Individual-Fair and Group-Fair Social Choice Rules under Single-Peaked Preferences

Extended Abstract

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ABSTRACT

We propose novel fairness notions for social choice under single-peaked preferences, for group-fairness as well as individual-fairness. Agents are assumed to be partitioned into logical groups, which could be based on natural attributes such as gender, race, or location. To capture fairness within each group, we introduce the notion of group-wise anonymity. To capture fairness across the groups, we propose a weak notion as well as a strong notion of fairness. The proposed fairness notions turn out to be natural generalizations of existing individual-fairness notions. We characterize the fair deterministic social choice rules and provide two separate characterizations of the fair random social choice rules: (i) direct characterization (ii) extreme point characterization (as convex combinations of deterministic rules). We also explore individual fairness by looking at the special case with singleton groups.

KEYWORDS

Random Social Choice; Single-Peaked ; Divisible PB; Characterize

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1 INTRODUCTION

Social choice involves aggregating the preferences of agents over a set of alternatives to decide an outcome. There are two natural families of social choice rules - deterministic (select a single alternative) and random (select a probability distribution over the alternatives). The two most desired properties in social choice, unanimity and strategy-proofness, are found to be incompatible, unless the rules are dictatorial [12, 13, 15]. Black [6] introduced the single-peaked domain, which is a special structure on rankings (ordinal preferences) over which unanimity and strategy-proofness become compatible. This resulted in the characterizations of unanimous and strategy-proof social choice rules in single-peaked domain.

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Another desirable property for social choice is *fairness*. Random social choice rules are often viewed as divisible PB rules [4], where the probability of each alternative is interpreted as the fraction of budget allocated to it. This interpretation motivated the study of fair social choice rules [1, 3, 5, 7]. The existing group-fairness notions guarantee fairness to every subset of agents and are satisfied only by the random dictatorial rule when applied to strict rankings [2, 3, 8, 10]. However, often in real-world, agents are naturally partitioned into groups based on gender, race etc., and it is adequate to guarantee fairness to these groups (called affirmative action or reservation in real-world [11]). Our work studies this model.

2 PREREQUISITES

Let $N = [n]$ be the set of agents and $A = \{a_1, \dots, a_m\}$ be the set of alternatives with a prior ordering $\prec$ given by $a_1 \prec \dots \prec a_m$. The min/max of a set of alternatives is derived w.r.t. $\prec$. We use $[a, b]$ to denote $\{c \mid a \preceq c \preceq b \text{ or } b \preceq c \preceq a\}$. Let $\mathcal{P}(A)$ denote the collection of all complete, reflexive, anti-symmetric, and transitive binary relations on A, where for $P \in \mathcal{P}(A)$, aPb is interpreted as "P prefers a over b". The k^{th} ranked alternative according to P is denoted by $P(k)$. We use $U(a, P)$ to denote $\{b \in S \mid bPa\}$.

Definition 1. *A preference $P \in \mathcal{P}(A)$ is called **single-peaked** if for all $a, b \in A$, $[P(1) \preceq a \prec b$ or $b \prec a \preceq P(1)]$ implies aPb.*

Let $\mathcal{D}$ be the set of all single-peaked preferences on A. Each agent i reports a preference $P_i \in \mathcal{D}$ and P_S denotes the collection of preferences of all agents in a set $S \subseteq N$. A *Deterministic Social Choice Function* (DSCF) on $\mathcal{D}^n$ is a function $f : \mathcal{D}^n \to A$, and a *Random Social Choice Function* (RSCF) on $\mathcal{D}^n$ is a function $\varphi : \mathcal{D}^n \to \Delta A$, where ΔA is the set of all probability distributions over A. For any $B \subseteq A$, we define $\varphi_B(P_N)$ as $\sum_{a \in B} \varphi_a(P_N)$, where $\varphi_a(P_N)$ is the probability of a at $\varphi(P_N)$.

There exist two kinds of characterizations of RSCFs in the literature: direct characterization and extreme point characterization (express RSCFs as convex combinations of DSCFs).

Definition 2. *An RSCF is said to be a **probabilistic fixed ballot rule (PFBR)** if there is a collection $\{\beta_S\}_{S \subseteq N}$ of probability distributions satisfying the following two properties:*

*(i) **Ballot Unanimity:** $\beta_\emptyset(a_m) = 1$ and $\beta_N(a_1) = 1$, and*

*(ii) **Monotonicity**: for all $a_t \in A, S \subset T \subseteq N \implies \beta_S([a_1, a_t]) \leq \beta_T([a_1, a_t])$*

such that for all $P_N \in \mathcal{D}^n$ and $a_t \in A$, we have

$$\varphi_{a_t}(P_N) = \beta_{S(t;P_N)}([a_1, a_t]) - \beta_{S(t-1;P_N)}([a_1, a_{t-1}]);$$

where $\beta_{S(0;P_N)}([a_1, a_0]) = 0$.

Lemma 1. *An RSCF on $\mathcal{D}^n$ is unanimous and strategy-proof if and only if it is a probabilistic fixed ballot rule [9].*

Definition 3. *A DSCF f is a **min-max** rule if for all $S \subseteq N$, there exists $\beta_S \in A$ satisfying*

$$\beta_\emptyset = a_m, \beta_N = a_1, \text{ and } \beta_T \preceq \beta_S \text{ for all } S \subseteq T$$

such that

$$f(P_N) = \min_{S \subseteq N}\left[\max_{i \in S}\{P_i(1), \beta_S\}\right].$$

A **random min-max rule** is a convex combination of min-max rules, which is expressed as $\varphi = \sum_{w \in W} \lambda_w \varphi_w$ where $\sum_{w \in W} \lambda_w = 1$, and for every $j \in W$, φ_j is a min-max rule and $0 \le \lambda_j \le 1$.

Lemma 2. *An RSCF on $\mathcal{D}^n$ is unanimous and strategy-proof if and only if it is a random min-max rule [14].*

3 GROUP-FAIRNESS

Let $G = [g]$ and $\mathcal{N}$ be a partition of N into g groups defined as $\mathcal{N} = (N_1, \ldots, N_g)$. A permutation π of N is *group preserving* if for all $q \in G$, $i \in N_q$ implies $\pi(i) \in N_q$. To achieve fairness within the group, we ensure that all the agents in it are treated symmetrically.

Definition 4. *An RSCF is **group-wise anonymous** if for all group preserving permutations π of N and all $P_N \in \mathcal{D}^n$, we have $\varphi(P_N) = \varphi(P_{\pi(N)})$ where $P_{\pi(N)} = (P_{\pi(1)}, \ldots, P_{\pi(n)})$.*

To ensure fairness across groups, we define two notions each with three parameters: $\kappa_G = (\kappa_q)_{q \in G}$, $\psi_G = (\psi_q)_{q \in G}$, and $\eta_G = (\eta_q)_{q \in G}$. For every group $q \in G$, ψ_q is a function that selects κ_q alternatives as the representatives of q. Assumptions on ψ_G and examples satisfying them are discussed in our long version [16]. Our weak fairness notion ensures that the κ_q representatives collectively receive a probability of at least η_q, while the stronger notion ensures that at least one of them gets a probability of at least η_q.

Definition 5. *An RSCF φ is **$(\kappa_G, \psi_G, \eta_G)$-weak fair** if for all $P_N \in \mathcal{D}^n$ and all $q \in G$, it holds that $\varphi_{\psi_q(P_{N_q})}(P_N) \ge \eta_q$.*

Definition 6. *An RSCF φ is **$(\kappa_G, \psi_G, \eta_G)$-strong fair** if for all $P_N \in \mathcal{D}^n$ and $q \in G$, there exists $a \in \psi_q(N_q)$ such that $\varphi_a(P_N) \ge \eta_q$.*

3.0.1 *Some notations.* Let Γ be the set of all g dimensional vectors such that $\gamma_q \in \{0, \ldots, |N_q|\}$ for all q. For $\gamma, \gamma' \in \Gamma$, we say $\gamma \gg \gamma'$ if $\gamma_q \ge \gamma'_q$ for all $q \in G$. Let $\underline{\gamma} = (0)_{q \in G}$ and $\overline{\gamma} = (|N_q|)_{q \in G}$. For a profile P_N and $1 \le t \le m$, let $\alpha(t; P_N) = (\alpha_q)_{q \in G}$ where $\alpha_q = |\{i \in N_q \mid P_i(1) \preceq a_t\}|$. For $t \le |N_q|$, we denote by $\tau_t(P_{N_q})$ the alternative at the t^{th} position when the top-ranked alternatives in P_{N_q} are arranged in increasing order (with repetition). A set of alternatives $\{a^1, a^2, \ldots, a^t\}$ **is feasible at** $(z_0, z_1, z_2, \ldots, z_t; \psi_q)$ if there exists a profile P_{N_q} such that $a^1 = \min\{\psi_q(P_{N_q})\}$, $|\{i \in N_q : P_i(1) \prec a^1\}| = z_0$, and $|\{i \in N_q : P_i(1) \preceq a^j\}| = z_j$ for every $j \in [t]$.

3.1 Direct Characterization

We first modify the idea of PFBRs to introduce **probabilistic fixed group ballot rules (PFGBRs)** which characterize PFBRs satisfying group-wise anonymity [16].

Definition 7. *An RSCF φ on $\mathcal{D}^n$ is said to be a **PFGBR** if there is a collection of probabilistic ballots $\{\beta_\gamma\}_{\gamma \in \Gamma}$ which satisfies*

*(i) **Ballot Unanimity:** $\beta_{\underline{\gamma}}(a_m) = 1$ and $\beta_{\overline{\gamma}}(a_1) = 1$, and*

*(ii) **Monotonicity**: for all $\gamma, \gamma' \in \Gamma$, $\gamma \gg \gamma'$ implies $\beta_\gamma([a_1, a_t]) \ge \beta_{\gamma'}([a_1, a_t])$ for all $t \in [1, m]$,*

such that for all $P_N \in \mathcal{D}^n$ and all $a_t \in A$,

$$\varphi_{a_t}(P_N) = \beta_{\alpha(t,P_N)}([a_1, a_t]) - \beta_{\alpha(t-1,P_N)}([a_1, a_{t-1}]);$$

where $\beta_{\alpha(0,P_N)}([a_1, a_0]) = 0$.

Proposition 1. *A PFGBR is $(\kappa_G, \psi_G, \eta_G)$-weak fair if and only if for all $q \in G$, for all $\gamma, \gamma' \in \Gamma$ such that $\gamma \gg \gamma'$, and for all $a_x \in A$ such that $\{a_x, a_{x+\kappa_q-1}\}$ is feasible at $(\gamma'_q, z, \gamma_q; \psi_q)$ for some z, we have $\beta_\gamma([a_1, a_{x+\kappa_q-1}]) - \beta_{\gamma'}([a_1, a_{x-1}]) \ge \eta_q$.*

Theorem 1. *A PFGBR is $(\kappa_G, \psi_G, \eta_G)$-strong fair if and only if for all $q \in G$, for all $\gamma^0, \gamma^1, \ldots, \gamma^{\kappa_q} \in \Gamma$ such that $\gamma^{\kappa_q} \gg \ldots \gg \gamma^1 \gg \gamma^0$, and for all $a_x \in A$ such that $\{a_x, a_{x+1}, \ldots, a_{x+\kappa_q-1}\}$ is feasible at $(\gamma^0, \gamma^1, \ldots, \gamma^{\kappa_q}; \psi_q)$, there exists $t \in [0, \kappa_q - 1]$ such that $\beta_{\gamma^{t+1}}([a_1, a_{x+t}]) - \beta_{\gamma^t}([a_1, a_{x+t-1}]) \ge \eta_q$.*

3.2 Extreme Point Characterization

We modify min-max rules to introduce group min-max rules.

Definition 8. *A DSCF f is called a **group min-max rule (GMMR)** if for every $\gamma \in \Gamma$, there exists $\beta_\gamma \in A$ satisfying $\beta_{\underline{\gamma}} = a_m, \beta_{\overline{\gamma}} = a_1$, and $\beta_\gamma \preceq \beta_{\gamma'}$ for all $\gamma \gg \gamma'$ such that*

$$f(P_N) = \min_{\gamma \in \Gamma}\left[\max\{\tau_{\gamma_1}(P_{N_1}), \ldots, \tau_{\gamma_g}(P_{N_g}), \beta_\gamma\}\right].$$

A **random group min-max rule (RGMMR)** is a convex combination of GMMRs, i.e., $\varphi = \sum_{w \in W} \lambda_w \varphi_w$ where $W = [q]$, $\sum_{w \in W} \lambda_w = 1$, and for every $j \in W$, $0 \le \lambda_j \le 1$ and φ_j is a GMMR. A rule is a PFGBR if and only if it is a RGMMR [16].

Theorem 2. *A RGMMR $\varphi = \sum_{w \in W} \lambda_w \varphi_w$ is $(\kappa_G, \psi_G, \eta_G)$-weak fair if and only if for all $q \in G$, for all $\gamma, \gamma' \in \Gamma$ such that $\gamma \gg \gamma'$, and for all $a_x \in A$ such that $\{a_x, a_{x+\kappa_q-1}\}$ is feasible at $(\gamma'_q, z, \gamma_q; \psi_q)$ for some z, we have*

$$\sum_{\{w \mid \beta^{\varphi_w}_{\gamma'} \succeq a_x,\ \beta^{\varphi_w}_{\gamma} \preceq a_{x+\kappa_q-1}\}} \lambda_w \ge \eta_q.$$

Theorem 3. *A RGMMR $\varphi = \sum_{w \in W} \lambda_w \varphi_w$ is $(\kappa_G, \psi_G, \eta_G)$-strong fair if and only if for all $q \in G$, for all $\gamma^0, \gamma^1, \ldots, \gamma^{\kappa_q} \in \Gamma$ such that $\gamma^{\kappa_q} \gg \ldots \gg \gamma^1 \gg \gamma^0$, and for all $a_x \in A$ such that $\{a_x, a_{x+1}, \ldots, a_{x+\kappa_q-1}\}$ is feasible at $(\gamma^0, \gamma^1, \ldots, \gamma^{\kappa_q}; \psi_q)$, there exists $t \in [0, \kappa_q - 1]$ such that*

$$\sum_{\left\{w \mid \beta^{\varphi_w}_{\gamma^t} \succeq a_{x+t},\ \beta^{\varphi_w}_{\gamma^{t+1}} \preceq a_{x+t}\right\}} \lambda_w \ge \eta_q.$$

4 SUMMARY

We introduced group-fairness notions and characterized the unanimous, strategy-proof, and group-fair RSCFs under single-peaked preferences. Individual-fairness is defined by considering a special case with only singleton groups. While weak individual-fairness ensures that $\varphi_{U(P_i(\kappa_i), P_i)}(P_N) \ge \eta_i$ for every i, strong fairness ensures that there exists $a \in U(P_i(\kappa_i), P_i)$ with $\varphi_a(P_N) \ge \eta_i$. Some computationally tractable group-fair rules and also characterizations of individually-fair rules are given in our long version [16].

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