Social Network Analysis for Judgment Aggregation

(Extended Abstract)

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ABSTRACT

The majority of work in judgment aggregation is devoted to the study of impossibility results. However the (social) dependencies that may exist between the voters has received less attention. In this extended abstract we use the degree centrality measure from social network analysis and obtain a correspondence between the *average voter rule* and this measure, and show that approach can lead to more resolute outcomes in the voting process.

Categories and Subject Descriptors

I.2 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems, Cooperation and coordination*

Keywords

Artificial social systems, Social and organisational structure, Collective decision making

1. JUDGMENT AGGREGATION

The problem of judgment aggregation investigates how to aggregate individual judgments on logically related propositions to a group judgment on these propositions [4]. Many judgment aggregation rules have been proposed in the literature, however all share the concern with the general problem of selecting outputs that are *consistent* with a set of constraints and *compatible* with individual judgments [3].

In this work, we use the recently introduced framework of binary aggregation with integrity constraints [2], where a judgment aggregation problem consists of a group of n agents $\mathcal{N}$ which have to jointly decide for which m issues $\mathcal{I}$ to choose "yes" and for which to choose "no". A ballot $B \in \{0,1\}^m$ associates either 0 ("no") or 1 ("yes") to each issue in $\mathcal{I}$. In general, not every possible ballot might be a *feasible* or *rational* due to a set of *integrity constraints* IC on the issues in the form of logical formulas. A *profile* is a vector of rational ballots $\mathbf{B} = (B_1, \ldots, B_n) \in Mod(IC)^n$, containing one ballot for each agent. A *voting rule* is a function that maps each profile $\mathbf{B}$ to a set of ballots. One of the most well-known voting rules is the (weak) *majority rule*, which accepts an issue if a weak majority of the agents accept it:

$$Maj(\mathbf{B})_j = 1 \text{ iff } |\{i \in \mathcal{N} \mid \mathbf{B}_{ij} = 1\}| \geq \left\lceil \frac{n}{2} \right\rceil.$$

Grandi and Pigozzi propose in [3] the *average voter rule (AVR)*, which selects the voter with the shortest Hamming distance to the voting profile:

$$AVR(\mathbf{B}) = \underset{\{B_i | i \in \mathcal{N}\}}{\operatorname{argmin}} \sum_{i' \in \mathcal{N}} H(B_i, B_{i'}).$$

EXAMPLE 1. *Suppose a judgment aggregation scenario consisting of six agents (a, b, c, d, e, f) voting on an agenda composed of four issues (p, q, r, z). The agenda is subject to the following integrity constraint: $IC = (p \wedge q \wedge r) \Leftrightarrow z$.*

Issue:	***p***	***q***	***r***	***z***
a	*0*	*1*	*1*	*0*
b	*1*	*0*	*0*	*0*
c	*1*	*1*	*1*	*1*
d	*1*	*0*	*0*	*0*
e	*1*	*0*	*1*	*0*
f	*0*	*0*	*1*	*0*
Maj	*1*	*0*	*1*	*0*
MRV	*1*	*0*	*1*	*0*

2. SOCIAL NETWORK ANALYSIS

Social network analysis (SNA) studies social relationships in terms of graph theory, consisting of vertices (representing individual actors) and edges (which represent relationships between the individuals). The centrality of vertices, identifying which vertices are more "central" than others, has been a key issue in network analysis. Freeman [1] originally formalised three different measures of vertex centrality: degree, closeness, and betweenness. We restrict our analysis to the degree centrality, which measures the local involvement of the vertex in the network by counting the number of vertices it is connected to. We consider a recent proposal [5] that uses a tuning parameter α to control the relative importance of number of edges compared to the weights on the edges. The degree centrality for a vertex i is computed as follows:

$$C_D^{W\alpha}(i) = k_i \times \left(\frac{s_i}{k_i}\right)^{\alpha} = k_i^{(1-\alpha)} \times s_i^{\alpha} \quad (1)$$

where W is the weight matrix of graph, α is a positive tuning parameter, k_i is the size of the neighbourhood of vertex i and s_i the sum of the weights of the incident edges. If α is between 0 and 1, then the size of the neighbourhood is

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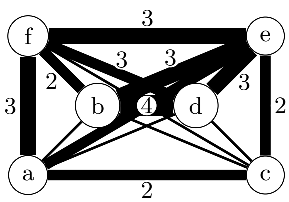
prioritised, whereas if it is set above 1, then the sum of the weights is.

Considering again Example 1, we can build a voter-to-voter matrix V by counting identical votes between voters (Figure 1a). V_{ij} denotes the number of times that both voters i and j voted "yes" or they both voted "no" for the same issue.

The voter-to-voter matrix V can be represented as a graph by letting the voters be the nodes and the value in each cell be the weight of the edge connecting the two nodes constituting the coordinate of the cell (Figure 1b). We can see in the voter-to-voter graph that the strongest connection is between agents b and d, representing the fact that their ballots are equivalent. Differently, agent c can be considered an outlier due to its weak connections with the other agents.

	a	b	c	d	e	f
a	4	1	2	1	2	3
b	1	4	1	4	3	2
c	2	1	4	1	2	1
d	1	4	1	4	3	2
e	2	3	2	3	4	3
f	3	2	1	2	3	4

(a) Voter-to-voter (V)



(b) Voter graph (G_V)

Figure 1: Relations among voters

3. ANALYSIS

There is an equivalence between the Hamming distance between two voters and the weight of the edge that connects the two voters in the corresponding voting graph, namely that the Hamming distance between two ballots B_i and B_j is equal to $m - V_{ij}$ in the corresponding voter graph V, i.e. $H(B_i, B_j) = m - V_{ij}$.

From this observation we have that the Hamming distance between a ballot B_i and a profile $\mathbf{B}$ is equal to $mn - s_i$, where s_i is the sum of the weights of the incident edges of vertex i in the voter graph constructed from $\mathbf{B}$, shortly: $\sum_{j\in\mathcal{N}} H(B_i, B_j) = mn - s_i$.

Since the average voter rule selects the voter that minimises the distance to the profile, it follows that AVR selects the voters corresponding to the maximum total weight vertices in the voter graph, i.e.: $AVR(\mathbf{B}) = \text{argmax}_{i\in\mathcal{N}}\, s_i$. Thus, the average voter rule corresponds to the node with the highest degree centrality when the tuning parameter $\alpha = 1$:

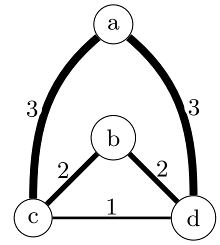
THEOREM 1. *The average voter rule selects those individual ballots that have the maximal degree centrality value when* $\alpha = 1$.

$$AVR(\boldsymbol{B}) = \underset{i\in\mathcal{N}}{\text{argmax}}\, C_D^{W\alpha}(i), \quad \textit{when } \alpha = 1.$$

Now that we have obtained a correspondence for $\alpha = 1$, we continue by varying this parameter. Consider the voting scenario in Figure 2a and the corresponding voter graph in Figure 2b. The outcome of the degree centrality for varying α are depicted in Figure 3. As we showed in Theorem 1, for $\alpha = 1$ the degree centrality measure corresponds to the s_i measure, so node a, c and d are all chosen as the most representative voter. When $\alpha < 1$ the amount of connections play are larger role and only c and d are chosen as the winner,

	1	2	3	4	5
a	0	1	1	1	1
b	1	0	0	0	0
c	0	1	1	0	0
d	0	0	0	1	1
Maj:	0	1	1	1	1
AVR:	0	1	1	1	1
	0	1	1	0	0
	0	0	0	1	1

(a) Profile



(b) Voter graph

Figure 2: Judgment aggregation example

while for $\alpha > 1$ the weight of the edges play are larger role and a is picked as the winner. This example shows that by exploiting the topology of the graph, we are able to obtain a more fine-grained outcome than the average voter rule.

Vertex	s_i	$C_D^{W\alpha}$ when $\alpha =$			
		0	0.5	1	1.5
a	**4**	2	2.5	**4**	**6.3**
b	2	2	2	2	2
c	**4**	**3**	**2.9**	**4**	5.5
d	**4**	**3**	**2.9**	**4**	5.5

Figure 3: Degree centrality scores when different values of α are used.

We plan to further investigate how the tuning parameter α influences the resoluteness of the results and to identify the properties of the new voting rules. Additionally, we deem interesting to investigate whether the issue-to-issue matrix, which represents correlation between issues instead of voters, can be used to identify additional voting rules. Finally, we are planning to analyse the other two centrality measures, closeness and betweenness, using empirical analysis on large networks, which is a typical SNA scenario.

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