Optimal Crowdfunding Design*

Extended Abstract

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1 INTRODUCTION

Crowdfunding has become an effective way of raising capital for developing and producing new products. In a typical crowdfunding campaign, a fundraiser (a seller) sets a product pre-buy price and a campaign success threshold. Consumers can indicate their willingness to pay the pre-buy price now, in exchange of a product in the future. Only when the number of consumers who commit to pre-buy exceeds the threshold, the crowdfunding is successful and the seller gets the corresponding pre-buy payments.

A series of works has modeled crowdfunding as imperfect information games, where each player (potential contributor) has a private valuation for the product, and has characterized the equilibrium behavior for some specific settings [1–4]. Other works focus on the effectiveness or the moral hazard of the crowdfunding campaigns by regarding it as an option to raise money for either private [5, 9, 10] or public projects [6–8].

However, the strategic aspect of crowdfunding from the seller's perspective hasn't attracted its deserved attention. In this paper, we take a mechanism design perspective and explore how a seller can design crowdfunding campaigns to maximize his profit. In addition to choosing the pre-buy price and the campaign success threshold for a crowdfunding campaign, the seller in our paper can consider two richer design variants: (1) choose two price-threshold pairs where a pre-buy price discount is given when the number of committed buyers exceeds the larger threshold, and (2) offer two differentiating products (simplified vs. standard) and set two success thresholds such that the advanced product will be delivered if the larger threshold is reached. We examine the optimal profit achieved in each scheme. Somewhat surprisingly, the richer design variants may not improve the seller's profit.

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2 STANDARD SCHEME: ONE PRICE-THRESHOLD PAIR

A crowdfunding campaign consists of n potential contributors, called players, and a pre-buy price τ and success threshold N pair. The seller sets the values of τ and N to raise fund for a product. Each player $i \in [n]$ has a private value v_i for the crowdfunded product. We assume for each $i \in [n]$, $v_i \in \{v_L, v_H\}$ follows the same binary distribution, denoted by p. Precisely, with probability p, $v_i = v_H$, and with probability $1-p$, $v_i = v_L$, where $0 < p < 1$ and $0 \le v_L < v_H$. Each player $i \in [n]$ chooses whether to contribute to the product: $a_i = 1$ represents a promise to contribute τ if the crowdfunding is successful and $a_i = 0$ represents not contributing. If there are at least N players who choose to contribute, the crowdfunding campaign succeeds, and these players make the payments. Then the seller will initiate the production with the raised money. The contributed players will receive a product in the future. Let a_{-i} denote the actions of players other than i. The utility of each player $i \in [n]$ satisfies:

$$u_i(a_i, a_{-i}) = \begin{cases} v_i - \tau \text{ if } a_i = 1, \sum_{j=1}^{n} a_j \ge N; \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

We overload our notation and use $a_i(\cdot)$ to denote a mapping $a_i : \{v_L, v_H\} \to \{0, 1\}$. Then $a_i(\cdot)$ is called the strategy of player i. A strategy profile (a_i, a_{-i}) is a *Bayesian Nash equilibrium* (BNE) if for any $i \in [n]$, any $v_i \in \{v_L, v_H\}$, and any other strategy $a'_i : \{v_L, v_H\} \to \{0, 1\}$,

$$E_{v_{-i}\sim p}[u_i(a_i(v_i), a_{-i}(v_{-i}))] \ge E_{v_{-i}\sim p}[u_i(a'_i(v_i), a_{-i}(v_{-i}))]. \tag{2}$$

A symmetric BNE is a BNE where every player adopts the same strategy. We call an equilibrium $\{a_i\}_{i=1}^{n}$ *non-trivial* if in expectation the campaign will succeed at the equilibrium,

$$E_{v_i\sim p}[\sum_{i=1}^{n} a_i(v_i)] \ge N.$$

In this paper, we only focus on non-trivial symmetric BNE as we are interested in understanding successful campaigns. There is a symmetric BNE, where for each player $i \in [n]$, $a_i(v_i) = 1$ if $v_i > \tau$, $a_i(v_i) = 0$ if $v_i < \tau$. Specifically, if $v_i = \tau$, although there is no difference between contributing or not for the player, we assume $a_i(v_i) = 1$. Ignoring the trivial equilibrium (e.g. $\forall i \in [n], a_i = 0$), this is the unique non-trivial symmetric BNE.

On behalf of the seller, there is a fixed cost B for initializing the production process, and a marginal cost τ_0 for producing each product sold during the crowdfunding campaign. After that, the production will continue and the future profit (revenue - marginal cost) of selling the product is A. In case of a failed crowdfunding campaign, or that the raised money from the crowdfunding campaign cannot afford the corresponding cost, the seller needs to consider an outside option, for example a bank loan, which has an interest rate of γ. We assume that the future profit A is not affected by whether the crowdfunding is successful or not. This certainly is not always true, but holds when participants of the crowdfunding are a different population from the major consumers of the product. Then with the players' actions $\{a_i\}_{i=1}^n$, the total profit of the seller can be calculated as

$$R = \begin{cases} A - \gamma \max\{B - k(\tau - \tau_0), 0\} + \max\{k(\tau - \tau_0) - B, 0\} & \text{if } k \geq N; \\ A - \gamma B & \text{otherwise}, \end{cases} \tag{3}$$

where $k = \sum_{j=1}^n a_j$.

By analyzing the optimal pair (N, τ), we know that the maximized expected profit of the seller should satisfy:

$$R^* = \max\{R(1, v_H), R(1, v_L)\} \tag{4}$$

where

$$\begin{aligned} R(1, v_H) =& (A - \gamma B)(1-p)^n + \sum_{k=1}^n \big(A - \gamma \max\{B - k(v_H - \tau_0), 0\} \\ &+ \max\{k(v_H - \tau_0) - B, 0\}\big)\tbinom{k}{n}(1-p)^{n-k}p^k \end{aligned} \tag{5}$$

$$R(1, v_L) = A - \gamma \max\{B - n(v_L - \tau_0), 0\} + \max\{n(v_L - \tau_0) - B, 0\} \tag{6}$$

3 VARIANT 1: TWO PRICE-THRESHOLD PAIRS FOR BULK DISCOUNT

In the first variant, we consider the possibility for the seller to offer bulk discount in crowdfunding. Formally, let N_2 $(1 \leq N \leq N_2)$ be the additional threshold for the discounted price τ_2 $(\tau_2 \leq \tau)$. If the number of players who promise to contribute is at least N_2, then these players only need to pay price τ_2. If the number is at least N but smaller than N_2, they pay the price τ. The utility of each player $i \in [n]$ becomes

$$u_i(a_i, a_{-i}) = \begin{cases} v_i - \tau \text{ if } a_i = 1, N \leq \sum_{j=1}^n a_j < N_2; \\ v_i - \tau_2 \text{ if } a_i = 1, \sum_{j=1}^n a_j \geq N_2; \\ 0 \text{ otherwise.} \end{cases} \tag{7}$$

With different choices of $(N, \tau), (N_2, \tau_2)$, the corresponding equilibria are summarized in the following theorem.

Theorem 3.1. *In a crowdfunding campaign using a scheme with two price-threshold pairs, (N, τ) and (N_2, τ_2), for non-trivial BNE:*

(1) If $\tau_2 \leq \tau \leq v_L$, there is a unique symmetric BNE where $a_i(v_i) = 1$, $\forall v_i$.

(2) If $v_L < \tau_2 \leq \tau \leq v_H$, there is a unique symmetric BNE where $a_i(v_i) = 1$ if and only if $v_i = v_H$.

(3) If $\tau_2 \leq v_L < \tau \leq v_H$ and $1 + (n-1)p \geq N_2$, there is a unique symmetric BNE where $a_i(v_i) = 1$, $\forall v_i$.

(4) If $\tau_2 \leq v_L < \tau \leq v_H$ and $1 + (n-1)p < N_2$, there are two symmetric BNE: (1) $a_i^{(1)}(v_i) = 1$ if and only if $v_i = v_H$, and (2) $a_i^{(2)}(v_i) = 1$, $\forall v_i$.

Based on the equilibrium analysis, the optimization for the expected profit, denoted by $R_2(N, \tau, N_2, \tau_2)$, can be divided into four corresponding cases. The maximal expected profit that can be achieved by any two pairs of $(N, \tau), (N_2, \tau_2)$ is

$$R_2^* = \max\{R_2(1, v_H, 1, v_H), R_2(1, v_L, 1, v_L)\},$$

where $R_2(1, v_H, 1, v_H) = R(1, v_H)$ and $R_2(1, v_L, 1, v_L) = R(1, v_L)$. This means $R_2^* = R^*$, so that setting an additional pair of threshold and price does not help increasing the seller's expected profit.

4 VARIANT 2: TWO THRESHOLDS FOR PRODUCT DIFFERENTIATION

Another variant of crowdfunding adopted in practice is to set two-level thresholds for product differentiation. The seller sometimes can choose to produce either a simplified version or a standard version of the product depending on how much funding is raised.

Let N_1 $(1 \leq N_1 \leq N)$ be an additional threshold. If the number of players who promise to buy the product at the price τ is at least N_1 but smaller than N, then the seller will produce only the simplified version of the product. But if the number is at least N, then the seller will produce the standard version of the product.

For this scheme, we need to further introduce each player i's private value $v_{i1} \in \{v_l, v_h\}$ for the simplified version. It is natural to consider that the private values of each player for the simplified and standard versions are correlated. Thus, for simplicity, we assume $v_l < v_L < v_h < v_H$, with probability p, $v_{i1} = v_h$, and $v_i = v_H$, and with probability $1 - p$, $v_{i1} = v_l$, and $v_i = v_L$.

The utility of each player $i \in [n]$ becomes

$$u_i(a_i, a_{-i}) = \begin{cases} v_{i1} - \tau \text{ if } a_i = 1, N_1 \leq \sum_{j=1}^n a_j < N; \\ v_i - \tau \text{ if } a_i = 1, \sum_{j=1}^n a_j \geq N; \\ 0 \text{ otherwise.} \end{cases} \tag{8}$$

Then with different choices of (N_1, N, τ), the corresponding equilibria are summarized in the following theorem.

Theorem 4.1. *In a crowdfunding campaign using a scheme with two thresholds for product differentiation (N_1, N, τ), the non-trivial strategies of players satisfy:*

(1) If $\tau \leq v_L$, there is a unique symmetric BNE where $a_i(v_i) = 1$, $\forall v_i$.

(2) If $v_L < \tau \leq v_h$ or $v_h < \tau \leq v_H$ and $1 + (n-1)p \geq N$, there is a unique symmetric BNE where $a_i(v_i) = 1$ if and only if $v_i = v_H$.

Finally, the key to analyzing the optimization for the expected profit $R_1(N, \tau, N_1)$ lies on the monotonicity of $R_1(N, \tau, N_1)$ w.r.t. N.

Lemma 4.2. *Consider any fixed τ and N_1. Let $\hat{N}$ be any integer satisfying $N \leq \hat{N} \leq n + 1$. Then*

$$\arg \max_{N_1 \leq N \leq \hat{N}} R_1(N, \tau, N_1) = N_1 \text{ or } \hat{N}.$$

This monotonicity means to maximize the profit, the seller should only provide the standard version, or only provide the simplified version. The corresponding optimal profit may be larger than the one in the standard scheme under some conditions, but the cause of such an increase in profit is not due to the additional threshold.

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