Separations and Collapses in Computational Social Choice

Doctoral Consortium

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ABSTRACT

Following the seminal work of Bartholdi et al. [2], there has been a slew of research on the complexity of constructive and destructive control for specific election systems (a.k.a. voting rules), which was driven by the field's desire to find a natural election system that is "resistant" to as many control attacks (types) as possible. While this race was happening, many proofs were devised for a variety of election systems, and yet unbeknownst to many, several control attacks were in fact *exactly* the same (when viewed as decision problems, which is the common framework). Hemaspaandra et al. [14] were the first to make this observation, demonstrating that there was a general lack of understanding of the standard control attacks. My work continues this line of research in three ways: (1) determining the relationships of electoral control types both in the "general" setting and in concrete settings, (2) finding axiomatic-sufficient conditions to determine if a particular equality between control types (a.k.a. collapse) occurs, and (3) linking results in the more abstract decision model to the more explicit search model.

KEYWORDS

Computational Social Choice; Elections and Voting; Electoral Control

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1 INTRODUCTION

Bartholdi et al. [2] started the study of electoral control from a computational perspective. They defined control as a form of attack in which an election chair seeks to make a given candidate a winner by (a) adding/deleting voters, (b) adding/deleting candidates, (c) partition of candidates, and (d) partition of voters. For each control type, they looked at the complexity of deciding, using a specific election system, if the chair can successfully make their preferred candidate win (this is known as constructive control). For example, they showed that under plurality, deciding if there is a successful constructive control action by adding candidates is NP-hard, and yet, deciding if there is a successful constructive control action by adding voters is in P. Given an election system and a control type, if the associated decision problem is NP-hard, then the election system is said to be *resistant* to that control type.[1]

This approach is certainly interesting from an electoral perspective; e.g., deleting voters models voter suppression and partition of voters can model gerrymandering. However, the approach itself is much more general. This study shows that certain preference aggregation schemes can be controlled, sometimes easily, to alter the output. For example, a scheme that seeks to gather information from agents (i.e., voters), so as to recommend a secure service (i.e., candidate) can be maliciously altered to artificially introduce agents with dummy information or even to delete a limited number of alternatives, thereby making a less secure service be recommended.

The original framework of Bartholdi et al. [2] was extended by Hemaspaandra et al. [15] who formulated the destructive variants of the control problems, i.e., variants in which the election chair seeks to prevent a candidate from winning.

What followed was a race to find a natural election system that is resistant to as many control types as possible.[2] It was both shocking and interesting when Hemaspaandra et al. [14] signaled to the COMSOC field that it had been doing duplicate work when they showed that, hiding in plain sight, were electoral control types that were exactly the same (when viewed as decision problems). Moreover, the findings of Hemaspaandra et al. [14] are also theoretically interesting.

We say that when two control types *collapse* when they are equal (when viewed as decision problems). Otherwise, they *separate*. My work looks at deepening our theoretical understanding of separations and collapses, either by direct arguments, through axiomatic-sufficient conditions, or by studying them through a framework that relates the hardness of search (sometimes also called functional) problems, and establishing, in some sense, an equivalence between those problems so as to translate the collapse/separation results from the decision model to the search model.

2 ESTABLISHED RESULTS

This section will go over results established in papers on which I am a coauthor [7–10].

Our "Separating and Collapsing Electoral Control Types" [8, 10] paper explores the line of work that Hemaspaandra et al. [14] started by fully determining the relationship that holds, for any election system, between each pair of (compatible, i.e., having the same input types) control types (322 pairs for each election system). We also extend that work to the most important, concrete election systems,

[1] This definition intentionally sweeps under the rug the notion of immunity, which would be outside the scope of this paper. For a more detailed definition, please refer to [15].

[2] By adapting complexity theory's join operator (⊕), Hemaspaandra et al. [16] gave an artificial election system that is resistant to every standard electoral control type.

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namely plurality, veto, and approval, and find 15 new collapsing pairs among the 1288 pairs that we study. Additionally, we establish some previously unknown relationships between control types, e.g., in some natural settings, a successful control action by partition candidates implies a success control action by partition of voters, which for many may sound artificial and yet it is not.

Furthermore, we provide some sufficient axiomatic conditions for our collapses. This axiomatic approach is even more powerful than restricting ourselves to studying only, one at a time, concrete election systems as it allows us to study a countably infinite number of elections in one go. We view it as a way of building strong theoretical tools to study separations and collapses in one fell swoop. I believe that this axiomatic approach has even more interesting applications in deepening our understanding of how specific control attacks "behave" differently under various election systems.

Finally, a less theoretical but yet very interesting contribution of that paper is the introduction of computer-aided search in finding concrete examples that witness that specific control types are not equal. Our programs do not explore the entire search space, but rather, randomly select elements of the search space to test. While we have not explored the theoretical guarantees of such an approach, we've found that in practice it works much faster than the deterministic methods we originally considered. The programs used have been made publicly available and have been written so as to be easily adaptable for other settings (i.e., other election systems). This takes away from the human the need to perform those tasks easily performed by a computer.

To supplement that work, we investigated whether our collapses translate from collapsing decision complexity to collapsing search complexity [7, 9]. Indeed, that connection is not a guaranteed one: Borodin and Demers [6] initially showed that under the assumption that $P \neq NP \cap coNP$, there is a problem that is decidable in polynomial-time and yet is not self-reducible (which can be a key property when showing that the search complexity of a problem is polynomially related to its decision complexity). Their assumption is reasonable in practice as it is well-known that the decision complexity of integer factorization is in $NP \cap coNP$, and so to be consistent with our desire for current cryptosystems to be secure, we hope that $P \neq NP \cap coNP$. To our knowledge, there are only three areas of research that have exploited this result and showed that there are problems that are expected to not be self-reducible: Bellare and Goldwasser [3] (whose focus is on search in cryptographic settings), Hemaspaandra and Narváez [19] (who study the complexity of finding nontrivial backbones in Boolean formulas), and Hemaspaandra et al. [14] (who showed settings where one can decide in polynomial-time if a successful attack exists and yet, the exact details of the attack cannot be computed in polynomial-time). And so, with the knowledge that the search complexities might differ, we build a methodology on top of work by Book et al. [5] and Megiddo and Papadimitriou [20] and show that each of our collapsing-control-type pairs share the same search complexity. For those collapsing pairs that hold for every election system, we show that the collapse in their search complexities does not rely on the election system. This has resulted in the paper "Search versus Search for Collapsing Electoral Control Types" [7, 9].

3 FUTURE DIRECTIONS

This section will outline future research lines that stem from the current projects, including the challenges that I faced in some occasions. I will first mention those problems that I wish to tackle during the remainder of my doctoral program.

(1) *Apply current methods to other election systems.* The first clear direction is to extend the study to include other election systems of interest, such as Copeland and Schulze. Such extensions can be done in two ways: (1) individually studying each election system and (2) complete axiomatizations of collapses. Thanks to our preexisting code, conducting the former can be simplified; we can indeed write programs to find separations for us when the election system's winner problem is polynomial-time computable. On the other hand, the latter is certainly more enticing, but the next point discusses some difficulties.

(2) *Complete axiomatic characterizations of currently known electoral control type collapses.* This will allow us to go from needing tedious proofs to show collapses, to using simple axiom-satisfiability tests, thereby making the results easier to use in practice. I have done some work in that direction, and one of the issues with complete characterizations of collapses is that the commonly used axioms do not follow naturally from electoral collapses. I suspect this might require devising new axioms, and that full characterizations might be the hardest task in this list.

(3) *Determining separations and collapses in online elections* (e.g., see [17, 18]). In some sense, these types of elections are closer in nature to the type of attacks experienced in practice and thus have the merit of benefiting a larger community.

(4) *Identifying the search complexities of online forms of electoral attacks.* Prior work shows hardness results beyond NP-hardness for concrete systems [17, 18], which suggests that computing successful attacks in practice can be very hard. Thus it would be interesting to investigate popular systems that have attacks with easy (in P) decision complexity, and yet the search complexity of the online variant of the attack is hard, which would add a new layer of protection.

4 CONCLUSION

Finally, it is worth mentioning that there are many other interesting studies that follow from this line of research, such as characterizations of collapsing electoral control types in the online setting, along with their analogous search-version results. In addition, one can depart from the standard control model and look at separations and collapses in the context of electoral manipulation [1]. In this setting, voters may vote insincerely in an attempt to make a candidate (not necessarily their favorite candidate) win. Various forms of manipulation have been studied form of attack and are discussed in depth in the excellent book chapter by Conitzer and Walsh [11]. Beyond that, the complexity of attacks in various excellent models have been studied over the years (some examples include weighted votes [12, 22] or coalitions of voters [4, 13, 21]). Extending the study of separations and collapses to those models can give us additional insights into the relationship between properties of election systems and attacks on elections.

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