Flexibly Priced Options: A New Mechanism for Sequential Auction Markets with Complementary Goods

(Extended Abstract)

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ABSTRACT

In this work, we propose a novel option pricing mechanism for reducing the exposure problem encountered by bidders with complementary valuations when participating in sequential, second-price auction markets. In our mechanism, both the option and the exercise price are determined dynamically, by the bids received in each auction. We show that our fl xible options model can achieve better market allocation efficien y, at an only marginal cost to seller revenues compared to existing state of the art option pricing models.

Categories and Subject Descriptors

I.2.11 [Distributed Artificial Intelligence]: Multiagent Systems

General Terms

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Keywords

auction theory, game theory, bidding strategies

1. INTRODUCTION

Sequential auctions form an important part of many problem settings in which agents could be realistically applied, and such settings cannot usually be mapped to one-shot, combinatorial auction mechanisms. Often, the goods to be allocated are offered by different sellers, sometimes in different markets, or in auctions with different closing times. Examples include inter-related items sold on eBay by different sellers in auctions with sequential closing times [1], decentralised transportation logistics, or dynamic electricity markets, with micro-embedded generation. In such settings, a bidder is often faced with complementary preferences, meaning bundles of goods have a higher value than the sum of their parts due to synergies between individual goods. In sequential auctions, this can result in the *exposure problem*. This occurs whenever an agent is faced with placing a bid for an item which is higher than its immediate marginal value, in the expectation of obtaining extra value through a synergy with another item sold later. However, if she fails to get the other item for a profitabl price, she risks making a loss. In this paper, we refer to such a bidder as a *synergy bidder*.

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To address this, we consider a mechanism which involves auctioning *options* for the goods, instead of the goods themselves. An option is a contract between the buyer and the seller of a good, where (1) the writer or seller of the option has the *obligation* to sell the good for the *exercise price*, but not the right, and (2) the buyer of the option has the *right* to buy the good for a pre-agreed *exercise price*, but not the obligation. Since the buyer gains the right to choose in the future whether or not she wants to buy the good, she pays for this right through an *option price* which she has to pay in advance, regardless of whether she chooses to exercise the option.

Existing literature on using options to reduce exposure in sequential auctions proposes two main types of option pricing mechanisms. One approach [1] is to have free options (i.e., an option price of zero), and then let the exercise price be determined by the market (i.e., the submitted bids). However, this approach enables self-interested agents to hoard those options, even if they are unlikely to exercising them, thus potentially reducing market allocative efficien y and seller revenues. Another approach [2] is to have a fi ed exercise price, set by the seller, and then have the market determine the option price. However, the exercise price can be perceived as a reserve price since no bidder with a valuation below that price has an incentive to participate. This negatively effects the market efficien y, by excluding some bidders from the market.

To address these problems, we introduce a novel option pricing mechanism, in which both the option and the exercise price are determined dynamically, by the bids placed in each auction. In the full paper we derive the optimal bidding strategy for a synergy bidder, both in this new model and in a previously proposed, fi ed exercise price model. Given the optimal strategies, we show that our fl xible options model can achieve better market allocation efficien y, at an only marginal cost to seller revenues.

2. FIXED VS. FLEXIBLE EXERCISE PRICES

We consider a setting with m second-price, sealed-bid auctions, each selling an option to buy a single item. In such auctions, bidders without synergies have a simple, dominant bidding strategy and, furthermore, they are strategically equivalent to the widely-used English auction. We assume that there exists a single *synergy* bidder who is interested in purchasing all of the items and receives a value of v if it succeeds, and 0 otherwise. Furthermore, every auction $j \in \{1, \ldots, m\}$ has N_j *local* bidders. These bidders only participate in their own auction, and are only interested in acquiring a single item. The values of this item for local bidders in auction j are i.i.d. drawn from a cumulative distribution function F_j. Bidders are assumed to be expected utility maximizers.

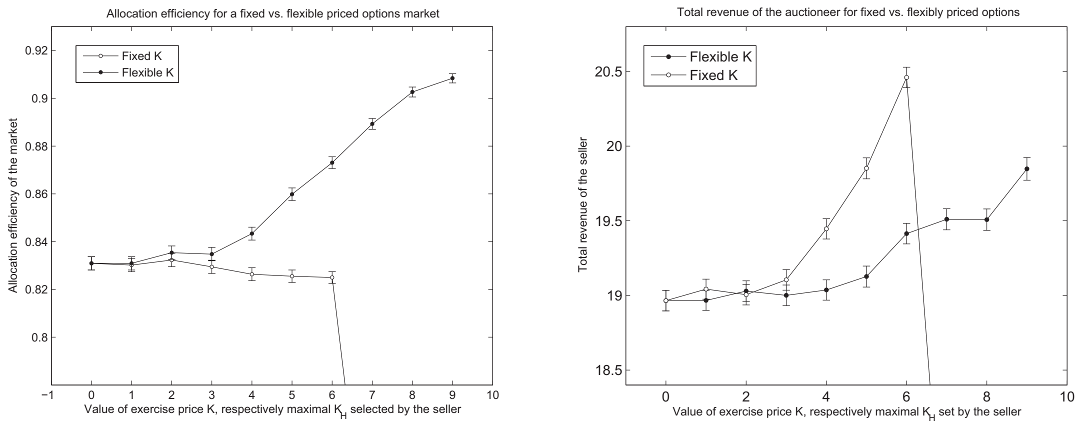

Figure 1: Allocative efficiency (left) and seller revenue (right) using the options mechanism with fixed vs. flexible exercise price options.

In a fi ed exercise price option model (such as the one proposed in [2]), the different exercise prices $\overrightarrow{K} = \{K_1, ..K_m\}$ for the auctions in the sequence are fi ed in advance, while the option prices are determined by the second-highest bid in the auction (if $K_j = 0$, this is equivalent to a *direct sale* auction, i.e., without any options).

In this model, however, the existence of the exercise prices created a secondary effect similar to having a reserve price in the auction. This is because any bidder with a private valuation lower than K_j will not participate in the auction and the same will happen if the synergy bidder has a valuation lower than the sum of the exercise prices. Therefore, although this reduces the exposure problem of the synergy bidder, at the same time setting this value too high may significantl reduce the market efficien y, and negatively affect seller revenues. In order to remove this effect, we introduce a novel model in which the exercise price of the options is also determined dynamically. Here, each seller sets a *maximum exercise price* K_j^H for each item $j = 1..m$, but the actual exercise price K_j depends on the bids placed by the bidders in the auction.

Definition 1. Flexible Exercise Price Options Mechanism Let each seller in a sequence of m second-price auctions select a parameter K_j^H, which is the maximum exercise price she is willing to offer for the item sold in auction j. Let b_j^{2nd} denote the second highest bid placed in this auction by the participating bidders. Then, the actual exercise price of the auction is given by:

$$K_j = \min\{K_j^H, b_j^{2nd}\}$$

The option price paid by the winning bidder is set to:

$$p_j = b_j^{2nd} - K_j$$

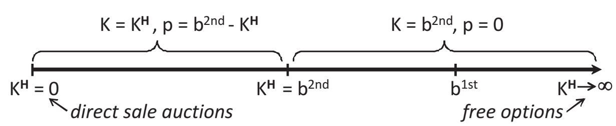

Figure 2: Relationship between the maximum exercise price, K^H, the second-highest bid, b^{2nd}, and their effect in determining the option price, p, and actual exercise price, K.

Figure 2 illustrates the features of this mechanism and how it compares to some existing approaches. As shown, depending on the values of K_j^H and b^{2nd}, one of two situations can occur. Either $K_j^H < b^{2nd}$, in which case the actual exercise price is set to K_j^H, and the winner pays $b^{2nd} - K_j^H$. Otherwise, if $K_j^H \geq b^{2nd}$, then the actual exercise price is set to the second highest bid and the option is given to the winning bidder for free. In both cases, however, the total payment of the winner (if she decides to exercise the purchased option) will be equal the second highest bid. Crucially, unlike the option mechanism with fi ed exercise price, from a local bidder's perspective, this auction is identical to a regular second-price auction, and there are no secondary effects on these bidders. Therefore, this options model only affects bidders with synergies.

Fig. 2 shows how this approach is a generalization of two other mechanisms. If the seller sets $K_j^H = 0$, the auction becomes identical to a direct auction (without options). If K_j^H is set at a high value (i.e. $K_j^H \to \infty$), then the exercise price is always equal to the second highest bid, and thus the option is always free.

3. RESULTS AND DISCUSSION

We derived and implemented the decision-theoretic optimal bidding strategies for a synergy bidder for both settings, when participating in markets where other bidders are local (i.e. require exactly one item). Figure 1 illustrates simulation results for a market with 3 auctions, each selling one item. Each auction involves 5 local bidders, with valuations randomly drawn from $N(2, 4)$ and a synergy bidder requiring all 3 items, with a valuation $N(20, 2)$ (the results above show the standard error for 2500 runs per setting). Note that direct sale appears as a particular case in both option models, when $K = K_H = 0$. As Figure 1 shows, the efficien y of the market can be improved considerably by using a model with fl xible prices. Here, the allocation efficien y is define as the ratio between the valuations of the agents that actually get the items and the valuation of agents in the optimal allocation. However, note that the sellers of the 3 items can achieve slightly higher total revenues using fi ed price options, for suitably chosen values of K. This is because fi ed exercise prices have a similar effect to setting a reservation price, which can increase the revenues of the auctioneer, although it decreases market efficien y.

In future work, we aim to study a more general option pricing model, in which the seller can specify both a minimal and a maximal exercise price for her option, thus combining the advantages of the fl xible option model proposed in this paper with the benefit of having a reservation value.

4. REFERENCES

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