An Efficient Algorithm for Taxi System Optimization (Extended Abstract)

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ABSTRACT

Taxi service is an important mode of modern public transportation. However, operated by a large number of self-controlled and profit-driven taxi drivers, taxi systems are quite in efficient and difficult to analyze and regulate. While there has been some work on designing algorithms for improving taxi system efficiency, the state of the art algorithm, unfortunately, cannot scale up efficiently. To address the inadequacy, we propose a novel algorithm—FLORA—in this paper. Using convex polytope representation conversion techniques, FLORA provides a *fully* compact representation of taxi drivers' strategy space, and avoids enumerating any type of schedules. Experimental results show orders of magnitude improvement of FLORA in terms of the complexity.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Intelligent agents, Multiagent systems

General Terms

Algorithms, Performance

Keywords

Taxi System, Game Theory, Optimization

1. INTRODUCTION

Taxi service has long been an indispensable part of public transportation in modern cities due to its high flexibility, great comfortableness, and easy accessibility. However, operated by a large number (e.g., around 66,000 in Beijing [2]) of self-controlled and profit-driven taxis, taxi systems are quiet inefficient and difficult to analyze and regulate. Besides, taxi systems can be affected by many factors ranging from road condition, customer demand, to fare price setting, which not only depend on each other in a very complex way, but also vary with time. How to analyze, regulate and optimize the taxi system are thus important but challenging problems, which have attracted many research interests over the past decades [4, 1, 3]. Among these problems, the taxi system optimization problem aims at improving efficiency of taxi systems though adjusting the taxi fare. Related work on this topic found in the transportation science community (e.g., [7, 8, 6]) mostly focused on modeling the interdependencies among factors in the system but ignored taxi drivers' behaviors which contribute the most to the characteristics of taxi systems. Specifically, profit-driven taxi drivers compete with each other for their maximum profits. These competing behaviors determines how taxi drivers react to fare structure changes and cannot be ignored in the analysis. In order to include the taxi drivers' behaviors, Gan et al. recently proposed a game theoretical approach to model a time-varying taxi market [5]. The key to such an approach is to address the scalability issue encountered in computing the taxi drivers' best strategy. A compact representation of the taxi drivers' strategy space, called *Atom Schedule Method* (ASM), was proposed to address this issue. Unfortunately, ASM is still inefficient in dealing with large-scale problems.

In our work, a more efficient algorithm—FLORA—is proposed to address the scalability issue. FLORA provides a novel compact representation of the taxi drivers' pure strategy space by utilizing convex polytope representation conversion techniques. Experiment were conducted to evaluate FLORA. The results show that FLORA can produce orders of magnitude improvement over existing algorithm in both time and space complexities.

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2. TAXI SYSTEM OPTIMIZATION

Derived from a multi-period model based on existing transportation research (e.g., [9, 10, 5]), the taxi system optimization problem is defined by the following *bilevel* program.

$$\max_{\boldsymbol{f}} \quad E\left(\boldsymbol{f}, \boldsymbol{p}(\boldsymbol{x}^*)\right) \tag{1}$$

$$\text{s.t.} \quad \boldsymbol{x}^* \in \arg\max_{\boldsymbol{x}} U\left(\boldsymbol{f}, \boldsymbol{p}(\boldsymbol{x})\right) \tag{2}$$

The term bilevel refers to the two levels of optimization programs. In the first level program (Eq. (1)), we maximize the efficiency E of the taxi system through adjusting the fare price $\boldsymbol{f}$. E is a function of $\boldsymbol{f}$ and the percentage $\boldsymbol{p}$ of working taxis, and $\boldsymbol{p}$ is furthermore a function of taxi drivers' strategy (we assume that all the taxis are identical, so that the taxi divers choose the same strategy). Note that in order to study a time-varying taxi system, the model divides the optimization horizon (e.g., a whole day) equally into a set of n periods, and treat the system in each period as a uniform system. Thus, $\boldsymbol{f}$ and $\boldsymbol{p}$ are vectors $\boldsymbol{f} = (f^1, \ldots, f^n)$ and $\boldsymbol{p} = (p^1, \ldots, p^n)$ with f^i and p^i corresponding to the i^{th} period. The second level program (Eq. (2)) indicates that taxi drivers choose the best strategy with respect to their utility U (which also depends on $\boldsymbol{f}$ and $\boldsymbol{p}$). A strategy $\boldsymbol{x}$ is a mixed strategy, i.e., a distribution over a set of pure strategies. Each pure strategy is a working schedule denoted by a vector $\boldsymbol{s} \in \{0,1\}^n$, where $s_i = 1$ (resp. $s_i = 0$) represents working (resp. not working) in the i^{th} period. We require that a schedule satisfy the following constraints.

C1: total working time should be at most n_w periods.

C2: continuous working time should be at most n_c periods.

Let the set of n-period schedules satisfying **C1** and **C2** be $\mathcal{S}$. It follows that $\boldsymbol{p}$ can be calculated as $\boldsymbol{p}(\boldsymbol{x}) = \sum_{\boldsymbol{s}\in\mathcal{S}} x_{\boldsymbol{s}}\boldsymbol{s}$, where $\boldsymbol{x} = \langle x_{\boldsymbol{s}} \rangle$, and $x_{\boldsymbol{s}}$ is the probability of schedule $\boldsymbol{s}$.

To resolve the bilevel program, we discretize the fare price space into a set $\mathcal{F}$ of candidate prices. For each price $\boldsymbol{f} \in \mathcal{F}$, we first solve the second level program to calculate $\boldsymbol{x}^*$, and then take it to the function of E to calculate the efficiency under price $\boldsymbol{f}$. In such a way, the price with maximum efficiency is the optimal price for the problem. Therefore, the key is to solve the second level program, where there is a scalability issue as the number of variables (i.e., $|\mathcal{S}|$) is exponentially large in terms of the number of periods.

To address this issue, we propose an algorithm called FLORA (FuLly cOmpact RepresentAtion), which is in contrast with the *partially* compact representation of ASM. FLORA reformulates the bilevel program as

$$\max_{\boldsymbol{f}} \quad E(\boldsymbol{f}, \boldsymbol{p}^*) \tag{3}$$

$$\text{s.t.} \quad \boldsymbol{p}^* \in \arg\max_{\boldsymbol{p}\in\mathbf{P}} U(\boldsymbol{f}, \boldsymbol{p}) \tag{4}$$

The idea is to compute the $\boldsymbol{p}^* = \boldsymbol{p}(\boldsymbol{x}^*)$ directly without computing the best strategy $\boldsymbol{x}^*$ first. To guarantee that the $\boldsymbol{p}^*$ obtained from the second level program can be implemented by schedules satisfying **C1** and **C2**, we utilize polytope representation conversion techniques (indeed, the feasible set of $\boldsymbol{p}$ is a convex polytope of the set $\mathcal{S}$ of n-dimensional points), and define the feasible set $\mathbf{P}$ by the following inequality set.

$$\left[\begin{array}{ccccc} & & I_n & & \\ \hdashline & & -I_n & & \\ \hdashline 1 & & \cdots & & 1 \\ \hdashline 1 & \cdots & 1 & & \\ & \ddots & & \ddots & \\ & & \underbrace{1 \quad \cdots \quad 1}_{n_c+1} & & \end{array}\right] \boldsymbol{p} \leq \left[\begin{array}{c} \mathbf{1} \\ \hdashline \mathbf{0} \\ \hdashline n_w \\ \hdashline n_c \\ \vdots \\ n_c \end{array}\right],$$

where I_n denotes an $n\times n$ identity matrix. In such a way, the FLORA formulation is equivalent to the original formulation in Eqs. (1) and (2). The proof of the equivalence is omitted due to page limit. The new formulation has only n variables and less than $3n$ constraints, and can be easily solved.

3. EVALUATION AND CHALLENGES

Experiments were conducted to compare the scalability of FLORA with ASM. Figure 1 depicts the runtime and memory use for solving the second level program. The results show that FLORA produces orders of magnitude improvement in both time and space complexities in comparison to ASM. FLORA is able to solve optimization problem with 100 periods very efficiently and still has the potential to handle even larger problems.

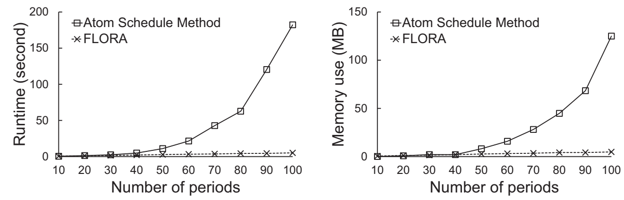

Figure 1: Runtime and memory use scalability.

A limitation of both ASM and FLORA is that they can only solve problems with constraints **C1** and **C2**, and cannot deal with situations where other types of practical scheduling constraints (e.g., minimum continuous working/resting time constraint or constraints resulted from market regulations) exist. Algorithm capable of handling additional constraints are needed to calculate more accurate optimal fares in these situations.

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