Time Constraints in Mixed Multi-unit Combinatorial Auctions

(Extended Abstract)

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ABSTRACT

We extend the framework of mixed multi-unit combinatorial auctions, which deals with transformations of goods rather than only with atomic goods, by allowing time constraints in the bids offering these transformations. This way, bidders can express their scheduling preferences, while previously the auctioneer alone could decide the order of transformations.

Categories and Subject Descriptors

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General Terms

Economics, Algorithms

Keywords

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1. INTRODUCTION

Cerquides et al. [1] have proposed an extension of the standard combinatorial auction model, called **mixed multi-unit combinatorial auctions** (or simply **mixed auctions**). In a mixed auction, bidders can offer transformations, consisting of a set of input goods and a set of output goods, rather than just plain goods. Bidding for such a transformation means declaring that one is willing to deliver the specified output goods after having received the input goods, for the price specified by the bid. Solving a mixed auction means choosing a sequence of transformations that satisfies the constraints encoded by the bids, that produces the goods required by the auctioneer from those he holds initially, and that maximizes the amount of money collected from the bidders (or minimizes the amount paid out by the auctioneer). Mixed auctions extend several other types of combinatorial auctions: direct auctions, reverse auctions, and combinatorial exchanges. A promising application is supply chain formation.

We propose extending the framework of mixed auctions by allowing bidders to specify constraints regarding the times at which they perform the transformations offered in their bids. The motivation for this extension is that, in a complex economy, the bidders (service providers) themselves may need services from others and have their own supply chains, so the bidders may have preferences over the timing of transformations and over their relative ordering. A notion of time is already implicit in the original framework as far as the auctioneer is concerned, who builds a sequence of transformations, but this is not the case for the bidders.

Our contribution covers four types of **time constraints**:

- *Relative time points:* associate each transformation with a time point and allow bidders to express constraints regarding their relative ordering, e.g., transformation X must be executed before Y.
- *Absolute time points:* additionally allow references to absolute time, e.g., execute X at time 15, or at most 3 time units after Y.
- *Intervals:* associate transformations with intervals and specify constraints, e.g., X must be executed *during* Y.
- *Intervals with absolute durations:* allow intervals with absolute time, e.g., X should take at least 5 time units.

These constraint types can be freely mixed to, for instance, express an interval taking place after a time point.

Furthermore, it is possible to model *soft constraints*, allowing bidders to offer discounts in return for satisfying certain time constraints, and to model the fact that an auctioneer may sometimes be able to quantify the monetary benefit resulting from a shorter supply chain.

For a full exposition, see [3, Ch. 6].

2. CORE BIDDING LANGUAGE

Let G be the finite set of all types of goods considered. A **transformation** is a pair $(\mathcal{I}, \mathcal{O}) \in \mathbb{N}^G \times \mathbb{N}^G$. An agent offering such a transformation declares that, when provided with the multiset of goods $\mathcal{I}$, he can deliver the multiset of goods $\mathcal{O}$. Let $\mathcal{T}$ be a finite (but big enough) set of **time point identifiers**. These time points are to be thought of merely as identifiers, not as variables having an actual value. Agents negotiate over sets of transformations with time point identifiers $\mathcal{D} \subset \mathbb{N}^G \times \mathbb{N}^G \times \mathcal{T}$, which we can write in the form

$$\mathcal{D} = \{(\mathcal{I}^1, \mathcal{O}^1, \tau^1), \dots, (\mathcal{I}^\ell, \mathcal{O}^\ell, \tau^\ell)\}.$$

For example, $\{(\{\}, \{q\}, \tau_1), (\{r\}, \{s\}, \tau_2)\}$ means that the agent in question is able to deliver q without any input at some time point τ_1, and to deliver s if provided with r at some time point τ_2.

A **time line** Σ (for a given bidder) is a finite sequence of transformations and "clock ticks" c (when no transformation

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is allocated to the bidder). That is, $\Sigma \in (\mathbb{N}^G \times \mathbb{N}^G \cup \{c\})^*$. A **valuation** v maps a time line Σ to a real number p. Intuitively, $v(\Sigma) = p$ means that an agent with valuation v is willing to make a payment of p for getting the task of performing transformations according to the time line Σ (p is usually negative, so the agent is *being paid*). We write $v(\Sigma) = \bot$ if v is *undefined* for Σ, i.e., the agent would be unable to accept the corresponding deal. For example, the valuation v given by

$$\begin{aligned} v((\{oven, dough\}, \{oven, cake\})) &= -2 \\ v((\{oven, dough\}, \{oven, cake\}); (\{\}, \{bread\})) &= -3 \\ v((\{\}, \{bread\}); (\{oven, dough\}, \{oven, cake\})) &= \bot \end{aligned}$$

expresses that for two dollars I could produce a cake if given an oven and dough, also returning the oven; for another dollar I could do the same and afterwards give you a bread without any input; but I could not do it the other way round.

A valuation v uses **relative time** if for all Σ we have that $v(\Sigma) = v(\Sigma - c)$, where $\Sigma - c$ stands for Σ with all clock ticks c removed. Otherwise v is said to use **absolute time**. That is, in the context of relative time, valuations depend only on the relative ordering of the transformations.

An **atomic bid** $\textsc{bid}(\mathcal{D}, p)$ specifies a finite set of finite transformations with time points and a price. For **complex bids**, we restrict ourselves to the XOR-*language*, which, for multi-unit mixed auctions, fully expresses most (if not all) intuitively sensible valuations [1]. Our framework can easily be extended to also handle the OR-operator. An XOR-bid,

$$Bid = \textsc{bid}(\mathcal{D}_1, p_1)\ \textsc{xor} \dots \textsc{xor}\ \textsc{bid}(\mathcal{D}_n, p_n),$$

states that the bidder is willing to perform *at most one* of the $\mathcal{D}_j$ and pay the associated p_j.

The **atomic constraints** for *relative time* are of the form $\tau < \tau'$; and for *absolute time*, with $\tau, \tau' \in \mathcal{T}$, $\xi, \xi' \in \mathbb{N}$:

$$\tau = \xi \qquad \tau < \xi \qquad \tau > \xi$$
$$\tau + \xi < \tau' + \xi' \qquad \tau + \xi = \tau' + \xi'$$

As an example, the atomic bid with time constraint

$$\begin{aligned} &\textsc{bid}(\{\ (\{oven, dough\}, \{oven, cake\}, \tau_1), \\ &\quad (\{\}, \{bread\}, \tau_2)\}, -3) \end{aligned} \qquad \tau_1 < \tau_2$$

expresses the above fact that I am willing to sell you the bread only *after* I have sold you the cake.

Time **constraint formulas** are of the form $\varphi = \gamma_1 \wedge \dots \wedge \gamma_\nu$ with atomic constraints γ_ι. A bidder submits a bid *Bid* together with a time constraint formula φ, expressing that he is willing to perform according to *Bid*, but *only under the condition* that φ is satisfied.

For the formal semantics of this bidding language, refer to [3, Ch. 6].

3. SYNTACTIC EXTENSIONS

The time constraint language may seem limited, allowing only conjunctions of atomic constraints. However, additional expressive power can be "borrowed" from the bidding language, as with the following three extensions.

While time constraints in the core bidding language are *hard*, **soft constraints** (associated with costs) can be expressed as well. For example, a bidder may want to bid on $(\mathcal{I}^1, \mathcal{O}^1)$ and $(\mathcal{I}^2, \mathcal{O}^2)$ for price p and offer a discount, i.e., raise his bid by δ, if he gets to do the first before the second:

$$\textsc{bid}(\{(\mathcal{I}^1, \mathcal{O}^1, \tau^1), (\mathcal{I}^2, \mathcal{O}^2, \tau^2)\}, p) \qquad (\tau^1 < \tau^2, \delta)$$

This expression can be translated:

$$\begin{aligned} &\textsc{bid}(\{(\mathcal{I}^1, \mathcal{O}^1, \vartheta^1), (\mathcal{I}^2, \mathcal{O}^2, \vartheta^2)\}, p) \\ &\quad \textsc{xor}\ \textsc{bid}(\{(\mathcal{I}^1, \mathcal{O}^1, \zeta^1), (\mathcal{I}^2, \mathcal{O}^2, \zeta^2)\}, p + \delta) \end{aligned} \qquad \zeta^1 < \zeta^2$$

Another bidder may want to use a **disjunctive constraint** and offer $(\mathcal{I}^1, \mathcal{O}^1)$, $(\mathcal{I}^2, \mathcal{O}^2)$ and $(\mathcal{I}^3, \mathcal{O}^3)$ for price p, where the third should take place after the first *or* the second, i.e.,

$$\begin{aligned} &\textsc{bid}(\{(\mathcal{I}^1, \mathcal{O}^1, \tau^1), (\mathcal{I}^2, \mathcal{O}^2, \tau^2), (\mathcal{I}^3, \mathcal{O}^3, \tau^3)\}, p) \\ &(\tau^1 < \tau^3) \vee (\tau^2 < \tau^3) \end{aligned}$$

This can be translated in a similar way as above.

Finally, we may want to use **intervals** rather than just time points to allow transformations to overlap or to have different durations. A transformation with start time and end time can be rewritten as follows:

$$(\mathcal{I}, \mathcal{O}, [\tau, \tau']) \quad \rightsquigarrow \quad \begin{aligned} &(\mathcal{I}, \emptyset, \tau), (\emptyset, \mathcal{O}, \tau') \\ &\tau < \tau' \end{aligned}$$

The usual interval relations can be rewritten as follows:

$$\begin{aligned} [\tau_1, \tau_1']\ \textsc{before}\ [\tau_2, \tau_2'] &\rightsquigarrow \tau_1' < \tau_2 \\ [\tau_1, \tau_1']\ \textsc{overlaps}\ [\tau_2, \tau_2'] &\rightsquigarrow \tau_1 < \tau_2 \wedge \tau_1' < \tau_2' \\ [\tau_1, \tau_1']\ \textsc{during}\ [\tau_2, \tau_2'] &\rightsquigarrow \tau_2 < \tau_1 \wedge \tau_1' < \tau_2 \end{aligned}$$

Absolute restrictions on the durations can also be used:

$$duration([\tau, \tau']) \circ \xi \rightsquigarrow \tau' \circ \tau + \xi, \qquad \circ \in \{<, >, =\}$$

In [3, Ch. 6] we give the full details of these translations, as well as a way to model the auctioneer's monetary benefit resulting from a shorter supply chain.

4. COMPUTATIONAL ASPECTS

Concerning **computational complexity**, as in the original model by Cerquides et al. [1], the winner determination problem for mixed auctions with time constraints is NP-complete. Our extension is modular in a way that facilitates transfer of analytical results. For example, Fionda and Greco [2] recently started charting the tractability frontier for a slightly simplified version of the original framework, using various criteria to restrict the class of allowed bids. Their results concerning the XOR-language still hold in our extended framework.

In [3, Ch. 6] we have formulated an **integer program** for solving a mixed auction with time constraints, building upon the original algorithm without time constraints. Like the theoretical framework, this extension is modular, facilitating integration with other extensions and algorithmic optimizations. An empirical evaluation is left for future work.

5. REFERENCES

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