Microbribery in Group Identification

Extended Abstract

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ABSTRACT

This paper studies the complexity of two microbribery problems under the model of group identification. In these problems, we are given a subset of distinguished individuals, and the questions are whether these individuals can be made socially qualified or whether they can be made exactly the socially qualified individuals, respectively, by modifying a limited number of entries in the qualifications-profile. For consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule, we obtain many NP-hardness results as well as polynomial-time solvability results. We also study the problems in r-profiles where each individual qualifies exactly r individuals.

KEYWORDS

group identification; complexity; bribery; consent rules; consensus-start-respecting rule; liberal-start-respecting rule

ACM Reference Format:
Gábor Erdélyi and Yongjie Yang. 2020. Microbribery in Group Identification. In *Proc. of the 19th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2020), Auckland, New Zealand, May 9–13, 2020,* IFAAMAS, 3 pages.

1 INTRODUCTION

In group identification, we are given a profile consisting of a set of individuals each of whom either qualifies (represented by 1) or disqualifies (represented by 0) every individual (including themselves). A subset of individuals, called socially qualified individuals, are identified by a certain social rule based on the valuations of all individuals. It may be relevant to the application where a group of autonomous agents are faced with the problem to select several agents to complete a particular task. Mathematically, group identification can be also regarded as a specific approval-based multiwinner voting model with two specifications: (1) voters and candidates coincide, and (2) there is no restriction on the number of winners.

Since the first work on group identification by Kasher [8], a number of social rules have been proposed, among which, the consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule have received a considerable amount of attention (see, e.g., [2, 3, 9, 10, 12]). Particularly, these rules have been well characterized by their axiomatic properties which provide significant guidance to evaluate these rules [3, 9, 12]. However, the resistance of these rules to strategic behavior has less been investigated in the literature. So far, only a few papers pertaining to this topic have been published dealing with manipulation, control, and bribery in group identification [4, 5, 13]. In this paper, we continue the line of this research by studying a more fine-grained version of bribery tailored especially for profiles with 0/1 entries, namely, microbribery in group identification for the aforementioned social rules. Our goal is to provide a more comprehensive guidance on the resistance of these rules to manipulative behaviors. Generally speaking, in microbribery (introduced in the context of voting by Faliszewski et al. [6] and has been investigated for several voting rules [1, 11]), there is a subset of distinguished candidates and we want to make all of them socially qualified (in a variant, we want these distinguished individuals exactly being the socially qualified individuals) by flipping at most ℓ entries in a given profile. For the aforementioned rules, we establish both NP-hardness results as well as many polynomial-time solvability results.

Proc. of the 19th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2020), B. An, N. Yorke-Smith, A. El Fallah Seghrouchni, G. Sukthankar (eds.), May 9–13, 2020, Auckland, New Zealand.

2 OUR RESULTS

Our main contributions are summarized as follows.

- We first adapt microbribery to the setting of group identification, and study the complexity of microbribery problems under important social rules.
- We introduce and investigate the complexity of the exact variant of microbribery, i.e., after bribery the set of socially qualified individuals has to exactly match the briber's distinguished set of individuals.
- In addition to the general profiles where every individual is allowed to qualify as many individuals as she/he wants, we also study r-profiles where every individual has to qualify exactly r individuals and this restriction should be maintained after bribery.
- For consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule, we solve the complexity of almost all problems, with only the complexity of microbribery restricted to r-profiles under consent rules remained open for general r (but we have a polynomial-time algorithm for $r = 1$).

In the following, we introduce the formal framework and present our results.

Let $N := \{a_1, \ldots, a_n\}$ denote the set of *individuals* (or *agents*). A *profile* over N is defined as a function $\varphi : N \times N \to \{0, 1\}$, where $\varphi(a_i, a_j) = 1$ means that individual a_i *qualifies* individual a_j, and $\varphi(a_i, a_j) = 0$ means that a_i *disqualifies* a_j. The profile can be represented as a matrix $(\varphi) \in \{0, 1\}^{n\times n}$, where $\varphi_{ij} := \varphi(a_i, a_j)$. In order to aggregate individuals' preferences, we need a *social rule* which is defined as a function f assigning to each pair (φ, N) a subset $f(\varphi, N) \subseteq N$ of individuals, referred to as the *socially qualified* individuals with respect to f and φ.

In this paper we are using the following three types of social rules. *Consent rules*, denoted by $f^{(s,t)}$, are specified by the *consent quotas* $s, t \in \mathbb{N}$. For each individual $a_i \in N$,

- if $\varphi(a_i, a_i) = 1$, then $a_i \in f^{(s,t)}(\varphi, N)$ if and only if
$$|\{a_j \in N \mid \varphi(a_j, a_i) = 1\}| \geq s,$$
- if $\varphi(a_i, a_i) = 0$, then $a_i \notin f^{(s,t)}(\varphi, N)$ if and only if
$$|\{a_j \in N \mid \varphi(a_j, a_i) = 0\}| \geq t.$$

For the *consensus-start-respecting rule*, denoted by f^{CSR}, we first identify the set of initially qualified individuals $K_0^C(\varphi, N)$ consisting of all the individuals qualified by every individual. Next, we iteratively increase the set of qualified individuals by adding all the individuals qualified by already socially qualified individuals, until there are no more changes to it.

For the *liberal-start-respecting rule*, denoted by f^{LSR}, we again first compute the set of initially qualified individuals by adding all the individuals to $K_0^L(\varphi, N)$ who qualify themselves. Next, we again iteratively increase the set of qualified individuals by adding all the individuals qualified by already socially qualified individuals, until there are no more changes to it.

For a social rule f, we study the following problems.

f-Constructive Group Microbribery	
Given:	A 4-tuple (N, φ, S, ℓ) of a set N of n individuals, a profile φ over N, a nonempty subset $S \subseteq N$ with $S \not\subseteq f(\varphi, N)$, and a positive integer ℓ.
Question:	Is there a way to change at most ℓ entries in the matrix φ such that $S \subseteq f(\varphi', N)$ where $\varphi' \in \{0,1\}^{n \times n}$ is the resulting new profile?

Our results regarding microbribery in group identification settings are summarized in Table 1.

$f^{(s,t)}$	$O(n^2)$-time solvable
f^{LSR}	NP-*complete*
f^{CSR}	NP-*complete*

Table 1: Results for microbribery. Here, n denotes the number of individuals.

Our first result shows that microbribery for consent rules is solvable in polynomial time. In contrast, standard bribery for consent rules is NP-complete for $t \geq 3$. The difference in the complexity of these problems follows from the fact that for each individual qualification only depends on the corresponding column in the profile φ and while in standard bribery the briber can only manipulate whole rows of the profile φ (thus facing a combinatorial problem that with one bribe the briber can potentially change several individuals' qualification), in microbribery the briber can make changes entry-wise (thus only concentrating on one individual at the time).

Sometimes a briber's goal is not just getting its preferred individuals socially qualified, but would like to have only exactly those individuals socially qualified.

Exact-f-Constructive Group Microbribery	
Given:	A 4-tuple (N, φ, S, ℓ) of a set N of $n$individuals, a profile φ over N, a nonempty subset $S \subseteq N$ with $S \neq f(\varphi, N)$, and a positive integer ℓ.
Question:	Is there a way to change at most ℓ entries in the matrix φ such that $S = f(\varphi', N)$ where $\varphi' \in \{0,1\}^{n \times n}$ is the resulting new profile?

Table 2 summarizes our results on exact-microbribery.

$f^{(s,t)}$	$O(n^2)$-time solvable
f^{CSR}	$O(n^3)$-time solvable
f^{LSR}	$O(n^4)$-time solvable

Table 2: Results for exact microbribery. Here, n denotes the number of individuals.

For consent rules, just as in the case of microbribery, we can provide a polynomial-time algorithm. Somewhat surprisingly, the exact versions of microbribery for both the consensus-start respecting rule and the liberal-start-respecting rule are polynomial-time solvable, standing in contrast to the NP-completeness of their nonexact versions. Roughly speaking, this is because that in the exact versions, we cannot resort to individuals not in S to make individuals in S socially qualified, which restricts the operations we need to consider and hence significantly shrinks the solution space to explore.

Furthermore, we study microbribery problems restricted to r-profiles where every individual has to qualify exactly r individuals. Note that in r-profiles, the bribery limit is always an even number $\ell = 2k$, as the briber has to keep the r-profiles. Our results are summarized in Table 3.

	$r = 1$	$r = 3$	$r \geq 4$
$f^{(s,t)}$	$O(n^2)$		
f^{LSR}	$O(n)$	NP-complete	
f^{CSR}	$O(n)$		NP-complete

Table 3: Results for microbribery restricted to r-profiles. Here, n denotes the number of individuals.

3 CONCLUSION

In this paper, we have studied the microbribery and exact microbribery problems in the setting of group identification. For the consent rules, the consensus-start-respecting rule, and the liberal-start-respecting rule, we identified their complexity, offering a guidance of whether these rules are resistant to microbribery behavior. We refer the reader to Tables 1–3.

For future research, one can study the parameterized complexity of these problems. The problems are clearly fixed-parameter tractable with respect to the number of individuals. An interesting parameter might be the number of distinguished candidates. Another interesting operation might be replacing qualified individuals, which means that after bribery, the number of qualified individual by a vote has to remain unchanged.

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