Revenue Maximization Mechanisms for an Uninformed Mediator with Communication Abilities

Extended Abstract

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ABSTRACT

Consider a market where a seller owns an item for sale and a buyer wants to buy an item, and both players have a private type. We study the problem of designing revenue-maximizing mechanisms for a mediator who has no private information but can privately communicate with players. We show that the mediator can, without loss of generality, focus on the set of direct and incentive-compatible mechanisms. Then we formulate this problem as a mathematical program. Moreover, we give an optimal solution to the optimization problem in closed form under certain technical conditions.

KEYWORDS

Mechanism Design; Revelation principle; Information Design

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1 INTRODUCTION

Consider a market where a seller wants to sell an item to a buyer. The item's quality is only known to the seller, while the buyer's value for the item depends on both the item's quality and the buyer's type. Each player wants to know the other player's private information but may be unwilling to reveal too much about their own. On one hand, revealing too much information gives the other player an information advantage. On the other hand, revealing no information may prevent a trade from happening. Therefore, it is difficult for them to come to a trade agreement on their own. Even after the purchase decision is made, the players also need to deal with a lot of paperwork, imposing high costs on both of them.

This problem is ubiquitous in real-world applications and gives rise to mediators between the two sides. For example, in real estate markets, there usually exists a broker or a realtor between a seller and a buyer. In fact, according to the National Association of Realtors [5], in 2021, 87% of buyers bought their homes, and 90% of sellers sold their homes through an agent or a broker in the US.

The above facts motivate us to consider how a mediator can maximize their revenue by charging prices for providing such a service. Formally, consider a market with two agents, a *seller s* and a *buyer b*. Let $t \in T$ be the private type of buyer. Denote by $q \in Q$ the quality of the item, which can be viewed as the private type of the seller. We assume both q and t are random variables independently drawn from publicly known distributions $G(q)$ and $F(t)$, with $[q_1, q_2]$ and $[t_1, t_2]$ being their supports, $g(q)$ and $f(t)$ being their corresponding probability density functions.

Let $v(t, q)$ be the valuation of the buyer if they are of type t and get an item of quality q. We assume that $v(t, q)$ is linear in t and has the form $v(t, q) = \alpha_1(q)t + \alpha_2(q)$ with $\alpha_1(q) > 0$ for all $q \in Q$. The seller has a reserve price $r(q)$ for the item, which we assume is proportional to the quality of the item, i.e., $r(q) = kq, k \geq 0$.

Now suppose that there is a mediator who has a private communication channel through which they can communicate with either the buyer or the seller privately. By using such a communication channel, the mediator can not only collect information from both sides but also send information to them. In the end, the mediator decides whether to recommend them to trade or not, and both players need to decide whether to follow the recommendation. We assume there is no outside option for both players since there can be a formidably high cost if they choose to trade on their own.

By revealing information strategically, the mediator can leverage the communication ability to make a profit. However, unlike most standard mechanism design problems, the mediator cannot force the players to follow the trade recommendation. In this work, we stand on the mediator's side, investigating how to design a communication protocol for the mediator in order to elicit and reveal information with both agents, and how to price for providing such a service to maximize the mediator's expected revenue.

2 MECHANISM SPACE

The mechanism space considered in this paper generalizes a similar definition called the "generic interactive protocol" proposed by Babaioff et al. [1]. Unlike their setting, there remain two players after the communication protocol has been designed. Because of the dynamics of the mechanism, we use the Perfect Bayesian equilibrium (PBE) as our solution concept, rather than BNE.

Before defining direct mechanisms, we first consider signaling schemes, which formalize how the mediator reveals information. Given a signal set Σ, a signaling scheme $\pi : T \times Q \mapsto \Delta(\Sigma)$ is a mapping from the players' type profile to a distribution over the signal set Σ. Actually, we can focus on the case where the mediator sends signals publicly and the signal set Σ has only two signals, i.e., $\Sigma = \{0, 1\}$, where 0 corresponds to "not trade" and 1 corresponds to "trade". This is because we assume that the two agents cannot trade

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without the mediator. Thus the mediator should clearly specify whether or not they would recommend them to trade.

Definition 2.1 (Direct Mechanism). A direct mechanism is described by a tuple (π, P_b, P_s), and proceeds as follows:

(1) The mediator announces π, P_b and P_s;
(2) The buyer and seller are asked to report their types t and q to the mediator privately;
(3) The mediator decides whether to recommend the agents to trade according to $\pi(t, q)$;
(4) Upon receiving the signal, the players decide whether to follow the recommendation;
(5) If the trade happens, the mediator charges the buyer $P_b(t)$ and pays the seller $P_s(q)$.

Definition 2.2 (Incentive Compatibility). A direct mechanism is incentive compatible if for both players, reporting their true types and following the mediator's recommendation form a PBE of the game induced by the mechanism.

We now show that the mediator can, without loss of generality, consider only direct and incentive compatible mechanisms.

THEOREM 2.3. *For any general mechanism M, there exists a direct, incentive compatible mechanism that achieves the same expected revenue as in any PBE of the game induced by M.*

3 PROBLEM ANALYSIS

For simplicity, we use $\pi(t, q)$ to denote the probability of sending signal 1 when the reported type profile is (t, q). The mediator's goal is to design $\pi(t, q), P_b(t), P_s(q)$ to maximize their revenue:

$$\int_{q\in Q}\int_{t\in T} \pi(t,q)[P_b(t) - P_s(q)]f(t)g(q)\,\mathrm{d}t\mathrm{d}q.$$

Individual rational (IR). Both players obtain 0 if they choose not to participate. Thus we need to ensure that after receiving signal 1, each player's expected utility of reporting truthfully and following the recommendation is no less than 0, that is:

$$U_b(t) = \int_{q\in Q} \pi(t,q)[v(t,q) - P_b(t)]g(q)\,\mathrm{d}q \geq 0, \tag{1}$$

$$U_s(q) = \int_{t\in T} \pi(t,q)[P_s(q) - r(q)]f(t)\,\mathrm{d}t \geq 0. \tag{2}$$

Incentive compatibility (IC). To satisfy the IC constraint, we need two steps. The first step is to ensure that following the mediator's recommendation is the best option for both agents assuming that they truthfully reported their types in previous steps. Interestingly, this turns out to be exactly the same as the IR constraints.

For the second step, we still only need to consider the case when signal 1 is received. We need to ensure that upon receiving signal 1, the maximum expected utility from misreporting is no more than truthfully reporting for both players, that is:

$$U_b(t) \geq \int_{q\in Q} \pi(t',q)[v(t,q) - P_b(t')]g(q)\,\mathrm{d}q, \tag{3}$$

$$U_s(q) \geq \int_{t\in T} \pi(t,q')[P_s(q') - r(q)]f(t)\,\mathrm{d}t. \tag{4}$$

4 THE OPTIMAL MECHANISM

We first introduce the following quantities: $R_b^\pi(t) = \int_{q\in Q} \alpha_1(q)\pi(t,q) g(q)\,\mathrm{d}q$, $R_s^\pi(q) = \int_{t\in T} \pi(t,q)f(t)\,\mathrm{d}t$. We call a mechanism (π, P_b, P_s) *feasible* if it satisfies constraints (1) - (4). The following lemma gives a characterization of feasible mechanisms.

LEMMA 4.1. *A mechanism (π, P_b, P_s) is feasible if and only if it satisfies the following constraints:*

$$R_b^\pi(t) \textit{ is monotone non-decreasing in } t. \tag{5}$$

$$R_s^\pi(q) \textit{ is monotone non-increasing in } q. \tag{6}$$

$$U_b(t) = U_b(t_1) + \int_{t_1}^{t} R_b^\pi(t)\,\mathrm{d}x \tag{7}$$

$$U_s(q) = U_s(q_1) - k\int_{q_1}^{q} R_s^\pi(q)\,\mathrm{d}x \tag{8}$$

$$U_b(t_1), U_s(q_2) \geq 0 \tag{9}$$

Let $\phi_b^-(t) = t - \frac{1-F(t)}{f(t)}$ be the buyer's virtual value function and $\phi_s^+(q) = q + \frac{G(q)}{g(q)}$ the seller's virtual cost function. Our main result is built upon the following regularity condition:

Definition 4.2 (Regularity). A problem instance is regular if both function $\phi_b^-(t)$ and function $\frac{k\phi_s^+(q)-\alpha_2(q)}{\alpha_1(q)}$ are monotone non-decreasing.

This regularity condition is also standard in the literature [2–4]. Our solution belongs to the following category of *threshold mechanisms* when the problem instance is *regular.*

Definition 4.3. A mechanism (π, P_b, P_s) is called a threshold mechanism if there exist monotone functions $\lambda(t)$ and $\eta(q)$, such that the mediator recommends "trade" as long as $\lambda(t) \geq \eta(q)$.

Now we are ready to present our optimal mechanism.

THEOREM 4.4. *Suppose that a problem instance satisfies the regularity condition. Then the threshold mechanism with threshold functions $\lambda(t) = \phi_b^-(t), \eta(q) = \frac{k\phi_s^+(q)-\alpha_2(q)}{\alpha_1(q)}$ and the following payment functions is an optimal mechanism:*

$$P_b^*(t) = \frac{1}{\int_{q\in Q}\pi^*(t,q)g(q)\,\mathrm{d}q}\left[\int_{q\in Q}\pi^*(t,q)v(t,q)g(q)\,\mathrm{d}q - \int_{t_1}^{t}\int_{q\in Q}\alpha_1(q)\pi^*(x,q)g(q)\,\mathrm{d}q\mathrm{d}x\right], \tag{10}$$

$$P_s^*(q) = \frac{1}{\int_{t\in T}\pi^*(t,q)f(t)\,\mathrm{d}t}\left[\int_{t\in T}\pi^*(t,q)r(q)f(t)\,\mathrm{d}t + k\int_{q}^{q_2}\int_{t\in T}\pi^*(t,x)f(t)\,\mathrm{d}t\mathrm{d}x\right]. \tag{11}$$

5 CONCLUSION

We studied the problem of designing revenue-maximizing mechanisms for the mediator. We defined the general communication protocol and proved that the mediator can focus on the set of direct and incentive-compatible mechanisms without loss of generality. Then we formulated this problem as a mathematical program. Moreover, we gave a closed-form solution to the optimization problem under the regularity condition.

REFERENCES

[1] Moshe Babaioff, Robert Kleinberg, and Renato Paes Leme. 2012. Optimal mechanisms for selling information. In *Proceedings of the 13th ACM Conference on Electronic Commerce*. 92–109.

[2] Yang Cai and Constantinos Daskalakis. 2011. Extreme-value theorems for optimal multidimensional pricing. In *2011 IEEE 52nd Annual Symposium on Foundations of Computer Science*. IEEE, 522–531.

[3] Vijay Krishna and Eliot Maenner. 2001. Convex potentials with an application to mechanism design. *Econometrica* 69, 4 (2001), 1113–1119.

[4] Roger B Myerson. 1981. Optimal auction design. *Mathematics of operations research* 6, 1 (1981), 58–73.

[5] National Association of Realtors. 2022. National Association of Realtors Statistics. https://www.nar.realtor. Accessed: 2022-8-5.