Incentives in Ridesharing with Deficit Control

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ABSTRACT

This paper proposes a novel market-based system for ridesharing, where commuters are matched based on their declared travel constraints, the number of available seats (which could be zero), and their costs. Based on this information, the system then designates commuters to be either drivers or riders, finds appropriate matches, and calculates rewards for drivers and payments for riders. We show that, for this system, the well-known Vickrey-Clarke-Groves (VCG) mechanism is incentive compatible (IC), individually rational (IR) and efficient (i.e., minimizing cost), but results in a very high deficit, thus requiring large subsidies. We therefore investigate alternative mechanisms. We first consider mechanisms with fixed prices and show that no such mechanism can be both efficient and IC. Thus, we propose an inefficient IC mechanism but which has deficit control. We then consider a VCG mechanism with two-sided reserve prices. We show that this mechanism is IC and IR for a certain range of reserve prices, and we analyse the deficit bounds and how these can be controlled. We furthermore show that the deficit can be controlled even further by limiting the (costly) detours taken by the drivers when computing the allocations, thereby trading off efficiency and deficit.

Categories and Subject Descriptors

I.2.11 [**ARTIFICIAL INTELLIGENCE**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Algorithms, Economics, Theory

Keywords

Ridesharing, combinatorial exchange, mechanism design

1. INTRODUCTION

Ridesharing began in the 1940s during World War II through car clubs in North America. Since then, it has been widely promoted to address problems such as fuel shortages, air pollution and traffic congestion [14, 5, 2]. More recently, there has been a surge in ridesharing platforms, such as *Zimride* and *Avego*, that leverage the power of online social networking services and smart phones. It was estimated that, in July 2011, there were 638 ridematching services in North America alone [2]. Despite the large number of initiatives over the years, both publicly and privately funded, there is still a lack of participation in those programs. According to the U.S. Census, only 10.7% of American workers commuted to work by ridesharing/carpool in 2008. The comparable figure is 7.7% in Canada in 2006. Reports also show that, in the US and Europe, most cars are driven less than 1 hour per day and carry only one person for 90% of trips.

In this paper, we use *mechanism design* to help address the above problem. Specifically, we propose a novel market-based ridematching model that incentivizes commuters to participate in the system and to truthfully declare their preferences and constraints, while limiting any deficit for the system in order to be economically sustainable. In the proposed model, each commuter is asked to declare their departure and arrival locations, and a travel time window. In addition they need to declare their costs, which corresponds to the travel costs incurred when not using the system (e.g., fuel costs, taxi costs, or the costs of public transportation). Finally, they declare the number of available seats (which is zero if they use e.g. public transport). Based on this information, the mechanism computes allocations specifying who is going to be a driver (i.e., selling any available seats), a rider (i.e., purchasing an available seat) or unmatched, a schedule for each commuter, and appropriate payments. The main goal of a ridesharing system is to minimize the overall costs (i.e., maximize efficiency), while incentivizing participation and truthful reporting. However, we show that it is not possible to satisfy these properties without the system incurring a significant deficit. Hence, such a system could not be economically sustainable. Thus, we propose several mechanisms that are less efficient but can control or even eliminate the deficit.

The problem of deficit control (also known as achieving weak budget balance) is well known in the mechanism design literature. Myerson [10] was one of the first to study this problem for bilateral trade. McAfee [8] proposed a trade reduction approach to achieve weak budget balance by sacrificing efficiency for a single-commodity exchange with multiple buyers and sellers. Gonen *et al.* [4] generalized

Appears in: *Alessio Lomuscio, Paul Scerri, Ana Bazzan, and Michael Huhns (eds.), Proceedings of the 13th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2014), May 5-9, 2014, Paris, France.*

McAfee's trade reduction idea, where they showed that efficiency can still be bounded for a class of single-valued domains. While these approaches also match buyers and sellers, they assume either a homogeneous good or a single-valued domain, which does not apply in our setting. Our setting is more akin to a combinatorial exchange. For this setting, Parkes *et al.* [15] build on the well-known Vickrey-Clarke-Groves (VCG) mechanism and change the payments to enforce (strict) budget balance, while sacrificing both efficiency and truthfulness. We take a different approach and try to control the deficit, rather than eliminating it completely, while maintaining truthfulness. Furthermore, so far there is very little literature that considers mechanism design specifically for the ridesharing problem. Exceptions are the work by Kleiner *et al.* [7], who propose a simple model based on the second-price auction, and Kamar *et al.* [6], who apply the payment mechanisms proposed in [15] to an experimental ridesharing system. Both of these papers study their models empirically in a dynamic ridesharing environment, but they contain no new theoretical contributions.

The remainder of the paper is organised as follows. Section 2 presents the ridesharing model. Section 3 demonstrates that efficient and truthful ridematching is only achievable with a large potential deficit. Then, Section 4 investigates two approaches for controlling deficit: fixed prices and VCG with two-sided reserve prices. We conclude and discuss future work in Section 5.

2. THE MODEL

We represent the commuters' possible routes as a graph $G = (L, E)$, where L is a set of stopping points and E is a set of edges (i.e., routes) between stopping points. There is a weight associated with each edge $e_{l_i,l_j} \in E$,[1] denoted by $w(e_{l_i,l_j})$, which indicates the time required to finish the travel on the route. Let $w_{min}(l_i, l_j)$ be the minimum weight of all paths between $l_i \in L$ and $l_j \in L$.

We consider discrete time periods $T = \{0, 1, \ldots, t_{end}\}$ for arriving at stopping points.[2] Furthermore, let N be the set of all commuters. Each commuter $i \in N$ has a privately observed **type**, $\theta_i = (l_i^d, l_i^a, t_i^d, t_i^a, c_i, q_i)$, where $l_i^d, l_i^a \in L$ are i's departure and arrival locations, $t_i^d, t_i^a \in T$ are i's earliest departure and latest arrival time (i.e., the time window available for travel), $c_i \in \mathbb{R}^+$ is i's travel cost to finish the trip, and $q_i \in \mathbb{N}$ is the number of seats available for any additional riders. Note that θ_i can represent commuters with their own transport, as well as commuters who would normally use public transportation, or even a taxi. The main difference is in terms of the parameter q_i. For example, if θ_i is based on public transportation, i.e. i has no seat to share with others, then $q_i = 0$. Otherwise, if they have car or are willing to share a taxi, then $q_i \geq 0$. Here, the cost parameter c_i represents the cost of commuting without participating in the system. For example, if the commuter normally uses public transport, this represents the cost of that transport. If he or she uses a car, this is the fuel cost for the commute. We assume that the cost is independent of the the occupancy of the vehicle.[3] However, as we will explain in detail below, an additional cost is incurred if the commuter has to make a detour to pick up other riders.

In the ridesharing system, each commuter i is required to report her type to the system. We denote the report by $\hat{\theta}_i$ and note that commuters could misreport their type if it is their best interest to do so. Let $\theta = (\theta_i)_{i \in N}$ be the type profile of all commuters, $\theta_{-i} = (\theta_1, \theta_2, \cdots, \theta_{i-1}, \theta_{i+1}, \cdots, \theta_n)$ be the type profile of all commuters except i. Given commuter i of type θ_i, we refer to Θ_i as the set of all possible type reports of i, and let Θ be the set of all possible type profile reports of all commuters with type profile θ.

Given report profile $\hat{\theta} \in \Theta$, a **ridesharing mechanism** will compute an **allocation** (i.e., matchings) $\pi = \{\pi_i(\hat{\theta})\}_{i \in N}$ and **payments** $x = \{x_i(\hat{\theta})\}_{i \in N}$. Here, $\pi_i(\hat{\theta}) = (d_i, s_i)$ where $d_i \in \{0, 1\}$ indicates whether or not i travels with her own transportation: $d_i = 1$ means i uses her own transportation (e.g., driving or public transport) and $d_i = 0$ means that i shares a ride with other commuters, and s_i is a schedule for i (defined below). Payment $x_i(\hat{\theta}) \in \mathbb{R}$ and if $x_i(\hat{\theta}) > 0$, i pays $x_i(\hat{\theta})$ to the mechanism, otherwise, i receives $-x_i(\hat{\theta})$ from the mechanism.

DEFINITION 1. *A **schedule** s_i for i is defined as a sequence $((l_i^1, t_i^{a_1}, t_i^{d_1}, w_i^1), (l_i^2, t_i^{a_2}, t_i^{d_2}, w_i^2), \ldots, (l_i^{k_i}, t_i^{a_{k_i}}, t_i^{d_{k_i}}, w_i^{k_i}))$, such that i has to arrive at $l_i^k \in L$ before $t_i^{a_k} \in T$ and depart from $l_i^k \in L$ with commuters $w_i^k \subseteq N$, where $i \in w_i^k$, no later than $t_i^{d_k} \in T$. k_i is the number of stops/transfers of s_i.*

Given i's schedule s_i, we define i's **schedule dependency** with respect to $j \in N \setminus \{i\}$ as:

$$s_i(j) = \{(l_i^k, t_i^{a_k}, t_i^{d_k}, w_i^k) \in s_i : j \in w_i^k\}.$$

Given allocation π we say that a commuter i is matched as a **driver** if she is allocated riders, i.e. if $\pi_i(\theta) = (1, s_i)$ and $\bigcup_{j \in N \setminus \{i\}} s_i(j) \neq \emptyset$; i is matched as a **rider** if $\pi_i(\theta) = (0, s_i)$. Otherwise, i is said to be **unmatched**, i.e. $\pi_i(\theta) = (1, s_i)$ but $\bigcup_{j \in N \setminus \{i\}} s_i(j) = \emptyset$.

Given report profile θ, allocation $\pi_i(\theta) = (d_i, s_i)$, an essential requirement is that the allocation is **feasible** for i. Before defining feasibility, we define **the driver of** s_i as $d(s_i) = \{d_k(s_i)\}_{k=1}^{k_i - 1}$, where $d_k(s_i) = \{j \in w_i^k : \pi_j(\theta) = (1, s_j)\}$. That is, $d_k(s_i)$ is the set of commuters who are matched as drivers travelling with i from the k-th stop of s_i.

DEFINITION 2. *Given commuter i of type θ_i, we say allocation (d_i, s_i) is **feasible** for i, if*

1. $l_i^1 = l_i^d$, $l_i^{k_i} = l_i^a$,
2. $t_i^d \leq t_i^{a_1}$, $t_i^{a_{k_i}} \leq t_i^a$, $t_i^{a_k} \leq t_i^{d_k}$ *for all* $1 \leq k \leq k_i$,
3. $t_i^{a_{k+1}} - t_i^{d_k} \geq w_{min}(l_i^k, l_i^{k+1})$ *for all* $1 \leq k < k_i$,
4. $d_k(s_i) = \{j\}$, $|w_i^k| - 1 \leq q_j$ *for all* $1 \leq k < k_i$, $w_i^{k_i} = \emptyset$,
5. $w_i^k \neq w_i^{k+1}$ *for all* $1 \leq k < k_i$.

We say allocation π is feasible if, for all $i \in N$, all profile θ, $\pi_i(\theta)$ is feasible and also consistent with other commuters' allocations[4].

[1] If there are multiple routes between two adjacent stops, we only need to model the shortest one.

[2] The number of discrete time periods depends on the specific applications. For instance, if we consider matchings in a 24-hour time window, and there is a time point for every 10 minutes, then $t_{end} = 24 * 6 = 144$.

[3] In reality, the fuel cost might increase with higher occupancy. At the same time, in some cities high-occupancy vehicles have special lane which might decrease the travel time/cost. Overall, the cost difference is not significant and is hard to model.

[4] For example, if i appears in the schedule of j, then j should appear in the schedule of i in the same way.

Constraint 1 above requires the first and the last locations to be i's departure and arrival locations. Constraints 2 and 3 require that the schedule to fit in i's travel time window and the time scheduled between stops to allow i to complete her travel. Constraint 4 means that there is always one and only one driver at each stop except the terminal, and the number of riders should be no more than the number of seats the driver has reported. The last constraint indicates that s_i should not include any stop on the way without transfer, pick-up or drop-off, i.e. all intermediate stops are necessary.

Given these constraints, the goal of the system is to maximize efficiency, which means (as we will show) minimizing overall costs. As already mentioned, additional costs are incurred if the driver requires to make a detour in order to pick up or drop off a rider. Now, an important feature of the model is that we assume the system is able to compute these costs based on the reported c_i, the schedule s_i, and the weights $w(e_{l_i,l_j})$ of graph G as defined earlier. For example, if c_i represents fuel costs of the journey without detour, the system estimates the fuel costs of the journey with a detour. Similarly, if c_i represents the costs of a taxi without a detour, the system extrapolates to compute the costs incurred by picking up additional passengers. Specifically, the detour cost, $c_{detour}(c_i, s_i)$, is calculated based on the additional driving time allocated to i proportional to the original driving time. Formally:

$$c_{detour}(c_i, s_i) = c_i\delta_i, \text{ where } \delta_i = \frac{w_{min}(s_i) - w_{min}(l_i^d, l_i^a)}{w_{min}(l_i^d, l_i^a)}.$$

Here, $w_{min}(s_i) = \sum_{k=1}^{k_i-1} w_{min}(l_i^k, l_i^{k+1})$ is the driving time of s_i with any detours. Note that the numeratour of δ_i denotes the additional driving time and $\delta_i \geq 0$. Also, for large enough detours, the detour costs can exceed the original costs. Then, the overall costs in the system given allocation $\pi(\theta)$ are given by:

$$c(\pi(\theta)) = \sum_{i \in N, \pi_i(\theta)=(1,s_i)} (c_i + c_{detour}(c_i, s_i)).$$

We say an allocation is **cost-efficient** if it minimizes costs for all type profile reports.

Given these detour costs, we define the **valuation** v of commuter i with type θ_i and allocation (d_i, s_i), as:

$$v(\theta_i, (d_i, s_i)) = \begin{cases} -c_{detour}(c_i, s_i) & \text{if } d_i = 1 \text{ with feasible } s_i, \\ c_i & \text{if } d_i = 0 \text{ with feasible } s_i, \\ -\infty & \text{if } s_i \text{ is infeasible.} \end{cases}$$

Intuitively, the valuation corresponds to the savings made by a commuter due to the allocation (without taking into account payments), where the savings are positive for a rider (since she no longer incurs her costs), and negative for a driver (if there is a detour). Note that, if i is unmatched, the commuter has no detour and her valuation is zero, which corresponds to the first case when $c_{detour}(c_i, s_i) = 0$.

We are now ready to define efficiency.

DEFINITION 3. *An allocation π is **efficient**, if for all θ, $\pi(\theta) \in \arg\max_{\pi' \in \Pi} \sum_{i \in N} v(\theta_i, \pi'_i)$, where Π is the set of all allocations. The sum of valuations is also referred to as the **social welfare**.*

It is easy to verify that minimizing the overall costs is equivalent to maximizing the social welfare (see Example 1 below).

PROPOSITION 1. *A ridesharing allocation is cost-efficient if and only if it is efficient.*

Given commuters' type profile θ, their reports $\hat{\theta} \in \Theta$, and mechanism $\mathcal{M} = (\pi, x)$, we define the **utility** of i as

$$u(\theta_i, \hat{\theta}, (\pi, x)) = v(\theta_i, \pi_i(\hat{\theta})) - x_i(\hat{\theta}).$$

Intuitively, the utility is the overall savings made. Without participating in the mechanism, there is no allocation, no detour and no payment, and so the utility is zero. We say a mechanism is **individually rational** if no commuter receives a negative utility by joining the system. This is also called the participation constraint.

In addition to the commuters, we are also interested in whether the system as a whole is profitable, or if it requires subsidies.

DEFINITION 4. *Given report profile θ and mechanism $\mathcal{M} = (\pi, x)$, the revenue of the market owner is*

$$Rev(\theta, (\pi, x)) = \sum_{i \in N} x_i(\theta).$$

*For all report profile $\hat{\theta}$, if $Rev(\hat{\theta}, (\pi, x)) = 0$ (≥ 0), we say $\mathcal{M}$ is **budget balanced** (weakly budget balanced). If $Rev(\hat{\theta}, (\pi, x)) < 0$, we say $\mathcal{M}$ runs a **deficit** $|Rev(\hat{\theta}, (\pi, x))|$.*

Following the tradition of mechanism design, preventing strategic manipulations of commuters is crucial to the system, which is known as *truthfulness* (or dominant-strategy incentive compatibility):

DEFINITION 5. *A mechanism $\mathcal{M} = (\pi, x)$ is **truthful** if $u(\theta_i, (\theta_i, \hat{\theta}_{-i}), (\pi, x)) \geq u(\theta_i, \hat{\theta}, (\pi, x))$ for all $i \in N$, all θ, all $\hat{\theta} \in \Theta$.*

That is, a mechanism is truthful if reporting the type truthfully is a dominant strategy. This property not only ensures that the system will receive truthful information, which is important to achieve efficiency, but also avoids the need for strategic reasoning by commuters.

We assume that commuters cannot misreport their departure and arrival locations, as the system can easily verify their locations via, for example, other commuters or mobile devices, and apply penalties for misreporting locations. Moreover, misreporting locations will lead to infeasible allocations which are not appealing to commuters. Therefore, a truthful mechanism should incentivize commuters to truthfully report their travel time window, costs, and the number of extra seats. Furthermore, in this paper we only consider deterministic mechanisms, where for all reports, the allocation and the payment outcomes are deterministic.

$$\theta_1 = (A, B, t_1^d, t_1^a, 5, 1)$$
$$\theta_2 = (A, B, t_2^d, t_2^a, 6, 1)$$
$$\theta_3 = (C, B, t_3^d, t_3^a, 7, 0)$$
$$\theta_4 = (D, B, t_4^d, t_4^a, 8, 0)$$

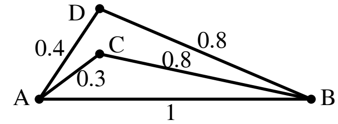

Figure 1: Ridesharing example

We finish this section by an example to clarify the model.

EXAMPLE 1. *Figure 1 shows an example with 4 commuters. The first two have one additional seat each and are travelling from A to B. The other two have no extra seat and travel from C to B and D to B respectively. The direct path from A directly to B requires a time of* 1*, and from A to B via C (D) is* 1.1 *(*1.2*). The efficient allocation is that 1 drives via D to take commuter 4, and 2 drives via C to take commuter 3, if this allocation is feasible. The total cost saved is the costs of* 3 *and* 4 *minus the detour costs of 1 and 2, which is* $7 + 8 - (0.2 * 5 + 0.1 * 6) = 13.4$*. The valuations for 1, 2, 3 and 4 are* $-0.2*5$, $-0.1*6$, 7 *and* 8*, respectively. Note that, if the cost of commuter 1 is* 15 *rather than* 5*, the efficient allocation will be that 2 takes 1 from A to B, and 3 and 4 are unmatched, where the total cost saved is* 15*. In other words, a commuter with extra seats to share can be matched as a driver, rider or unmatched, depending on her type and others' types. More precisely, a commuter with extra seats can be both a seller and a buyer.*[5]

3. TRUTHFUL & EFFICIENT MECHANISM

In this section we consider the problem of finding a truthful and efficient mechanism, and the implications for the deficit in our system. A well-known class of mechanisms that is efficient and incentivizes truthful reporting in many domains is the set of Groves mechanisms [11, 12]. In fact, in unrestricted domains, the Groves mechanisms are the *only* mechanisms which are both efficient and truthful [11, Th.2.8]. The classic mechanism from this class is called the *Clarke mechanism*, also referred to as the Vickrey-Clarke-Groves (VCG) mechanism, which has some nice properties, such as individual rationality, and also has no deficit in many domains. In this mechanism, the payments are based on the marginal contribution of an agent to the social welfare, and are computed as follows:

Clarke pivot payment x^{Clarke}

Given reported profile θ and efficient allocation π, the payment for commuter i is given by:

$$x_i^{Clarke}(\theta) = V(\theta_{-i}, \pi) - V_{-i}(\theta, \pi)$$

where

- $V(\theta_{-i}, \pi) = \sum_{j \in N \setminus \{i\}} v(\theta_j, \pi_j(\theta_{-i}))$, i.e. the social welfare of the efficient allocation when excluding agent i's report from the allocation.
- $V_{-i}(\theta, \pi) = V(\theta, \pi) - v(\theta_i, \pi_i(\theta))$, i.e. the social welfare of the efficient allocation excluding the valuation of agent i.

Note that, for riders, the first term is no smaller than the second term, and so their payment is positive. For drivers, the opposite is true and so they always *receive* payment. Intuitively, a driver i will receive the amount of the cost saved for other commuters because of i's participation, while a rider j will pay the amount of the cost increased by j's participation for other commuters.

[5]Note that, although our setting is similar to a combinatorial exchange in many ways, different to our setting, in typical combinatorial exchanges an agent cannot be both a seller and a buyer of the same item or bundle.

The following theorem (Theorem 1) shows that an efficient allocation with payment x^{Clarke} is indeed truthful and also individually rational in our setting. To prove that, we will use the following characterization of truthfulness, which is directly based on Proposition 9.27 from [12] and the proof is omitted here:

LEMMA 1. *A ridematching* (π, x) *is truthful if and only if, for all* i *of type* θ_i *and all* θ_{-i}*, it satisfies:*

1. *The payment* $x_i(\theta_i, \theta_{-i})$ *does not depend on* θ_i*, but only on the alternative allocation* $\pi(\theta_i, \theta_{-i})$*. That is, for all* $\hat{\theta}_i \neq \theta_i$*, if* $\pi(\theta_i, \theta_{-i}) = \pi(\hat{\theta}_i, \theta_{-i})$*, we have that* $x_i(\theta_i, \theta_{-i}) = x_i(\hat{\theta}_i, \theta_{-i})$.
2. *The utility of* i *is optimised. That is,* $\pi(\theta_i, \theta_{-i})$ *and* $x(\theta_i, \theta_{-i})$ *maximize* i*'s utility among all alternatives in the range of* $\pi(., \theta_{-i})$*, i.e.* $\theta_i \in \arg\max_{\hat{\theta}_i \in \Theta_i} u(\theta_i, (\hat{\theta}_i, \theta_{-i}), (\pi, x))$.

THEOREM 1. *Efficient allocation* π^{eff} *combined with payment* x^{Clarke}*, i.e. VCG, is truthful and individually rational.*

PROOF. First we consider truthfulness. We show that the two conditions of Lemma 1 hold for this mechanism.

Given θ_{-i}, for all $\hat{\theta}_i \neq \theta_i$, if $\pi(\theta_i, \theta_{-i}) = \pi(\hat{\theta}_i, \theta_{-i})$, we get $x_i^{Clarke}(\theta_i, \theta_{-i}) = x_i^{Clarke}(\hat{\theta}_i, \theta_{-i})$, because $V(\theta_{-i}, \pi^{eff})$ and $V_{-i}(\theta, \pi^{eff})$ only depends on θ_{-i} and the allocation. Therefore, the first condition of Lemma 1 is satisfied.

Next, given θ_{-i} and commuter i of type θ_i, we need to show that the utility of i via reporting θ_i is maximized by the mechanism, i.e. $\theta_i \in \arg\max_{\hat{\theta}_i \in \Theta_i}(v(\theta_i, \pi_i^{eff}(\hat{\theta}_i, \theta_{-i})) - x_i^{Clarke}(\hat{\theta}_i, \theta_{-i}))$. Let $\pi_i^{eff}(\theta_i, \theta_{-i}) = (d_i, s_i)$ and $\pi_i^{eff}(\hat{\theta}_i, \theta_{-i}) = (\hat{d}_i, \hat{s}_i)$. We assume that $(\hat{d}_i, \hat{s}_i)$ is feasible for θ_i, otherwise, i's utility for misreporting $\hat{\theta}_i$ is $-\infty$. To simplify the notation, we omit the mechanism in the utility. We have $u(\theta_i, \theta) = v(\theta_i, (d_i, s_i)) - x_i^{Clarke}(\theta_i, \theta_{-i}) = v(\theta_i, (d_i, s_i)) - V(\theta_{-i}, \pi^{eff}) + V_{-i}(\theta, \pi^{eff})$, and $u(\theta_i, (\hat{\theta}_i, \theta_{-i})) = v(\theta_i, (\hat{d}_i, \hat{s}_i)) - V(\theta_{-i}, \pi^{eff}) + V_{-i}((\hat{\theta}_i, \theta_{-i}), \pi^{eff})$. We know that $v(\theta_i, (d_i, s_i)) + V_{-i}(\theta, \pi^{eff}) \geq v(\theta_i, (\hat{d}_i, \hat{s}_i)) + V_{-i}((\hat{\theta}_i, \theta_{-i}), \pi^{eff})$ because π^{eff} is efficient, otherwise $\pi_i^{eff}(\theta_i, \theta_{-i}) \neq (d_i, s_i)$. Thus, $u(\theta_i, \theta) - u(\theta_i, (\hat{\theta}_i, \theta_{-i})) \geq 0$ for all $\hat{\theta}_i \in \Theta_i$.

For individual rationality, it is evident that $v(\theta_i, \pi_i^{eff}(\theta)) - x_i^{Clarke}(\theta) = V(\theta, \pi^{eff}) - V(\theta_{-i}, \pi^{eff}) \geq 0$ because π_i^{eff} is efficient. □

Furthermore, we can easily show that, in our model, if we do not charge unmatched commuters, truthfulness implies individual rationality, because commuters can always misreport so that any match becomes infeasible, and thus they get a utility zero.

PROPOSITION 2. *Any truthful ridesharing mechanism without charging unmatched commuters is also individually rational.*

Despite the fact that VCG is both truthful and efficient in our setting, we show that it can lead to severe deficit, which limits its applicability to real-life applications.

PROPOSITION 3. *The deficit generated by the VCG ridematching with Clarke pivot payment can be as much as* m *times of the cost saved by the mechanism, where* $m + 1$ *is the maximum number of stops on a simple path (i.e., with no repeated stops) in the given route map.*

PROOF. The proof is by example. A large deficit occurs when every driver receives the amount of the total cost saved by the system, and the payments for the riders are zero. This situation occurs, for example, when each commuter is critical for the system to achieve a positive social welfare. Figure 2 shows an example of such an extreme situation, where each driver travels between two adjacent stops and there is no overlap where several drivers travel on the same section. All sections are connected together to form a line (type θ_1 to θ_m), and there is one rider who needs to travel on the entire path (type θ_0). Assume that, apart from the setting where no commuters are matched, the only other feasible allocation is where the rider travels with all drivers on the path. If one of the drivers is removed, there is no sharing, so each driver receives an amount of transfer equal to the cost of the rider (since they all contribute equally to the social welfare). The rider pays nothing because there is no competition with other riders, and her participation does not reduce anyone's valuation. □

$$\begin{aligned}
\theta_0 &= (l_1, l_{m+1}, t_1, t_{m+1}, c_0, 0)\\
\theta_1 &= (l_1, l_2, t_1, t_2, c_1, 1)\\
\theta_2 &= (l_2, l_3, t_2, t_3, c_2, 1)\\
&\vdots\\
\theta_m &= (l_m, l_{m+1}, t_m, t_{m+1}, c_m, 1)
\end{aligned}$$

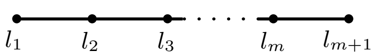

Figure 2: An example for the proof of Proposition 3

The above proposition shows that, not only can the deficit be very high, but it can be many times higher than the social welfare obtained by the system. Therefore, it makes no economic sense, even for a government, to implement such a mechanism. Now, the problem of having a deficit is well known in two-sided markets (with buyers and sellers) such as ours. In fact, Myerson's impossibility result for bilateral trade also holds here (since bilateral trade is a special case of our setting, where individual drivers are matched to individual riders), which states that a deficit is unavoidable when designing a truthful, efficient and individually rational mechanism [10]:

THEOREM 2 ([10]). *It is impossible to have a truthful, efficient and individually rational ridesharing mechanism without outside subsidies.*

4. TRUTHFULNESS & DEFICIT CONTROL

In the previous section, we have shown that VCG is truthful and efficient, but it cannot avoid large deficits. Furthermore, no efficient and truthful mechanism can be weakly budget balanced (i.e., no deficit) in our setting. In order to develop a practical and economically sustainable system, in this section we take a different approach and relax efficiency to obtain better deficit control, while maintaining truthfulness. We start, in Section 4.1, by considering fixed payment mechanisms. Then, we introduce VCG with two-sided reserve prices in Section 4.2.

4.1 Fixed payments

Fixed prices are widely used due to their truthfulness, and the ability to get revenue guarantees. Hence, we analyse these types of payments and their implications for efficiency and deficit control. There are many ways to define fixed payments. In this paper, we allow different payments for drivers and riders. We also consider payments which can be a function of the schedule, e.g. the length of the detour. These payments are fixed in the sense that they exclusively depend on the current allocation (and not e.g. on the commuters' valuations). Fixed payments are defined below:

Fixed payment $x^{fixed}(p^0, p^1)$

Given predefined values $p^0 \geq 0$ and $p^1 \leq 0$,

- The payment for rider i given schedule s_i is:

$$\begin{cases} p^0 & \textit{allocation independent.}\\ p^0 \cdot w_{min}(l_i^d, l_i^a) & \textit{location dependent.} \end{cases}$$

- The payment for driver j given schedule s_j is:

$$\begin{cases} p^1 & \textit{allocation independent.}\\ p^1 \cdot \delta_j \cdot w_{min}(l_j^d, l_j^a) & \textit{detour dependent.}\\ p^1 \cdot t_j^{share} & \textit{sharing dependent.} \end{cases}$$

  where t_j^{share} is the total travel time j shared with riders under s_j counting the number of riders in her car, i.e. if j travelled t_j time units with two riders, then $t_j^{share} = 2t_j$.
- The payment for any unmatched commuter is zero.

Note that $x^{fixed}(p^0, p^1)$ has two parameters: p^0 for riders and p^1 for drivers, and we only consider positive payments for riders, and negative for drivers (i.e., they receive payments). We do not consider the case where $p^0 < 0$ or $p^1 > 0$, because this will lead to severe deficit and/or violate individual rationality. Note that the above payments can be used in various combinations, and also can be added together to build multi-part tariffs (e.g., an allocation independent component, and a detour component).

We now analyse the efficiency of truthful mechanisms using such payments. Recall from Proposition 2 that truthfulness necessitates individual rationality in our setting (subject to having zero payment when unmatched, which is the case here). Therefore, it is clear that we cannot obtain efficiency in general, since this violate individual rationality in some instances (when the fixed payments are higher than the savings). However, we can show that, even if we limit the set of possible allocations to those that provide non-negative utility for each commuter, obtaining efficiency within that remaining set of allocations is not feasible. Formally, let $\Pi(p^0, p^1)$ denote the set of allocations where individual rationality is satisfied given the payment $x^{fixed}(p^0, p^1)$.

THEOREM 3. *Given any fixed payment $x^{fixed}(p^0, p^1)$, there exists no truthful mechanism such that:*

$$\pi(\theta) \in \arg\max_{\pi' \in \Pi(p^0, p^1)} \sum_{i \in N} v(\theta_i, \pi'_i).$$

PROOF. The proof is by example based on allocation independent payment and it can be extended for any other fixed payments. Consider an example where there are only two commuters i and j who travel on the same route during the same time window and both can take one rider. Furthermore, let $c_i < c_j$.

If $c_j < p^0$, then the only allocation without violating individual rationality is where both of them are unmatched.

If $c_j \geq p^0$, then the only allocation without violating individual rationality and minimizing cost is where i drives with payment p^1 and j rides with payment p^0. The utility for i is $u_i^1 = 0 - p^1$ and for j is $u_j^0 = c_j - p^0$.

Now let us consider the situation where $c_j > c_i \geq p^0$. The allocation is still the same as above for $c_j \geq p^0$. Under this situation, if i (j) misreported a different cost $\hat{c}_i > c_j$ ($\hat{c}_j < c_i$), then the allocation will be that j drives with payment p^1 and i rides with payment p^0. The corresponding utilities are $u_i^0 = c_i - p^0$ and $u_j^1 = 0 - p^1$. Therefore, if $u_i^0 > u_i^1$, i.e. $c_i > p^0 - p^1$, i is incentivized to misreport, and if $u_j^1 > u_j^0$, i.e. $c_j < p^0 - p^1$, j is incentivized to misreport. Thus, the only situation where both i and j will not misreport is when $c_i \leq p^0 - p^1 \leq c_j$. Since p^0 and p^1 are predefined and c_i, c_j are variables, there is no way to guarantee $c_i \leq p^0 - p^1 \leq c_j$ for all possible reports of i and j. Note that, if we introduce $c_i \leq p^0 - p^1 \leq c_j$ as a constraint on the allocation, then not only the cost will not be minimized, but also commuters will have incentives to manipulate if their costs do not satisfy $c_i \leq p^0 - p^1 \leq c_j$ and it is beneficial for them to do so. □

Note that, Theorem 3 still holds even if detours are not allowed, or the number of seats of each commuter is known by the system (which is achievable by requiring commuters to register these in advance), as the manipulations shown in the example are independent of these factors.

The consequence of Theorem 3 is that, with fixed payments, we lose the ability to minimize cost. The main reason is that the allocation is not able to maximize each commuter's utility (violates the second condition of Lemma 1). Therefore, we need to reduce the outcome space and/or add more constraints on the allocation. The following mechanism shows an example that does both: it first partitions commuters into drivers and riders (not based on the reports), and then allocates according to a predefined order.

Serial dictatorship $(\pi^{(D,\succ_D,R,\succ_R)}, x^{fixed}(p^0,p^1))$

Given predefined subset of commuters $D \subseteq N$, where D can be allocated as drivers but not as riders, while $R = N \setminus D$ can be allocated as riders but not drivers, and predefined orders $\succ_D, \succ_R$, do the following:

- Let $\Pi(p^0,p^1)$ denote the set of allocations where individual rationality is satisfied given $x^{fixed}(p^0,p^1)$.
- Following the ordering $\succ_D$, for each successive potential driver $i \in D$:
  - Allocate the set of unmatched riders $R' \subseteq R$ to i that maximizes i's utility such that: (1) riders in R' can complete their journey with i alone, and (2) the allocation is in $\Pi(p^0,p^1)$. If there are ties, choose the set of riders with the highest ranking order $\succ_R$.
  - Remove the set of allocated riders from R before proceeding to the next driver.
- Set the payment for each rider and driver to be $x^{fixed}(p^0,p^1)$.

THEOREM 4. *$(\pi^{(D,\succ_D,R,\succ_R)}, x^{fixed}(p^0,p^1))$ is truthful and individually rational.*

PROOF. We will show that the two conditions in Lemma 1 are both satisfied here. The first condition clearly holds because all fixed payments only depend on the allocation, and not on the commuters' report.

We will show that the utility for each commuter $i \in D$ is maximized. Since (1) $i \in D$ cannot change her ranking in D, (2) the utility of i is maximized whenever its her turn to get riders, and (3) i cannot influence the allocations of drivers allocated before her because their allocations only depend on their own and riders' constraints, therefore i's utility is maximized.

Finally, we show that the utility of every commuter $j \in R$ is also maximized. Firstly, note that, neither payment p^0 nor $p^0 \cdot w_{min}(l_j^d, l_j^a)$ depends on j's schedule, so her utility is maximized if she is allocated as a rider with a feasible schedule no matter with which driver. Secondly, j cannot change her ranking in R and it is not worth for her to misreport to enable infeasible allocations (giving her negative utility) or disable feasible allocations (reducing her chance to win). Therefore, j's chance to be allocated as a rider with a positive utility is maximized via truthful reporting.

Individual rationality follows directly since only the set of individually rational allocations $\Pi(p^0,p^1)$ are considered. □

The serial dictatorship mechanism is clearly very inefficient. In fact, it is easy to see that the mechanism can be arbitrarily inefficient in the worst case (e.g., when riders and drivers are wrongly partitioned). Nevertheless, there are some interesting observations from this mechanism. First of all, even with such a restrictive mechanism, the payments of the riders cannot depend on the schedule. Otherwise, the riders can misreport in order to become infeasible for some drivers, and be allocated to a different driver instead (to pay less). Also, relaxing some of the restrictions of the mechanism is not trivial. In particular, to ensure truthfulness, we need to ensure that a driver later in the sequence cannot influence the allocation of drivers earlier in the sequence. As a result, we can only consider allocations where each rider is matched with up to one driver. These observations suggest that it is difficult to obtain meaningful (deterministic) mechanisms using fixed prices, although we leave a full characterisation for future work.

We now consider the deficit of this mechanism. Due to the limit of space, we select some typical payments to analyse the deficit.

PROPOSITION 4. *Given $\mathcal{M} = (\pi^{(D,\succ_D,R,\succ_R)}, x^{fixed}(p^0,p^1))$, if the payment for riders is location dependent and the one for drivers is sharing dependent, without detour, we get:*

1. *if $p^0 = -p^1$ (or $p^0 > -p^1$), $\mathcal{M}$ is budget balanced (or weakly budget balanced).*
2. *if $p^0 < -p^1$, the deficit is bounded by $\frac{-p^1-p^0}{c^{min}} C^{saved}$, where $c^{min} > 0$ is the minimum travel cost per time unit and C^{saved} is the travel cost saved by $\mathcal{M}$.*

PROOF. Without detour, we get $\sum_{i \in N, \pi_i(\theta)=(0,s_i)} w_{min}(l_i^d, l_i^a) = \sum_{i \in N, \pi_i(\theta)=(1,s_i)} t_i^{share}$. Thus, the mechanism is budget balanced (or weakly budget balanced) if $p^0 = -p^1$ (or $p^0 > -p^1$).

Let $T^{share} = \sum_{i \in N, \pi_i(\theta)=(1,s_i)} t_i^{share}$. Since there is no detour cost, the cost saved is the costs of riders, which is at least $T^{share} c^{min}$, i.e. $C^{saved} \geq T^{share} c^{min}$. Therefore, the deficit is $(-p^1 - p^0) T^{share} \leq (-p^1 - p^0) \frac{C^{saved}}{c^{min}}$. □

Proposition 4 reveals that, if the prices satisfy, say $\frac{-p^1-p^0}{c^{min}} \leq \frac{1}{k}$, then we can guarantee that the cost saved by the system is at least k times the amount invested in the system. For instance, if $k = 2$ and we invest \$100 in the system, the community will receive at least \$300 of benefit, where \$200 comes from the cost saved for riders and \$100 comes from the payment the system distributed. That is, the serial dictatorship mechanism can guarantee a promising return on investment (ROI), if the market owner, say a local government is willing to invest some money in the system.[6]

4.2 VCG with two-sided reserve prices

We have seen that fixed prices can control the deficit but can lead to large inefficiencies. We now try and combine the benefits of fixed prices and VCG by introducing reserve prices. Reserve prices are commonly used in single-sided auctions [9, 3, 1]. Here, we introduce two-sided reserve prices, i.e. different reserves for drivers and riders, in order to control the deficit and the efficiency of the mechanism. The mechanism proceeds as follows.

VCG with two-sided reserve prices $\mathcal{M}^{VCG}(r^0, r^1)$

Initialise $D = N$ and $R = N$ to be the set of possible drivers and riders respectively. Then, given commuters' report profile θ and predefined values $r^0 \geq 0$, $r^1 \leq 0$, where r^0 is a reserve price for the rider, and r^1 for the driver, do the following:

1. For each commuter $i \in N$: if $c_i > -r^1$, remove i from the set of drivers D; if $c_i < r^0$, remove i from the set of riders R. Note that it is possible for a commuter to be removed from both sets.
2. Apply efficient allocation π^{eff} subject to drivers D and riders R, denoted by $\pi^{eff}(\theta, D, R)$.
3. For each rider i, the payment is $\max(r^0, x_i^{Clarke}(\theta, D, R))$, where $x_i^{Clarke}(\theta, D, R)$ is the Clarke pivot payment when the allocation is $\pi^{eff}(\theta, D, R)$.
4. For each driver j, the payment is $\max(\delta_j r^1, x_j^{Clarke}(\theta, D, R))$.

THEOREM 5. *$\mathcal{M}^{VCG}(r^0, r^1)$ is truthful and individually rational if and only if $r^0 \geq -r^1$. If $\mathcal{M}^{VCG}(r^0, r^1)$ is not truthful, the manipulation gain for each commuter i is bounded by $\max(-r^1 - r^0, \delta_i^{max}(-r^1 - r^0))$, where δ_i^{max} is the maximum detour for i.*

PROOF. We first show that, if $r^0 \geq -r^1$, $\mathcal{M}^{VCG}(r^0, r^1)$ is truthful. Commuter i is not incentivized to change D and R, because all the disabled allocations would have given her non-positive utilities. It is also easy to check that any commuter i can be either in D or R, but not both, as c_i will satisfy either $c_i > -r^1$ or $c_i < r^0$, given that $r^0 \geq -r^1$. If i is allocated as a rider, we show that she cannot misreport to gain a better utility. The payment for rider i is $\max(r^0, x_i^{Clarke}(\theta, D, R))$. If $r^0 \leq x_i^{Clarke}(\theta, D, R)$, then Clarke pivot payment guarantees i's utility is maximized. Otherwise, i's payment is fixed by r^0 and she cannot change it by misreporting given that i can only be allocated as a rider or unmatched. Thus, rider i's utility is maximized. Similarly, we can show that the utility for each driver is also maximized. For unmatched commuters, they cannot misreport to gain anything because VCG is truthful.

Next, we show that, if $\mathcal{M}^{VCG}(r^0, r^1)$ is truthful, then $r^0 \geq -r^1$. By contradiction, assume $\mathcal{M}^{VCG}(r^0, r^1)$ is truthful but $r^0 < -r^1$. It is easy to see that there are situations where a commuter will be in both D and R when $r^0 \leq c_i \leq -r^1$. Given that, we can always find the following example where such a commuter is incentivized to misreport. Consider the example in Figure 3 with two commuters travelling on two different routes in a triangle route map with costs $c_1 < c_2$. They each have one extra seat and they are both able to take a detour to take each other with the same detour parameter δ. Given the setting in Figure 3, the efficient allocation is that commuter 1 drives with 2 as a rider, and the payment for 1 is $\max(\delta r^1, -c_2) = \delta r^1$. However, if 1 misreports $\hat{c}_1 > c_2$, she will be a rider and pay $\max(r^0, \delta c_2) = \delta c_2$. It is evident that if $c_1 - \delta c_2 > -\delta c_1 - \delta r^1$, 1 is incentivized to misreport. It is easy to check that, for any $r^0 < -r^1$, we can always find an example such that $c_1 - \delta c_2 > -\delta c_1 - \delta r^1$, which contradicts the truthfulness assumption.

As we show in the above, $\mathcal{M}^{VCG}(r^0, r^1)$ is not truthful when $r^0 < -r^1$, because commuters with cost $r^0 \leq c_i \leq -r^1$ might misreport to switch between rider and driver to gain better utility. However, the possible gain by misreporting is limited by $\max_{c_i} |(c_i - r^0) - \delta_i^{max}(-r^1 - c_i)|$, which is $\max(-r^1 - r^0, \delta_i^{max}(-r^1 - r^0))$. □

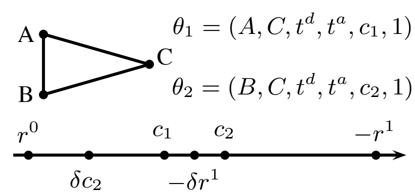

Figure 3: Example for the proof of Theorem 5

THEOREM 6. *$\mathcal{M}^{VCG}(r^0, r^1)$ is weakly budget balanced if there is no detour. Otherwise, the deficit is bounded by $-n_d \delta^{max} r^1 - n_r r^0$, where n_d and n_r are respectively the number of drivers and riders allocated by $\mathcal{M}^{VCG}(r^0, r^1)$, and δ^{max} is the maximum detour.*

PROOF. It is evident that the mechanism is weakly budget balanced when there is no detour, because the payment for drivers are zero. When considering detour in the allocation, since δ_i is bounded by δ^{max}, the payment that each driver receives is no more than $-\delta^{max} r^1$ and each rider pays r^0. □

We can remove the deficit entirely by restricting allocations such that each rider is riding with at most one driver,[7] since we then get $n_d \leq n_r$. Therefore, the deficit bound $-n_d \delta^{max} r^1 - n_r r^0 \leq 0$ for $\delta^{max} \leq 1$ and $-r^1 \leq r^0$, i.e.

[6]We note that, under the same conditions, VCG will still generate a potential deficit as large as the cost saved. This situation happens, for example, when there are no competitors for both drivers and riders, and each driver receives all the cost saved for her riders and the riders pay nothing.

[7]Limiting one rider to one driver is reasonable in real-life applications and also more reliable as there are always unpredictable delays which will make switching drivers infeasible. It also reduces the computation required to find the allocation. Moreover, the mechanism remains truthful since it is maximal in its range [13].

the mechanism is weakly budget balanced. Finally, we can further reduce the deficit by limiting δ^{max}, which can be directly controlled through the allocation computation (where setting $\delta^{max} = 0$ corresponds to no detour).[8]

We now briefly discuss how we can control the efficiency.

PROPOSITION 5. *By fixing r^0 or r^1, $\mathcal{M}^{VCG}(r^0, r^1)$ becomes more efficient as $r^0 + r^1$ decreases.*

PROOF. Assume r^0 is fixed. Then, $r^0 + r^1$ decreases iff r^1 is decreasing, i.e. $-r^1$ is increasing. When $-r^1$ increases, fewer commuters will satisfy $c_i > -r^1$. That is, fewer commuters will be removed from D, providing a larger set of possible allocations to optimise over. The argument is analogous for r^1. $\square$

Clearly, by reducing r^0 and r^1, the efficiency of $\mathcal{M}^{VCG}(r^0, r^1)$ approaches the VCG mechanism, but the deficit is also growing. We leave finding a clear relationship between efficiency and deficit in these mechanisms for future work.

Finally, we note that, similar to the fixed price mechanism, $\mathcal{M}^{VCG}(r^0, r^1)$ can be extended to allow reserve functions which depend (to some extent) on the schedule. In particular, we can define reserve prices to be dependent on journey length:

$$r^0(s_i) = r^0 \cdot w_{min}(l_i^d, l_i^a), \text{ and } r^1(s_i) = r^1 \cdot w_{min}(l_i^d, l_i^a).$$

It is easy to see that Theorem 5 still holds for this extension.

5. CONCLUSION

We have proposed a novel market-based model for ridesharing, which requires each commuter to report their travel constraints and preferences, and uses this information to allocate drivers and riders in order to reduce overall travel costs and so improve social welfare. Specifically, we have focused on mechanisms that incentivize commuters to participate in the system and to be truthful about their information. At the same time, we require that the system has deficit guarantees, so that it is economically sustainable. We showed that it is impossible to minimize cost (i.e., maximize social welfare) and provide incentives for commuters without generating a large deficit for the system owner. Therefore, we proposed less efficient alternatives, but which have deficit control, based on fixed prices and two-sided reserve prices, without sacrificing commuters' incentives. We analysed how the deficit can be flexibly controlled with various price settings, and proved bounds on the achieved deficit.

This is the first time that such an extensive ridesharing model is proposed and analysed using mechanism design, and there are many directions for future work. First, we have focused on deficit bounds, but it would be interesting to analyse bounds on the efficiency, and the trade-off between efficiency and deficit. This can be done theoretically and through simulations based on a realistic setting. Second, we have only briefly touched on the computational issues in this paper. The problem of finding optimal schedules is computationally hard. As we have seen, we can easily bound the possible allocations, such as allowing each rider to be matched to at most one driver, or limiting the drivers' detour. These limitations simplify the problem, while maintaining truthfulness. It would be interesting to consider other mechanisms that are computationally tractable, and see how this affects efficiency as well as the deficit. Finally, we have assumed that the system receives information from all commuters before making allocation decisions. Another approach is to use online mechanism design, and allow commuters to submit requests dynamically over time.

[8] It is worth mentioning that, even without detour, the deficit of the VCG without reserves can still be as bad as in Proposition 3, since the example in the proof has no detour.

6. ACKNOWLEDGMENTS

This work was partially supported by JSPS KAKENHI Grant (24220003) and JST PRESTO program.

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