ABSTRACT
Although plenty of qualitative logical frameworks have been proposed to evaluate and model trust in multi-agent settings [2, 6, 9, 11], these approaches generally ignore reasoning about quantitative aspects such as degrees of trust. In this paper, we address this limitation from the modelling and verification perspectives. We construct TCTL\textsuperscript{G}, a logical language to represent the quantitative aspect of trust. Moreover, we develop and implement a new symbolic model checking algorithm for quantifying the relationships among the interacting agents. Finally, we evaluate the tool and report experimental results using a healthcare scenario.

CCS CONCEPTS
- Theory of computation:

KEYWORDS
Degrees of Trust, Temporal Logic, Model Checking

ACM Reference Format:

1 PRELIMINARIES
To describe a MAS, we employ the formalism of Vector-based Interpreted Systems VIS introduced in [3], which is an extended version of the original Interpreted Systems, introduced in [5] and widely used for model checking MAS [4], for instance in [7, 8].

The formalism of VIS includes the notion of agents’ vector \( v \). That is, for each agent \( i \in \text{Agt} \), a vector \( v^i \) of size \( |\text{Agt}| \) is associated with each local state \( l_j \in L_j \) of this agent. \( v^i(j), v^i(k) \) are the components of the vector \( v^i \) where \( (i,j,k) \in \text{Agt}^3 \). The vector \( v \) is used to define the trust accessibility relation \( \sim_{i,j} \). Intuitively, the relation \( \sim_{i,j} \) relates the states that are considered to be trustful from the vision of agent \( i \) with regard to agent \( j \). Specifically, for two global states \( s, s' \in S \) where \( s' \) is reachable from \( s \), \( s \sim_{i,j} s' \) is obtained by comparing the value \( v^i(j) \) in the local state \( l_j \) at the global state \( s \) (denoted by \( l_j(s)(v^i(j)) \)) with \( v^i(j) \) in the local state \( l_j \) at the global state \( s' \) (i.e., \( l_j(s')(v^i(j)) \)). Thus, the trust accessibility of agent \( i \) towards agent \( j \) (i.e., \( \sim_{i,j} \)) does exist only if the element values that we have for agent \( j \) in the vector of the local states of agent \( i \) for both global states \( s \) and \( s' \) are the same, i.e., \( l_j(s)(v^i(j)) = l_j(s')(v^i(j)) \) (we refer to [3] for more details about the formalism). We use this formalism to define the model of TCTL\textsuperscript{G}.

Definition 1 (Model of TCTL\textsuperscript{G}). The model of TCTL\textsuperscript{G} is a tuple: \( M_G = (S_G, I_G, R_G, \sim_{i,j} \mid (i, j) \in \text{Agt}^2), V_G \) where: \( S_G \) is a non-empty set of reachable global states of the system; \( I_G \subseteq S_G \) is a set of initial global states; \( R_G \subseteq S_G \times S_G \) is the transition relation; \( \sim_{i,j} \subseteq S_G \times S_G \) is the direct trust accessibility relation for each truster-trustee pair of agents \( (i, j) \in \text{Agt}^2 \) defined by \( s \sim_{i,j} s' \) iff: \( l_j(s)(v^i(j)) = l_j(s')(v^i(j)) \), and \( s' \) is reachable from \( s \) using transitions from the transition relation \( R; V_G : S_G \to 2^{AP} \) is a labeling function, where \( AP \) is a set of atomic propositions.

In this model, infinite sequences of states linked by transitions define paths. If \( \pi \) is a path, then \( \pi(i) \) is the \((i+1)^{th}\) state in \( \pi \).

2 GRADED TRUST TEMPORAL LOGIC

2.1 Syntax and Semantics
In this section, we present the syntax and semantics of TCTL\textsuperscript{G}.

Definition 2 (Syntax of TCTL\textsuperscript{G}). The syntax of TCTL\textsuperscript{G} is defined recursively as follows:

\[
\varphi ::= \rho \mid \neg \varphi \mid \varphi \lor \varphi \mid \text{E} \varphi \mid \text{E}(\varphi \lor \varphi) \mid A(\varphi \lor \varphi) \mid T
\]

\[
T ::= T_p^{\Delta_k} (i, j, \varphi) \mid T_c^{\Delta_k} (i, j, \varphi, \varphi)
\]

The formula \( \varphi \) is one of the following: an atomic proposition, a negated formula, or two formulae connected by the connective \( \lor \). Moreover, \( E \) and \( A \) are the existential and universal quantifiers on paths. \( X \) and \( U \) are CTL path connectives standing for “next”, and “until” respectively. The trust operator \( T \) represents the trust relationship between two agents. There are two trust modalities: \( T_p^{\Delta_k} \) and \( T_c^{\Delta_k} \), that represent respectively preconditional and conditional graded trust. From the syntax perspective, \( T_p^{\Delta_k} (i, j, \varphi, \psi) \) expresses that “the trustor \( i \) trusts the trustee \( j \) to bring about \( \psi \) given that the precondition \( \varphi \) holds with a degree of trust \( \Delta_k \)”, where \( k \) is a rational number in \([0, 1]\), and \( \Delta \) is a relation symbol in the set \( \{\leq, \geq, <, >, =\} \). While the formula \( T_c^{\Delta_k} (i, j, \varphi, \psi) \) reads as “agent \( i \) trusts agent \( j \) about the consequent \( \psi \) when the antecedent \( \varphi \) holds with a degree of trust \( \Delta_k \)”. It is worth pointing that when \( k = 0 \), it means the trust has not been achieved, however, when \( k = 1 \), the trust has been perfectly fulfilled. Moreover, when the degree of trust \( k = 1 \), the standard trust operators \( T_p(i, j, \varphi, \psi) \) and \( T_c(i, j, \varphi, \psi) \) can be obtained as abbreviations:

\[
T_p(i, j, \varphi, \psi) \triangleq T_p^{1}(i, j, \varphi, \psi) \quad \text{and} \quad T_c(i, j, \varphi, \psi) \triangleq T_c^{1}(i, j, \varphi, \psi)
\]

For example the following formula specifies that it is not possible, with degree at least 0.95, for the buyer to trust the seller to deliver
the requested items if the payment has not been made.

\[ \neg EF T^c_{\leq 0.95} \text{(buyer, seller, \neg payment, deliver)} \] (1)

**Definition 3 (Semantics of TCTL^G).**

Excluding the graded trust, the semantics of TCTL^G is defined in the standard manner (see for example [1]). The intuition behind the semantics of TCTL^G is: the degrees of trust that an agent associates to a formula \( \varphi \) in a global state \( s \) is the ratio between the number of states \( s' \) distinguishable and accessible from \( s \) and satisfying \( \varphi \) (i.e., \(| s \sim_{i \rightarrow j} s': s' \neq s & s' \models \varphi \)) and the total number of distinguishable and accessible states from \( s \) (i.e., \(| s \sim_{i \rightarrow j} s': s' \neq s \)). This would be formalized as follows:

- \((M_G, s) \models T^M_p(i, j, \psi, \varphi)\) if \( s \models \psi \land \neg \varphi \) and \( \exists s' \neq s \) such that \( s \sim_{i \rightarrow j} s' \) and \(| s \sim_{i \rightarrow j} s': s' \neq s & s' \models \varphi \Delta k; \)
- \((M_G, s) \models T^M_c(i, j, \psi, \varphi)\) if \( s \models \neg \psi \land \neg \varphi \) and \( \exists s' \neq s \) such that \( s \sim_{i \rightarrow j} s' \) and \( \neg \psi \models \varphi \) and \(| s \sim_{i \rightarrow j} s': s' \neq s & s' \models \varphi \Delta k; \).

### 3 MODEL CHECKING TCTL^G

Model checking is the problem of automatically establishing whether or not a formula is satisfied on a given model. In this section, we present an efficient algorithm for the TCTL^G model-checking problem. Indeed, we extend the standard symbolic model checking algorithm for CTL [1] by simply adding procedures that compute the set of states that satisfy the graded trust formula.

#### 3.1 BDD-based Algorithm of Graded Trust

This section introduces the model checking algorithms for both the \( T^M_p \) and \( T^M_c \) operators. Given a TCTL^G formula \( \Phi \) and a TCTL^G model \( M_G \) over the vector-based interpreted system, the two algorithms compute the set of states of \( M_G \) in which \( \Phi \) holds. Algorithm 1 describes the procedure \( SMCMC_T(i, j, \psi, \varphi, \Delta k, M_G) \). This procedure returns the set of states in which the preconditional graded trust formula holds. First, the algorithm starts by computing the set \( Y \) of states in which the negation of the formula \( \varphi \) holds. Afterwards, the procedure calculates the set \( X_1 \) (the set of states satisfying \( \psi \land \neg \varphi \)). Thereafter, it assigns to the set \( X_2 \) the set of states where the formula \( \varphi \) holds. Thereafter, the algorithm proceeds to build and return the set \( Z \) by computing the set of states in \( X_1 \) such that their number of accessible states that are in \( X_2 \) over the total number of their accessible states minus 1 satisfies the appropriate relation \( \Delta k \).

To compute the formula \( T^M_c(i, j, \psi, \varphi) \), we follow the same steps in Algorithm 1, except lines 2 and 3 which assign to the set \( X_1 \) the set of states satisfying \( \neg \varphi \), and to the set \( X_2 \) the set of states satisfying \( \psi \Rightarrow \varphi \). Indeed, this is based on our proposed semantics of conditional graded trust where the set of global states satisfying the formula \( T^M_c(i, j, \psi, \varphi) \) in a given model \( M_G \) is computed by calculating and checking if the ratio between the number of states satisfying \( \psi \Rightarrow \varphi \) over the total number of all states that can reach and see such states through the accessibility relation \( \sim_{i \rightarrow j} \) satisfies the appropriate relation \( \Delta k \).

#### 4 IMPLEMENTATION AND EXPERIMENTS

We implemented our algorithms by extending the model checker MCMAS [10]. We consider the Breast Cancer Diagnosis and Treatment (BCDT) protocol as an illustrative application example to show how our model checking technique can efficiently be applied on a medical health care platform to check the trust transactions against some quantified temporal trust conditions. In [7, 12], the authors formalized this scenario in terms of commitments, identifying the contractual business relationships among the parties involved. Indeed, such relationships can be founded as a basic of defining trust specifications as requirements for engineering contracts among parties. We use our formal model \( M_G \) associated to the vector-based interpreted systems introduced earlier in Section 1 to formally model the BCDT protocol. Moreover, to check the correctness of the process model, we consider some protocol properties that are expressed in the TCTL^G logic.

#### 4.1 Experimental Results

In order to assess the scalability of our technique and implementation, we measured the model checking processing time to construct the model and the BDD memory usage to successfully perform the verification task when running on a machine Intel(R) Core(TM) i7-6700 CPU @ 3.40GHZ with 16 GB memory. We run our experiments with a number of agents ranging from 6 to 30. The experiments revealed that all the tested formulae are satisfied. Table 1 recorded the verification results along with the number of agents and the reachable states in the model constructed. We can observe that the number of reachable states reflects the fact that the state space increases exponentially with the number of agents. It is also worth noticing that the memory consumption increases polynomially. However, the program timed out when the number of agents exceeds 30. Yet, it is still acceptable for detecting design errors in scalable models.

### Table 1: Verification results of the BCDT protocol

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1Available at: http://aspe.hhs.gov/sp/reports/2010/PathRad/index.html
REFERENCES


