Routing Games with an Unknown Set of Active Players

Itai Ashlagi Dov Monderer Moshe Tennenholtz
Technion–Israel Institute of Technology
Faculty of Industrial Engineering and Management
Haifa 32000, Israel
ashlagii@tx.technion.ac.il
{dov,moshet}@ie.technion.ac.il
Full version in: http://ie.technion.ac.il/Moshet.phtml

ABSTRACT

In many settings there exists a set of potential participants, but the set of participants who are actually active in the system, and in particular their number, is unknown. This topic has been first analyzed by Ashlagi, Monderer, and Tennenholtz [AMT] in the context of simple routing games, where the network consists of a set of parallel links, and the agents can not split their jobs among different paths. AMT used the model of pre-Bayesian games, and the concept of safety-level equilibrium for the analysis of these games. In this paper we extend the work by AMT. We deal with splitable routing games, where each player can split his job among paths in a given network. In this context we generalize the analysis to all two-node networks, in which paths may intersect in unrestricted manner. We characterize the relationships between the number of potential participants and the number of active participants under which ignorance is beneficial to each of the active participants.

Categories and Subject Descriptors

K.4 [**Social and Behavioral Sciences**]: Economics; I.2 [**Artificial Intelligence**]: Distributed Artificial Intelligence —*Multiagent systems*

General Terms

Economics, Theory

Keywords

Pre-Bayesian games, Safety-level equilibrium, Routing games

1. BACKGROUND AND DESCRIPTION OF RESULTS

The study of congestion games [16, 12, 11] is central to game theory, computer science, and electronic commerce. Indeed, the study of congestion games has become a central ingredient in work connecting the above disciplines (see e.g. [14, 13, 9, 2, 8, 7, 3]). Most of the related studies assume complete information, and, to the best of our knowledge, all of them, except for [1], are assuming complete information about the set (and in particular the number) of participants in the system.[1] However, in many settings, although the set of registered/potential participants may be known, the actual set of active participants is unknown. Hence, incorporating uncertainty about the set of actual participants into congestion settings is a desirable task.

Routing games are defined by congestion networks, and as such they are a special of congestion games, which are defined by congestion forms.[2] In this paper we focus on symmetric congestion games, and therefore we focus on two-node congestion networks only.[3] In a two-node congestion network there is a two-terminal directed graph, and every edge in the graph is associated with a per-unit cost function. The per-unit cost of moving one unit of good through this edge is a function of the number of units that are moved through this edge. Distinct edges may have distinct cost functions. In this paper we assume that the edge cost functions are continuously differentiable, convex, and increasing. There are two types of games that can be associated with a congestion network and a finite set of users. In a routing game with divisible units, each unit of good is continuously divisible, and hence every user should decide about the proportion of her good to be moved through each route. In the other model, the goods are not divisible. In this paper we focus on routing games with divisible units. In a routing game with complete information every player knows the network structure, the cost functions, and the number of users. In a routing game with incomplete information discussed in this paper, every active player knows all of the above except for the number of active players; She does know the number of potential players.[4]

[1]Some recent works deal with incomplete information about other parameters in the Bayesian setting [4, 5].

[2] A congestion form is a particular structure consisting of a set of resources together with a class of subsets of this set. Every congestion form, and a finite set of users uniquely define a congestion game. However, distinct congestion forms can generate identical congestion games. A particular type of form is the one generated by a network. Games defined by congestion networks are called routing games.

[3]It is obvious, and also can be derived from [10] that every symmetric congestion game is a symmetric routing game, which can be derived from a two-node congestion network.

[4]In our model there is a finite number of agents. The initial

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Games with incomplete information with a commonly known prior probability over the set of states of nature are called Bayesian Games. When such a prior probability does not exist in a given game, and we want to stress its non-existence we call this game pre-Bayesian.[5] In previous work Ashlagi, Monderer and Tennenholtz [AMT] [1] suggested to model behavior in pre-Bayesian games by the concept of safety-level equilibrium. A safety-level equilibrium is a strategy profile in which each agent minimizes his worst case cost over all possible states of the environment, assuming the other agents stick to their prescribed strategies. In the above context the possible states of the environment correspond to the possible sets of active players. AMT concentrated on simple (parallel) routing games with nondivisible units. They showed that in the linear case the lack of information about the number of active participants is beneficial for the players. Their result was obtained under the assumption that the participants use a symmetric equilibrium in both, the complete and incomplete information games.

As we said the focus of this paper is on symmetric routing games with divisible units. Therefore, we deal with general two-node networks. Unlike [AMT] We do not assume symmetric behavior but rather prove that such behavior holds in equilibrium. We characterize the relationships between the number of potential participants and the number of active participants under which ignorance is beneficial to each of the active participants.

In order to study the value of ignorance one needs to define a corresponding index for the value of ignorance. Let $c(k)$ be the cost of each player in equilibrium in the complete information case when there are k players, and $c(k,n)$ be the cost of each player in a safety-level equilibrium in the related game with incomplete information when there are k active players and n potential players. We define the value of ignorance to be $\nu(k,n) = c(k) - c(k,n)$. If this value is non-negative, ignorance is beneficial for the players. In order for the above index to make sense the cost in equilibrium at each of the above settings should be uniquely defined. As we will show, this is indeed the case in the model discussed in this paper.

We show that for simple linear routing games with divisible units, for sufficiently large k, $\nu(k,n) \geq 0$ for $k < n < k(k+1) - 1$. That is, each of the k active players "enjoys" ignorance when the number of potential players, n, is in the above interval. Under the minor assumption that there are two linear edge-cost functions which have different additive coefficients, each agent strictly gains due to that uncertainty, i.e. $\nu(k,n) > 0$. In addition we show that $\nu(k,n) \leq 0$ for $n > k(k+1) - 1$, and $\nu(k,n) = 0$ for $n = k(k+1) - 1$.

Next we deal with general linear two-node networks. We show that $\nu(k,n) \geq 0$ for $k < n \leq 2k-1$. Moreover we show that $\nu(k,n-1) \leq \nu(k,n)$ for every $k < n \leq 2k-1$ and that the reverse inequalities hold for every $n > 2k-1$. In particular for a fixed k, $\nu(k,n)$ is minimized at $n = 2k-1$.

research of congestion games [16], as well as much of the recent research [15, 13] discuss congestion games with continuum of agents, which are called non-atomic congestion games.

[5]In the economics' literature the term " game with incomplete information" has been used as a synonym to "Bayesian Game". Only recently researchers have been worked on games without priors, and such games received several titles in distinct papers. In this paper we follow the terminology of [6], and we refer to such games as pre-Bayesian.

Our results have interesting implications in the context of protocol design in congestion settings with incomplete information. Consider an organizer who knows the number of participants at each given point, and wishes to maximize social surplus. That is, the organizer's goal is to minimize the agents' costs. In ranges in which the value of ignorance is positive (i.e. when the number of potential participants is not too large with respect to the number of active participants) the organizer should not reveal the number of actual participants. Analogously, in ranges in which the value of ignorance is negative the organizer should reveal the number of actual participants. Note that if the costs are paid to a revenue-maximizing organizer, the above policies should be reversed.

2. AN EXAMPLE

As this version is restricted to 3 pages, we have decided to use the space for presenting an example from the full paper.

In the following example we demonstrate the value of ignorance in a simple congestion network.

EXAMPLE 1. *Consider the congestion network $\mathcal{N}$ in Figure 2. Let $I = \{1,2,3\}$, i.e. there are 3 potential players. Let the real state be $K = \{1,2\}$. Hence, there are two active players. First we find the equilibrium in the routing game with complete information with 2 players. Assume the first player sends $y \geq 0$ on the upper edge and $1 - y \geq 0$ on the lower edge. Then the second player's objective is to minimize $x(x+y) + (1-x)(1-x+1-y+1)$, where x is the amount she will send on the upper edge. The solution to this is $x = \frac{2-y}{2}$. Since the induced edge flow profile in equilibrium is symmetric (see Theorem 1) it must be that $x = y$. Therefore $x = \frac{2}{3}$. The cost for each of the players in this case is $c(2) =$*

$$\frac{2}{3} * \frac{4}{3} + \frac{1}{3} * (\frac{2}{3} + 1) = \frac{13}{9}.$$

We next find the equilibrium in the routing game with complete information with 3 players. Assuming the total amount two players send in the upper edge is $y \geq 0$, then the third player's objective is to minimize $x(x+y)+(1-x)(1-x+2-y+1)$, where x is the amount she will send on the upper edge. The solution to this is $x = \frac{5-2y}{4}$. By the symmetry of the induced edge flow profile in equilibrium we obtain $x = \frac{5-4x}{4}$ and therefore $x = \frac{5}{8}$. If the state K is not known to the players then by playing the safety-level equilibrium each of the players in K will send $\frac{5}{8}$ in the upper edge an therefore their costs will be $c(2,3) =$

$$\frac{5}{8} * \frac{10}{8} + \frac{3}{8}(\frac{6}{8} + 1) = \frac{23}{16}.$$

Hence the value of ignorance is $\nu(2,3) = \frac{13}{9} - \frac{23}{16} > 0$.

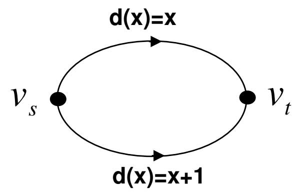

Figure 2.

3. SUMMARY

In many congestion settings there exists a set of potential users, but the set of users which are actually active in the system is unknown. Routing games naturally fall into these settings.

The value of ignorance is an index, which measures how much the users "enjoy" the ignorance about the actual set of players. AMT [1] analyzed the value of ignorance in routing games in which players cannot split their goods and the underlying network is simple - all paths are parallel. In this paper we significantly extend their work. We analyze the value of ignorance in all symmetric routing games, in which players can split their goods. We deal both with simple and two-node networks. We show that in both types of networks ignorance is helpful if the number of potential participants is not too big with respect to the number of active participants. Our results in simple networks differ from the results obtained in AMT. We show how the value of ignorance changes as a function of the number of potential players. As in AMT we use the concept of pre-Bayesian games, and the concept of safety-level equilibrium for the analysis of the value of ignorance.

The number of active players in a routing game is just one example for data that may not be available to the players. Other natural examples are network structure, edge cost functions, and job sizes. The study of the Pre-Bayesian games associated with lack of information about such data is an interesting direction for future research.

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