Proportional Fairness in Obnoxious Facility Location

Extended Abstract

Haris Aziz
UNSW Sydney
Sydney, Australia
haris.aziz@unsw.edu.au

Alexander Lam
UNSW Sydney
Sydney, Australia
alexander.lam1@unsw.edu.au

Bo Li
Hong Kong Polytechnic University
Hong Kong, Hong Kong SAR
comp-bo.li@polyu.edu.hk

Fahimeh Ramezani
UNSW Sydney
Sydney, Australia
ramezani81@googlemail.com

Toby Walsh
UNSW Sydney
Sydney, Australia
t.walsh@unsw.edu.au

ABSTRACT

We consider the obnoxious facility location problem (in which agents prefer the facility location to be far from them) and propose a hierarchy of distance-based proportional fairness concepts for the problem. These fairness axioms ensure that groups of agents at the same location are guaranteed to be a distance from the facility proportional to their group size. We consider deterministic and randomized mechanisms, and compute tight bounds on the price of proportional fairness. In the deterministic setting, not only are our proportional fairness axioms incompatible with strategyproofness, the Nash equilibria may not guarantee welfare within a constant factor of the optimal welfare. On the other hand, in the randomized setting, we identify proportionally fair and strategyproof mechanisms that give an expected welfare within a constant factor of the optimal welfare.

KEYWORDS

Facility location; Fairness; Social choice

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1 INTRODUCTION

In the *obnoxious facility location problem (OFLP)*, some undesirable facility such as a garbage dump or an oil refinery is to be located on a unit interval (i.e. the domain of locations is $[0, 1]$), and the agents along the interval wish to be as far from the facility as possible [2–5]. In this problem, agents have single-dipped preferences, contrasting with the single-peaked preferences of agents in the classic facility location problem (in which agents prefer to be located as close as possible to the facility).

In this work, we pursue notions of *proportional fairness* as a central concern for the problem. Specifically, we formulate a hierarchy of proportional fairness axioms which guarantee that each group of agents at the same location are a distance from the facility proportional to the relative size of the group. While proportional fairness axioms have been formulated and studied in the classic facility location problem [1], they have not yet been applied to the OFLP. Our paper provides a comprehensive overview of proportionally fair solutions for the obnoxious facility location problem, examining the interplay between proportional fairness and utilitarian/egalitarian welfare, and investigating concerns of agent strategic behaviour in both the deterministic and randomized settings.

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2 MODEL

Let $N = \{1, \dots, n\}$ be a set of agents, and let $X := [0, 1]$ be the domain of locations. Our results can be naturally extended to any compact interval on $\mathbb{R}$. Agent i's location is denoted by $x_i \in X$; the profile of agent locations is denoted by $x = (x_1, \dots, x_n) \in X^n$. We also assume the agent locations are ordered such that $x_1 \leq \cdots \leq x_n$. A *deterministic mechanism* is a mapping $f : X^n \to X$ from a location profile $\hat{x} \in X^n$ to a facility location $y \in X$. We define a *randomized mechanism* as a probability distribution over deterministic mechanisms. Given a facility location $y \in X$, agent i's utility is equal to its distance from the facility $u(y, x_i) := |y - x_i|$. We are interested in maximizing the objectives of *Utilitarian Welfare* (UW), defined for a facility location y and location profile x as the sum of agent utilities $\sum_i u(y, x_i)$, and *Egalitarian Welfare* (EW), defined as the minimum agent utility $\min_i u(y, x_i)$.

In mechanism design, it is typically desirable for mechanisms to satisfy *strategyproofness*, meaning that no agent can improve their (expected) distance from the facility by lying about its location.

2.1 Proportional Fairness Axioms

We introduce adaptations and approximations of the proportional fairness axioms studied by Aziz et al. [1], for the obnoxious facility location problem. We show that the 2-approximations are the tightest approximations for a solution to be guaranteed to exist.

Definition 2.1 (2-Individual Fair Share (IFS)). Given a profile of locations x, a facility location y satisfies 2-*Individual Fair Share (2-IFS)* if $u(y, x_i) \geq \frac{1}{2n} \quad \forall i \in N$.

Definition 2.2 (2-Unanimous Fair Share (UFS)). Given a profile of locations x, a facility location y satisfies 2-*Unanimous Fair Share (2-UFS)* if for any set of agents S with identical location, $u(y, x_i) \geq \frac{|S|}{2n} \quad \forall i \in S$.

Table 1: Table of price of fairness and welfare approximation results.

		Price of Fairness		Best Known Approx.
		2-IFS	2-UFS	by 2-UFS SP Mech.
Deter.	UW	2	2	**Incompatible**
	EW	1	n-1	
Rand.	UW	12/11	1.09384...	1.5
	EW	1	1	1

Randomized mechanisms satisfy these axioms *in expectation* if the *expected utility* of each agent satisfies the respective axioms.

For these axioms, we denote the polynomial time mechanism which computes the optimal 2-IFS/UFS facility location in terms of utilitarian welfare as f^*_{2IFS} and f^*_{2UFS}, respectively.

In this paper, we consider the *price of 2-IFS/UFS fairness*, which measures the loss of efficiency from imposing a certain fairness constraint. The price of fairness can also be interpreted as the approximation ratio for the respective optimal mechanism satisfying the fairness constraint. We formally define it below.

A *fairness property P* is a mapping from an agent location profile $x \in X^n$ to a (possibly empty) set of facility locations $P(x) \in X$. Every facility location $P(x)$ satisfies the fairness property P. The price of fairness for property P is the worst case ratio between the optimal welfare and the optimal welfare from a facility location satisfying P.

Definition 2.3 (Price of Fairness for Utilitarian/Egalitarian Welfare). Let $\{f^*_{UW}, f^*_{EW}\}$ be the mechanism that returns the solution maximizing utilitarian/egalitarian welfare. For UW/EW and fairness property P, we define the price of fairness as the worst case ratio (over all location profiles) between the optimal UW/EW and the optimal UW/EW achieved by a facility location satisfying fairness property P:

$$\max_{x\in[0,1]^n} \frac{\sum_i u(f^*(x), x_i)}{\max_{y\in P(x)} W(y, x_i)}.$$

For UW, $f^*(x) := f^*_{UW}(x)$ and $W(y, x_i) := \sum_i u(y, x_i)$.
For EW, $f^*(x) := f^*_{EW}(x)$ and $W(y, x_i) := \min_i u(y, x_i)$.

3 RESULTS

We prove tight bounds on the price of 2-IFS and 2-UFS fairness for utilitarian and egalitarian welfare, and that in the deterministic setting, no strategyproof mechanism can satisfy any approximation of our proportional fairness axioms. For the randomized setting, our price of fairness bounds are derived from the approximation ratios of the optimal 2-IFS/UFS mechanisms for the respective welfares. We also give examples of randomized strategyproof 2-UFS mechanisms with a constant approximation ratio for utilitarian welfare, or are optimal for egalitarian welfare. Full details of the mechanisms can be found in the full paper. A summary of these results can be found in Table 1.

Since strategyproofness is incompatible with our fairness axioms, we are interested in the performance of proportionally fair mechanisms in our model when accounting for agent strategic behaviour. Such performance can be quantified by the *price of anarchy*, which quantifies the worst-case degradation of efficiency from a pure Nash equilibrium of reports. We show that for f^*_{2IFS} and f^*_{2UFS} a pure Nash equilibrium may not exist, but the approximate *pure ϵ-Nash equilibrium* always exists for all $\epsilon > 0$.

Definition 3.1 (Tardos and Vazirani [6]). A *pure ϵ-Nash equilibrium* is a profile of reported agent locations $x' = (x'_1, \dots, x'_n)$ such that no single agent can improve its own utility (with respect to its true location) by strictly more than ϵ by changing its reported location. A pure Nash equilibrium is a pure ϵ-Nash equilibrium where $\epsilon = 0$.

As a pure Nash equilibrium may not exist, the price of anarchy is not well-defined and thus we consider the ϵ-price of anarchy, defined as the worst case ratio (over all location profiles x) between the utilitarian welfare corresponding to all agents reporting truthfully and the minimum utilitarian welfare corresponding to agents reporting in a pure ϵ-Nash equilibrium.

Definition 3.2. Given f and x, define the set of pure ϵ-Nash equilibria location profiles as ϵ-$Equil(f, x)$. The price of anarchy for utilitarian welfare is defined as:

$$\epsilon\text{-}PoA(f) := \max_{x\in X^n} \frac{\sum_i u(f(x), x_i)}{\min_{x'\in \epsilon\text{-}Equil(f,x)} \sum_i u(f(x'), x_i)}.$$

Theorem 3.3. *For any $\epsilon \in (0, \frac{1}{n})$, the ϵ-price of anarchy for f^*_{2IFS} and f^*_{2UFS} of utilitarian welfare is at least $\frac{2n-1+n\epsilon}{1-n\epsilon}$. The price of anarchy is unbounded for $\epsilon \geq \frac{1}{n}$.*

Theorem 3.4. *For any $\epsilon \in (0, \frac{1}{2n})$, the $\epsilon-$price of anarchy for f^*_{2IFS} and f^*_{2UFS} of utilitarian welfare is at most $\frac{2n}{1-2n\epsilon}$.*

By setting $\epsilon = 0$ in the ϵ-price of anarchy bounds of Theorems 3.3 and 3.4, we achieve the following result.

Corollary 3.5. *If a pure Nash equilibrium exists, the price of anarchy for f^*_{2IFS} and f^*_{2UFS} of utilitarian welfare is between $2n-1$ and $2n$.*

One downside of the 2-UFS definition is that agents located near each other but not at the same location are considered to be in separate groups. An axiom which accounts for groups of agents located relatively close to each other is 2-Proportional Fairness (2-PF), adapted from [1]. We define it as follows, and prove that it is the tightest approximation which guarantees a solution.

Definition 3.6 (2-PF). Given a profile of locations x, a facility location y satisfies *2-PF* if for any set of agents S within range r,
$u(y, x_i) \geq \frac{1}{\alpha}(|S|/(n)) - r \quad \forall i \in S.$

Note that $\alpha-$PF implies $\alpha-$UFS, and therefore also implies $\alpha-$IFS.

In the hybrid model, agents either want to be located close to the facility (as in the classic model), or wish to be located far away from the facility (as in our obnoxious model). We adapt IFS and UFS to this model and prove that a solution always exists for these axioms.

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