Limiting the Deviation Incentives in Resource Sharing Networks*

Extended Abstract

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1 INTRODUCTION

Crowdsourcing, Peer-to-Peer (P2P) resource networks and sharing economies have created a sequence of new waves [11, 13, 16], to explore the power to match the needs of users and supplies from providers through the Internet and mobile networks [7, 10, 15]. To motivate sharing, [12] pioneered the use of incentive techniques to drive cooperation and to promote voluntary contributions by participating agents. Arguably, the BitTorrent protocol, created by Cohen [9], has been well recognized, and then the BitTorrent network becomes one of few Internet wide successful P2P systems [17].

Pioneered by the idea of BitTorrent, Wu and Zhang [19] proposed the proportional response protocol (PRP) to allocate the bandwidth resource on P2P networks, under which each peer contributes its bandwidth to its neighbors in proportion to the amount received from them. The PRP not only is local information based, but also converges to a fixed point solution, which is exactly the same with a market equilibrium [19]. Those classic works developed a theory on the economic property of PRP except its incentive compatibility open. However, how to provide agent incentives to follow the designed protocols stands out the most important factor dictating the success of the designed protocols. The work in [19] left an open problem whether an agent would gain more utility if it cheats to report false information about its private information.

Recent effort has been made to show the PRP protocol promotes voluntary participations of network agents. [5, 6] have demonstrated that no agent can gain more utility by manipulative actions, such as cutting communication links or misreporting the amount of resource it owns. Obviously, the more manipulation a protocol can withstand, the better the protocol will perform in the realistic environment.

Unfortunately, in further analysis, it was shown that an agent can make such deviations to change the resource allocation for its benefit [1, 14, 18]. Past progress in this direction has been slow. Recently, several studies [2–4, 8] have been conducted on the incentives from Sybil attack, a grave threat in P2P system, for the resource sharing on some special networks such as trees, cliques and cycles, and characterize how much one can improve its utility at most and formalize the improvement formally by the concept of incentive ratio.

The contribution in this paper is to establish the first result with a constant upper bound of incentive ratio on general networks. Our work settles a standing open problem along the line of research for bounding the incentive ratio for Sybil attack to the BitTorrent system.

2 PRELIMINARIES

The resource sharing problem on P2P network is modeled on an undirected graph $G = (V, E, w)$, on which each vertex represents an agent with an upload resource amount $w_v$. Denote the neighborhood of $v$ by $\Gamma(v)$. Let $X = \{x_{uv}\}$ be an allocation of resource in the system, where $x_{uv}$ is the amount of resource agent $v$ allocates to its neighbor $u$. The utility of agent $v$ from allocation $X$ is $U_v(X) = \sum_{u \in \Gamma(v)} x_{uv}$.

Naturally, the resource sharing on P2P network can be understood as a pure exchange market, in which agent brings divisible resource to the market and exchanges its resource with neighbors to derive utility. As market equilibrium is well known as a standard economic notion for characterizing efficient allocations, we would like to pursue an allocation from a market equilibrium for resource sharing on P2P networks. Many works focused on the computation of market equilibrium. Specially for the resource sharing problem on P2P network, Zhang and Wu [19] proposed a proportional response dynamics to fairly allocate resource, which converges to a market equilibrium, and the corresponding equilibrium allocation can be obtained by a combinatorial mechanism.

Bottleneck Decomposition and BD Allocation Mechanism. For a subset $S \subseteq V$, let us define $w(S) = \sum_{v \in S} w_v$, and $\Gamma(S) = \cup_{v \in S} \Gamma(v)$. Note that $\Gamma(S) \cap S = \emptyset$ if and only if $S$ is independent. Define $\alpha(S) = w(\Gamma(S))/w(S)$, named as the $\alpha$ ratio of $S$. A set $B$ is

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called a bottleneck, if $\alpha(B) = \min_{S \subseteq V} \alpha(S)$. In addition, the bottleneck with the maximal size, is called the maximal bottleneck.

The bottleneck decomposition of a graph is obtained as follows. Given $G = (V, E, w)$, start with $V_1 = V$, $G_1 = G$, and $i = 1$. Find the maximal bottleneck $B_i$ of $G_i$ and let $G_{i+1}$ be the induced subgraph on $V_{i+1} = V_i - (B_i \cup C_i)$, where $C_i = \Gamma(B_i) \cap V_i$. Repeat the process until $V_{k+1} = \emptyset$ for some $k \geq 1$. $B = \{B_1, C_1, \ldots, B_k, C_k\}$ is the bottleneck decomposition of $G$, in which $(B_i, C_i)$ is the $i$-th bottleneck pair and $\alpha_i = w(C_i)/w(B_i)$ is the $\alpha$-ratio of $(B_i, C_i)$. Based on the bottleneck decomposition, we call vertex $v$ a B class (or a C class) vertex if $v \in B_i$ (or $v \in C_i$), $i \leq k$, respectively.

From a bottleneck decomposition, an allocation (named as BD allocation) can be explored from BD Allocation Mechanism as follows. For a pair $(B_i, C_i)$, construct network $N = (V_N, E_N)$, with $V_N = \{s, t\} \cup B_i \cup C_i$ and directed edges $(s, u)$ with capacity $w_u$ for $u \in B_i$, $(u, t)$ with capacity $w_t/\alpha_i$ for $v \in C_i$ and $(u, v)$ with capacity $+\infty$ for $(u, v) \in (B_i \times C_i) \cap E$. The max-flow min-cut theorem ensures a maximal flow $\{f_{uv}\}$, $u \in B_i$ and $v \in C_i$, such that $\sum_{u \in \Gamma(u) \cap C_i} f_{uv} = w_u$ and $\sum_{v \in \Gamma(v) \cap B_i} f_{uv} = w_t/\alpha_i$. Let $x_{uv} = f_{uv}$ and $x_{vu} = \alpha_i f_{uv}$. For any other edge $(u, v) \in (B_i \times C_i) \cap E$, let $x_{uv} = 0$.

Wu and Zhang [19] proposed some important results showing that, if set $p_u = \alpha_i w_u$, when $v \in B_i$; and $p_v = w_t$ when $v \in C_i$, then the price vector $p = (p_u)$ together with BD allocation $X$ is a market equilibrium. Moreover, $U_v(X) = \alpha_i w_v$ if $v \in B_i$ and $U_v = w_t/\alpha_i$ if $v \in C_i$.

Resource Sharing Game. Some works are interested in the resource sharing game, in which an agent may manipulate BD allocation mechanism by different strategies to change the resulting allocation. We mainly study the impact of Sybil attack on the resource sharing game, which is a more computationally incentive ratio of BD allocation mechanism to characterize the extent to which utilities can be increased. Sybil attack is modeled as: agent $v$ splits itself into $m$ nodes $v^1, \ldots, v^m$, $1 \leq m \leq d_v$ ($d_v$ is the degree of $v$) and assigns amount $w_{v^i}$ of resource to each node $v^i$, satisfying $0 \leq w_{v^i} \leq w_v$ and $\sum_{i=1}^{m} w_{v^i} = w_v$.

We observed that the maximum utility from Sybil attack can be achieved by splitting into $d_v$ nodes and each node is connected to one neighbor. Such an observation simplifies the discussion and make the strategic agent only focus on the weight assignment among $d_v$ nodes to obtain the optimum. In the resulting network $G'$, $v$’s new utility $U'_v$ is the sum of utilities from all copied nodes, denoted by $U'_v$. Then the incentive ratio of $v$ is informally defined as $\xi_v = \max(U'_v/U_v)$ and the incentive ratio of BD allocation mechanism is $\xi = \max \xi_v$. The main contribution in this paper is the following.

Theorem 2.1. The incentive ratio of BD allocation mechanism against Sybil attack on general network is no more than three, i.e., $\xi \leq 3$.

3 INCENTIVE RATIO ON GENERAL NETWORKS

From the observation in Section 2, we only need to study the strategy that $v$ splits into $d$ nodes, $\{v^1, \ldots, v^d\}$, each node being adjacent to one neighbor, and assigns weight $(w_{v^1}, \ldots, w_{v^d})$ to each node. To obtain the result in Theorem 2.1, we shall prove for any $(w_{v^1}, \ldots, w_{v^d})$, $v$ can not get more than three times of its original utility, i.e., $U'_v \leq 3 \cdot U_v$. Our main technique is to decompose the whole process from the initial network $G$ to the ultimate one $G'$ into $d - 1$ step. In each step, $v$ plays once Resource Reserved Binary (RRB) split. Given allocation $X$ on current network, we say an agent plays RRB split to split one node out, if it partitions its neighborhood into two disjoint subsets: $N_1$ and $N_2$; and then splits itself into two nodes $v'$ and $v''$, along with weights $w_{v''} = \sum_{u \in N_1} x_{uv''}$, such that $v'$ is adjacent to all neighbors in $N_1$, $i = 1, 2$. In addition, between two adjacent steps, we shall adjust the weights of nodes properly. Therefore, we develop a successive process to help us to compute the incentive ratio.

Transform Process. Given the initial network $G$ with weight profile $w$, BD allocation $X$ of $G$ and a weight assignment $\{w_{v^1}, \ldots, w_{v^d}\}$, transform process includes:

- Pre-Processing: For each neighbor $u'$, if $x_{uv'} = w_{u'}$, let $v$ split node $v''$ out and connect $v'$ to $u'$ with weight $w_{v'} = w_{u'}$. At the end of pre-processing, some neighbors of $v$ are adjacent to nodes one-to-one, and others are connected to one node $\tilde{v}$. Set the the resulting network to be $G$ and $v' := \tilde{v}$.

- RRB Split: Partition $\Gamma(v)$ into two disjoint subsets: $\tilde{N} = \{u' | x_{uv'} < w_{u'}\}$ and $\tilde{N} = \{u'' | x_{uv''} > w_{u''}\}$. Let $v$ play RRB split to split itself into two nodes $\tilde{v}$ and $\tilde{v}$, such that all neighbors in $\tilde{N}$ or $\tilde{N}$ are connected to $\tilde{v}$ or $\tilde{v}$, respectively.

- Increasing process: Increase $w_{\tilde{v}}$ continuously. During the increasing process, BD allocation $X$ changes and at least one neighbor $u'' \in N$ obtains more resource. Once the updated allocation $x_{uv'''} = w_{uv''}$, then let $\tilde{v}$ split $u''$ out and connect $u''$ to $\tilde{v}$. Denote $\tilde{v}$ to be the node that the rest neighbors in $\tilde{N}$ are connected to. Set $\tilde{v} := \tilde{v}$ and continue to increase $w_{\tilde{v}}$ until each split out node is a leaf and has a weight of $w_{u''}$.

- Decreasing process: Decrease $w_{\tilde{v}}$ continuously. During the decreasing process, BD allocation $X$ changes and at least one neighbor $u'' \in \tilde{N}$ obtains less resource. Once the updated allocation $x_{uv'''} = w_{uv''}$, then let $\tilde{v}$ split $u''$ out and connect $u''$ to $\tilde{v}$. Denote $\tilde{v}$ to be the node that the rest neighbors in $\tilde{N}$ are connected to. Set $\tilde{v} := \tilde{v}$ and continue to decrease $w_{\tilde{v}}$ until each split out node is a leaf and has a weight of $w_{u''}$.

In our constructive proof, we first execute the increasing process to increase $w_{\tilde{v}}$ and split some node out iteratively, then execute the decreasing process to decrease $w_{\tilde{v}}$ and split some node out iteratively. Note that there is a interim network $G$, at which the increasing process has finished and then the decreasing process is about to begin. Conveniendly, we denote $U_v, \tilde{U}_v$ and $U'_v$ to be the initial utility on $G$, the interim utility on $G$ and the ultimate utility on $G'$.

Our proof for Theorem 2.1 includes two parts. The first is to prove $\tilde{U}_v \leq 2 \cdot U_v$ if $v$ is in $B$ class; and $\tilde{U}_v \leq 3 \cdot U_v$ if $v$ is in $C$ class, at the end of the increasing process. The second it to prove $U'_v \leq \tilde{U}_v$ at the end of the whole process.

Proof for Theorem 2.1. In summary, if $v$ is a C-class vertex, then $U'_v \leq \tilde{U}_v \leq 3 \cdot U_v$; if $v$ is a B-class vertex, then $U'_v \leq \tilde{U}_v \leq 2 \cdot U_v$. □
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