 local groups organized into 326 regional councils. Local groups include from a minimum of five to several dozen members. Each farm is reviewed by three to five peer reviewers (according to the group). In the case of small groups, members from other PGS groups are invited to perform reviews. Each producer participates to at least one farm review. Reciprocal review between two producers of the same group is not allowed. A single skill is taken into account: at least one reviewer must be literate to be able to fill the evaluation report: \( E = \{ \text{literate} \} \).

5.2 Typical data Experimental evaluation of an ASP encoding
In 2018, SPG Agroécologie had 16 producers and 10 consumers, of which 11 (7 producers and 4 consumers) were skilled in reviewing and 12 producers skilled in agroecology. One member was in an unusual position that generated an important number of vetoes. In 2019, including newcomers, the PGS had 27 producers and 19 consumers. In total, taking into account new skills acquired by initial members, 16 were experienced in reviewing and 16 in agroecology. To solve this case, we added the 2018 review assignment as vetoes. All interpersonal vetoes from the previous year were lifted, except the one concerning two brothers. Finally, we simulated a review round for 2020 adding the assignment of 2019.

In 2019, in the specific Nature & Progrès group (from Hérault region, South of France) we tested the model with 20 producers (of which 15 skilled in review) and 8 consumers (all skilled in review).

For PGS India Organic, we tested the model by simulating a group from South Andaman Islands of 20 members, 12 of whom are literate. We ran 3 rounds to simulate 2019, 2020, and 2021.

6 CONCLUSION AND FUTURE WORKS
This paper focused on peer review selection in PGS. It emerged from real-life PGS stakeholders’ demand and was modeled according to authors’ knowledge of such systems. We proposed a formal model encompassing the diversity of PGS local situations. While we showed that peer review selection in PGS may lead to computationally hard problems, we identified tractable cases. Finally, our encoding in ASP shows that modern solvers can handle this problem in practice.

The proposed model and its ASP implementation are now being tested with stakeholders in real PGS settings - first in February 2020 in Morocco - to ensure they are adapted to practical use. During these tests, new development needs will certainly emerge. We can already foresee future works that comprise extensions of our formal model and results on parameterized complexity.

Dynamic model Regular monitoring reviews, generally on a yearly basis, introduce a dynamic component to PGS. The stakeholders population can change, typically new producers may join an emerging PGS, or a PGS could split when it becomes too large to be managed locally. A dynamic model would allow us to model constraints over multiple years, e.g., ensuring that producers are not reviewed by the same producers each year.

Weighted model PGS are considered cheaper and less time consuming than traditional third-party certification systems. Our model can be extended by adding weights between producers which would correspond to the cost of making one producer review the other. An interesting solution would then minimize some functions of the costs that it induces.

Few vetos Producers usually express few vetos towards other producers. A study on parameterized complexity with respect to the number of vetos would help us identify cases that are tractable in practice.

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\(^4\)http://reseauriam.org/upload/documents/rispgmarocdef5juin.pdf
\(^5\)https://www.natureetprogres.org/
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