Coalitional Games with Stochastic Characteristic Functions Defined by Private Types

Extended Abstract

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ABSTRACT

This paper studies a coalitional game of task allocation where the characteristic function is not known and it is controlled by some private information from the players. Hence, the challenge here is twofold: (i) incentivize players to reveal their private information truthfully, (ii) incentivize them to collaborate together. Existing reward distribution mechanisms or auctions cannot solve the challenge. Hence, we propose a novel mechanism for the problem from the perspective of both mechanism design and coalitional games.

KEYWORDS

coalitional games, stochastic characteristic functions, incentive compatibility, task allocation

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1 INTRODUCTION

In this paper, we study a coalitional game where the characteristic function is defined by some private information owned by the coalition. Specifically, we study a task allocation problem, where a group of players collaborate to accomplish a sequence of ordered tasks and there is a deadline to finish all the tasks [2].

Due to the deadline, we need to find the best set of players to perform the tasks. Each player's capability is modelled by how much time she needs to complete a task. However, the completion time is a random variable following some distribution, known only to the player. Because of the uncertainty, the objective is to maximize the probability to meet the deadline [3]. In order to find the best set of players to meet the objective, we need to first know their private distributions (which defines the probability to meet the deadline for each coalition). That is, the characteristic function is defined by the private distributions of the players in each coalition.

We model the problem as a coalitional game, but the value for each coalition (the characteristic function) is controlled by the players' private information. On one hand, we want the players to expose their true private information in order to make the best decision, and on the other hand, we want the reward to be fairly distributed among the players. Coalitional games are often used to take care of the reward distribution to enforce collaboration, while mechanism design is good at eliciting private information in a competitive environment [5]. We show that the challenge cannot be solved by just using techniques from one of the two paradigms.

Thus, our goal is to design new reward sharing mechanisms such that players are incentivized to report their private distributions truthfully, and the reward are distributed fairly among all players. To combat this problem, we propose a novel mechanism to solve both challenges using the techniques from both cooperative game theory and mechanism design. Our solution is based on a modified Shapley value that distributes the reward according to the players' capabilities, but at the same time incentivizes all players to reveal their true capabilities. The price to solve both challenges is overpayment, i.e., the total reward distributed by the mechanism might be larger than the value the coalition can get. However, the overpayment is bounded and it is fairly low according to our simulations. For comparison, we also studied a solution from a non-cooperative perspective by using Vickrey-Clarke-Groves (VCG) mechanism [1, 4, 7]. VCG is very good at eliciting private information, but it does not distribute the reward properly.

2 THE MODEL

We consider herein a task allocation problem. The problem is of a project that consists of a sequence of m different tasks $T = (\tau_1, \cdots, \tau_m)$ to be finished in order, i.e. τ_i cannot be started until all tasks before τ_i have been finished. There is a deadline d to finish the entire project. Finishing all tasks before the deadline generates a value V, otherwise, the vlaue is zero.

There are n agents (players) denoted by $N = \{1, \cdots, n\}$ who can perform the tasks with different capabilities. We assume that each player is only capable of doing one of the tasks (our results can be generalized to the general case). Let $N_{\tau_i} \subseteq N$ be the set of players who can handle task τ_i. We have $N_{\tau_i} \neq \emptyset$ for all $\tau_i \in T$, and $\cup_{\tau_i \in T} N_{\tau_i} = N$.

For each player $i \in N_{\tau_j}$, her capability to handle task τ_j is measured by the time she needs to complete τ_j. However, player i does not know the exact time she will need to finish τ_j, but she does privately know a duration distribution.

Let the discrete random variable E_i denote the execution time of player i on task τ_j. We use $e_i \in \{1, 2, \cdots\}$ to denote a realization of E_i. Let f_i be the probability mass function of E_i, i.e., $Pr[E_i = e_i] = f_i(e_i)$. There might be a cost c_i for i to execute task τ_j. We assume the cost is public and it can be ignored for the current analysis.

The objective here is to find the optimal task allocation (one task is allocated to at most one player) such that the tasks T can be

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finished before deadline d with the highest probability. This would generate the highest expected value for the players. That is, the characteristic function $v : 2^N \to \mathbb{R}$ is defined by

> $v(S)$ equals the highest probability that the coalition $S \subseteq N$ can finish all the tasks T alone before d.

Since f is private and $v(S)$ is not publicly known, if we use standard techniques for coalitional games, the players can misreport f to influence the outcomes. *Hence, the goal is to design new reward sharing mechanisms such that players are incentivized to report their time distributions truthfully.*

The reward sharing mechanism requires each player to report her execution time distribution, but the player may not necessarily report her true distribution. For each player $i \in N$, let f_i be the density function of her true distribution, and f_i' be her report. Let $f = (f_1, \cdots, f_n)$ be the true density function profile of all players and $f' = (f_1', \cdots, f_n')$ be their report profile. We also denote f by (f_i, f_{-i}) and f' by (f_i', f_{-i}'). Let $\mathcal{F}_i$ be the density function space of f_i and $\mathcal{F} = (\mathcal{F}_1, \cdots, \mathcal{F}_n)$ be the space of density function profile f.

Definition 2.1. A **reward sharing mechanism** is defined by $x = (x_i)_{i \in N}$, where $x_i : \mathcal{F} \to \mathbb{R}$ defines the reward player i receives.

3 TRUTHFUL SHAPLEY VALUE

Shapley value is a well-known solution concept for cooperative games [6]. It divides the reward in a coalition according to their marginal contributions. It has many desirable properties such as *efficiency* (the reward is fully distributed to the players), *symmetry* (equal players receive equal rewards) and *null player* (dummy players receive no reward).

If we simply apply the Shapley value in our setting, players can report a more promising execution time distribution to receive a higher value. This is because the Shapley value mechanism only depends on their reports. In reality, we could actually observe how much time a player has actually spent to finish her task. Therefore, we can pay them according to their execution outcomes [8].

Definition 3.1. Given all players' report profile $f' = (f_1', \cdots, f_n')$, for each coalition $S \subseteq N$, $\pi_S^{f'} : T \to N \cup \{\perp\}$ is the **task assignment** to define $v(S)$. $\pi_S^{f'}(\tau_j) = \perp$ means that τ_j has not been assigned to any player under coalition S with reports f'.

Next are some specific notations for the new mechanism.

- Let $v(S, f')$ be the highest probability to finish all the tasks before the deadline under the report profile f'.
- Let $v(\pi_S^{f'}, f'')$ be the probability to finish all the tasks before the deadline given that the task assignment is defined by $\pi_S^{f'}$ but the actual probability to finish the tasks is calculated by f''. In our mechanism, f' represents their reports and f'' represents what we observed.

Our first modification is defined as follows.

Shapley Value with Execution Verification (SEV)
Given all players' report profile f':

- For each player i who has been assigned a task under $\pi_N^{f'}$, if her realised execution time is e_i, then her Shapley value is updated:

$$x_i^{sev}(f', e_i) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(\pi_{S \cup \{i\}}^{f'}, (f_i^{e_i}, f_{-i}')) - v(S, f')) \tag{1}$$

where $f_i^{e_i}$ represents the realization e_i and is defined as:

$$f_i^{e_i}(e) = \begin{cases} 1 & \text{if } e = e_i \\ 0 & \text{otherwise} \end{cases}$$

- For each player j who has not been assigned a task in the assignment $\pi_N^{f'}$, her Shapley value stays the same (as we cannot observe j's execution time):

$$x_j^{sev}(f') = \sum_{S \subseteq N \setminus \{j\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{j\}, f') - v(S, f')) \tag{2}$$

Theorem 3.2. *The SEV mechanism is incentive compatible for all players who are assigned a task, but it is not incentive compatible for players who are not assigned a task.*

Following Theorem 3.2, players who were not assigned a task can misreport to gain a higher reward under the SEV mechanism, because there is a lack of verification on their reports. It is certainly not ideal to assign each task to all its players to try, which is also not practical. Instead, we do the manipulation on behalf of the players to maximize their rewards they could gain. For each player who is not assigned the task, we treat this player as good as the player who is assigned the task to calculate a new Shapley value as her reward. This reward is the best the player could get by misreporting, and therefore, there is no incentive for misreporting anymore. The updated mechanism is defined as follows.

Shapley Value with Execution Verification and Bonus (SEVB)
Given all players' report profile f':

- For each player i who is assigned a task under $\pi_N^{f'}$, if her realised execution time is e_i, her Shapley value is defined as the same as in SEV (Equation (1)), i.e., $x_i^{sevb}(f', e_i) = x_i^{sev}(f', e_i)$.
- For each player j (assume $j \in N_{\tau_i}$) who is not assigned the task τ_i in $\pi_N^{f'}$, her Shapley value is upgraded as $x_j^{sevb}(f') =$

$$\sum_{S \subseteq N \setminus \{j\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{j\}, (f_j^*, f_{-j}')) - v(S, (f_j^*, f_{-j}'))) \tag{3}$$

where $f_j^* = f_{i^*}'$ and $i^* = \pi_N^{f'}(\tau_i)$, i.e., i^* is the player who is assigned τ_i.

Theorem 3.3. *The SEVB mechanism is incentive compatible.*

Since the SEVB mechanism may pay more than their actual Shapley values for players who did not receive a task, the total payment together might be greater than the total value the grand coalition can get. Theorem 3.4 shows that the overpayment is bounded.

Theorem 3.4. *Given any execution time distribution report profile f, the total reward distributed under the SEVB mechanism is bounded by* $\arg\max_k (1 + \sum_{i=1}^{k} \frac{i!(m-1)!}{(m+i)!}(n - m - k + 1))v(N, f)$.

4 CONCLUSION

We have studied a task allocation setting that merges the information revelation challenge in mechanism design and the payoff distribution challenge in cooperative game theory. We proposed a solution which guarantees that players will truthfully reveal their private information and that the rewards they receive from the coalition are fairly distributed. The price for achieving this is potential overpayment, but the extra payment is bounded.

REFERENCES

[1] Edward H Clarke. 1971. Multipart pricing of public goods. *Public choice* 11, 1 (1971), 17–33.

[2] Liat Cohen, Tal Grinshpoun, and Roni Stern. 2019. Assigning Suppliers to Meet a Deadline. In *Twelfth Annual Symposium on Combinatorial Search*. 170–171.

[3] Harary Frank. 1969. Shortest paths in probabilistic graphs. *Operations Research* 17, 4 (1969), 583–599.

[4] Theodore Groves. 1973. Incentives in teams. *Econometrica: Journal of the Econometric Society* (1973), 617–631.

[5] Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay V Vazirani. 2007. *Algorithmic game theory*. Vol. 1. Cambridge University Press Cambridge.

[6] Lloyd S Shapley. 1953. A value for n-person games. *Contributions to the Theory of Games* 2, 28 (1953), 307–317.

[7] William Vickrey. 1961. Counterspeculation, auctions, and competitive sealed tenders. *The Journal of Finance* 16, 1 (1961), 8–37.

[8] Dengji Zhao, Sarvapali D Ramchurn, and Nicholas R Jennings. 2016. Fault tolerant mechanism design for general task allocation. In *Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems*. International Foundation for Autonomous Agents and Multiagent Systems, 323–331.