

# Practical Argumentation Semantics for Socially Efficient Defeasible Consequence

Hiroyuki Kido and Katsumi Nitta  
Interdisciplinary Graduate School of Science and Engineering  
Tokyo Institute of Technology, Japan  
{kido, nitta}@ntt.dis.titech.ac.jp

## ABSTRACT

An abstract argumentation framework and the semantics, often called Dungean semantics, give a general framework for nonmonotonic logics. In the last fifteen years, a great number of papers in computational argumentation adopt Dungean semantics as a fundamental principle for evaluating various kinds of defeasible consequences. Recently, many papers address problems not only with theoretical reasoning, i.e., reasoning about what to believe, but also practical reasoning, i.e., reasoning about what to do. This paper proposes a practical argumentation semantics specific to practical argumentation. This is motivated by our hypothesis that consequences of such argumentation should satisfy Pareto optimality because the consequences strongly depend on desires, aims, or values an individual agent or a group of agents has. We define a practical argumentation framework and two kinds of extensions, preferred and grounded extensions, with respect to each group of agents. We show that evaluating Pareto optimality can be translated to evaluating preferred extensions of a particular practical argumentation framework. Furthermore, we show that our semantics is a natural extension of Dungean semantics in terms of considering more than one defeat relation. We give a generality order of four practical argumentation frameworks specified by taking into account Dungean semantics and Pareto optimality. We show that a member of preferred extensions of the most specific one is not just Pareto optimal, but also it is theoretically justified.

## Categories and Subject Descriptors

I.2.3 [Deduction and Theorem Proving]: Nonmonotonic reasoning and belief revision

## General Terms

Theory

## Keywords

Argumentation, Collective decision making, Reasoning, Logic-based approaches and methods

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## 1. INTRODUCTION

An abstract argumentation framework and the semantics, often called Dungean semantics, give a general framework for nonmonotonic logics [6]. In the last fifteen years, a great number of papers in computational argumentation adopt Dungean semantics as a fundamental principle for evaluating states of arguments. Dungean semantics is defined on an abstract argumentation framework, denoted by  $AF$ , consisting of a set of arguments and a defeat relation on the set of arguments. Its main feature is that nonmonotonic reasoning can be realized without any internal structures of arguments such as languages or inferences. Recently, many papers address problems not only with theoretical reasoning, i.e., reasoning about what to believe, but also practical reasoning, i.e., reasoning about what to do, and apply Dungean semantics to these problems described as instances of  $AF$  or their expansions.

This paper shows that there exists a different kind of semantics specific to practical argumentation. Practical argumentation is known as the form of argumentation which aims at answering the question: ‘What is to be done [11]?’ The declaration is motivated by our hypothesis that decisions by practical argumentation must satisfy Pareto optimality. Consequences of practical argumentation are decisions of a course of action that an agent or a group of agents takes, and the decisions strongly depend on desires, aims, or values that it has. In such argumentation, agents are certain to avoid Pareto improvable decisions because if it is not Pareto optimal, there exists another decision that makes some agents better off and no one worse off. From this standpoint, there is no basis for believing that Dungean semantics gives an adequate principle for evaluating practical argumentation because it does not explain a relationship to social efficiency. The same holds true for the modification of Dungean semantics defined on a value-based argumentation framework [3]. Furthermore, many argument-based approaches for practical reasoning do not provide a sufficient explanation for applying Dungean semantics. In our view, Dungean semantics is specialized in evaluating acceptance of propositions as true, but it is insufficient for evaluating acceptance of actions as desirable.

In this paper, we propose practical argumentation semantics specific to practical argumentation. Practical argumentation semantics is defined on a practical argumentation framework consisting of a set of arguments without any internal structures, a set of agents, and a function from the set of agents to the power set of a binary relation on the set of arguments. The function outputs a defeat relation

that an inputted agent has. On the framework, we define two kinds of extensions, preferred and grounded extensions, with respect to each group of agents. In order to show the correctness of our theory, we show that evaluating Pareto optimality can be translated to evaluating preferred extensions of a particular practical argumentation framework. Furthermore, we show that evaluating defeasible consequences with Dungean semantics can also be translated to evaluating extensions of a particular practical argumentation framework. We give a generality order of four practical argumentation frameworks specified by taking into account Dungean semantics and Pareto optimality. We show that a member of preferred extensions of the most specific one is not just Pareto optimal, but also it is theoretically justified.

This paper is organized as follows. Section 2 shows a motivational example for addressing practical argumentation semantics. Section 3 gives preliminaries. In Section 4, we propose practical argumentation semantics, and in section 5, we show properties of the semantics. Section 6 gives an order relation of practical argumentation frameworks and Section 7 shows illustrative examples. Section 8 shows related works and Section 9 describes conclusions and future works.

## 2. MOTIVATIONAL EXAMPLE

Let us consider simple deliberative argumentation by which agents  $i$  and  $j$  try to decide what to do about buying an apartment. Agent  $i$  has concerns about safeness and quietness, and she prefers getting a safe neighborhood, avoiding an unsafe neighborhood, getting a quiet place, and avoiding a noisy place, in this order. In contrast, agent  $j$  has concerns about access to transportation, sunlight and safeness, and he prefers getting good access to transportation, avoiding bad access to transportation, getting a place with sufficient sunlight, and getting a safe neighborhood, in this order. Consider the following arguments put forward by agents  $i$  and  $j$  at some point in argumentation.

- $A_i$  : We ought to buy apartment ‘ $a$ ’ because it is located in a safe area.
- $B_j$  : We ought to buy apartment ‘ $b$ ’ because it is quiet and it has sufficient sunlight.
- $C_j$  : We ought not to buy ‘ $a$ ’ because it has bad access to transportation.
- $D_i$  : We ought not to buy ‘ $b$ ’ because the public security is poor and the access to transportation is bad.

What is the consequence of the argumentation? In other words, what actions would be taken by rational agents. We think that rational agents are certain to decide to take socially efficient actions. Pareto optimality is a formal criterion for evaluating efficiency, and a solution is Pareto optimal if no agents can be made better off without making someone else worse off. Our idea here is that we evaluate efficiency of practical argumentation semantics, proposed in this paper, by checking whether the consequences defined by the semantics are Pareto optimal or not. However, it is difficult to evaluate the above argumentation in terms of Pareto optimality because it differs completely from the problem setting that Pareto optimality assumes. It assumes that each agent has his/her individual preferences on outcomes

and implicitly assumes that any two distinct outcomes are incompatible. Our detailed idea is that we reduce the original argumentation to restricted ones that can be handled in a problem of Pareto optimality, and conclude that our semantics is efficient based on the fact that the consequences of the restricted argumentation are identical to Pareto optimal solutions. For example, consider the situation that they evaluate the arguments based on his/her own preference on the arguments. If we consider the restricted argumentation consisting of arguments  $A$  and  $B$ , then both  $A$  and  $B$  are Pareto optimal because agent  $i$  prefers  $A$  to  $B$  and agent  $j$  prefers  $B$  to  $A$ . If we consider the restricted argumentation consisting of arguments  $B$  and  $D$ , then only  $D$  is Pareto optimal because both agents prefer  $D$  to  $B$ . Our practical argumentation semantics must define defeasible consequences that are consistent with the evaluation of Pareto optimality in each restricted argumentation.

We have to take into account arguments about not only what to do, but also, what to believe in practical argumentation. Consider the following arguments put forward by agent  $j$  at the end of argumentation.

- $E_j$  : ‘ $a$ ’ is not located in a safe area because a murder occurred and the murderer is still at large.
- $F_j$  : It takes five minutes from ‘ $b$ ’ to the closest station and the station has two train lines. Further, there is a police office near the station. Therefore, the public security and the access to transportation are not bad.

In this situation, what is the consequence of the argumentation, or what actions would be taken by rational agents?  $A$  and  $D$  fail to justify their own actions because they cannot defeat the defeating arguments  $E$  and  $F$ , respectively. Therefore, the effects of these arguments on the decision should be canceled. We benefit from Dungean semantics for evaluating this kind of arguments, and combine our practical argumentation semantics and Dungean semantics in order to handle not only practically efficient, but also theoretically justified arguments.

## 3. PRELIMINARIES

Let  $G$  be a set and  $R$  be a binary relation on  $G$ , i.e.,  $R \subseteq G \times G$ .  $R$  is called reflexive if  $(x, x) \in R$ , for all  $x \in G$ , transitive if whenever  $(x, y) \in R$  and  $(y, z) \in R$  then  $(x, z) \in R$ , for all  $x, y, z \in G$ , and antisymmetric if whenever  $(x, y) \in R$  and  $(y, x) \in R$  then  $x = y$ , for all  $x, y \in G$ .  $R$  is called quasi-order if it is reflexive and transitive, and partial order if it is reflexive, transitive, and antisymmetric. The inverse relation of  $R$ , denoted by  $R^{-1}$ , and the complement relation of  $R$ , denoted by  $\bar{R}$ , are defined as  $R^{-1} = \{(x, y) \mid (y, x) \in R\}$  and  $\bar{R} = \{(x, y) \mid (x, y) \notin R\}$ , respectively. The inverse complement relation of  $R$  is the complement relation of the inverse relation of  $R$ , i.e.,  $\overline{R^{-1}}$ .

Welfare economics is a branch of economics that is concerned with the evaluation of alternative economic situations (states, configurations) from the point of view of the society’s well being [10]. One of the prominent measures for evaluating society’s well being is Pareto optimality defined as follows.

*Definition 1.* An outcome  $o_1 \in \mathcal{O}$  is Pareto optimal (or Pareto efficient) if there is no other outcome  $o_2 \neq o_1$  such that  $\forall i \in I, o_2 \succeq_i o_1$  and  $\exists j \in I, o_2 \succ_j o_1$ .

In other words, a solution is Pareto optimal if no agents can be made better off without making someone else worse off.

The abstract argumentation framework [6] is one of the argument-based approaches for nonmonotonic reasoning. Its main feature is that nonmonotonicity arises from the interactions between conflicting arguments, not in the process of constructing arguments. The abstract argumentation framework is especially abstract because it takes no account of the internal structures of arguments and only takes account of the external structures between arguments, i.e., defeat relation. The framework allows us to define various semantical notions of argumentation extensions. These notions are intended to capture various types of nonmonotonic consequence. The basic formal notions, with some terminological changes, are as follows.

*Definition 2.* [6] The abstract argumentation framework is defined as a pair  $AF = \langle AR, defeat \rangle$  where  $AR$  is a set of arguments, and defeat is a binary relation on  $AR$ , i.e.  $defeat \subseteq AR \times AR$ .

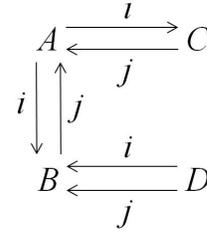
- A set  $S$  of arguments is said to be conflict-free if there are no arguments  $A, B$  in  $S$  such that  $A$  defeats  $B$ .
- An argument  $A \in AR$  is acceptable with respect to a set  $S$  of arguments iff for each argument  $B \in AR$ : if  $B$  defeats  $A$  then  $B$  is defeated by an argument in  $S$ .
- A conflict-free set of arguments  $S$  is admissible iff each argument in  $S$  is acceptable with respect to  $S$ .
- A preferred extension of an argumentation framework  $AF$  is a maximal (with respect to set inclusion) admissible set of  $AF$ .

For argumentation framework  $AF$ , an argument is justified with respect to  $AF$  if it is in every preferred extension of  $AF$ , and is defensible with respect to  $AF$  if it is in some but not all preferred extensions of  $AF$  [13].

## 4. PRACTICAL ARGUMENTATION SEMANTICS

Practical argumentation semantics is a general rule for defining notions of defeasible consequences of a practical argumentation. Practical argumentation is known as the form of argumentation which aims at answering the question: ‘What is to be done [11]?’ Practical argumentation as shown in Section 2 handles two different kinds of arguments. One is the argument concluding actions that a group of agents should do or should not do, and the other is the argument concluding truth of propositions. We call these two kinds of arguments practical and theoretical arguments, respectively. In this paper, we assume that a set  $Args$  of arguments is divided into a set  $Pargs$  of practical arguments and a set  $Targs$  of theoretical arguments where  $Args = Pargs \cup Targs$  and  $Pargs \cap Targs = \emptyset$  hold. The assumption is based on the observation that these two kinds of arguments should be formally distinguished not at the level of abstract arguments without any internal structures of arguments, but at the level of internal structures of arguments such as logical languages or inferences. We define a practical argumentation framework as follows.

*Definition 3.* A practical argumentation framework, denoted by  $PRAF$ , is a pair  $PRAF = \langle Args, Agents \rangle$ ,



**Figure 1: Arguments and subjective defeat relations**

*Defeat*  $\succ$ , where  $Args$  is a set of arguments,  $Agents$  is a set of agents, and *Defeat* is a function that maps  $Agents$  into  $2^{Args \times Args}$ .

*PRAF* characteristically has each agent  $i$ 's defeat relation defined by  $Defeat(i)$ . This reflects the fact that defeat relations between practical arguments are subjective because they strongly depend on preferences, desires, aims, values, morality, or ethics that an individual agent has. The individual agent's defeat relation might be substantiated by subjective preferences, values, and/or ethics, objective logical contradiction, or any combination thereof. *PRAF* abstracts any such internal information about arguments, and it consists of a minimal number of elements that practical argumentation semantics can be defined. In what follows, we say that  $x$  defeats  $y$  under  $i$  if there exist  $i \in Agents$  and  $(x, y) \in Defeat(i)$ .

*Example 1.* The following is the practical argumentation framework consisting of some arguments and defeat relations shown in Section 2.

$$\begin{aligned} PRAF &= \langle \{A, B, C, D\}, \{i, j\}, Defeat \rangle \\ Defeat(i) &= \{(A, B), (A, C), (D, B)\} \\ Defeat(j) &= \{(B, A), (C, A), (D, B)\} \end{aligned}$$

The arguments and the defeat relations can be shown in Figure 1. There exists an arrow from  $x$  to  $y$  with label  $i$  if  $x$  defeats  $y$  under  $i$ .

In what follows, we assume an arbitrary but fixed practical argumentation framework. Consequences of practical argumentation are decisions of a course of action that an agent or a group of agents takes. Therefore, the consequences must be consistent. One of the properties that a set of arguments has is conflict-freeness.

*Definition 4.* A set  $S \subseteq Args$  of arguments is conflict-free to a set  $N \subseteq Agents$  of agents if for all arguments  $A, B \in S$ ,  $A$  does not defeat  $B$  under any agent  $i \in N$ .

We define a notion of acceptability. The basic idea of acceptability is that a set  $N$  of rational agents would accept an argument  $A$  if each argument defeating  $A$  under some agent is defeated by some argument under an agent in  $N$ .

*Definition 5.* An argument  $A \in Args$  is acceptable to a set  $N \subseteq Agents$  of agents with respect to a set  $S \subseteq Args$  of arguments if each argument defeating  $A$  under an agent  $i \in Agents$  is defeated by an argument  $B \in S$  under an agent  $j \in N$ .

In contrast to acceptable arguments defined in Dungean semantics, our acceptable arguments differ from one set of agents to another. Note that acceptability does not require that each argument defeating  $A$  is defeated by an argument  $B \in S$  under all agents  $j \in N$ . The notion of admissibility is defined on the basis of conflict-freeness and acceptability.

*Definition 6.* A set  $S \subseteq \text{Args}$  of arguments is admissible to a set  $N \subseteq \text{Agents}$  of agents if  $S$  is conflict-free to  $N$  and each argument in  $S$  is acceptable to  $N$  with respect to  $S$ .

Self-admissibility is defined in this paper. Intuitively, every argument  $A$  in a self-admissible set can defeat every argument defeating  $A$  by  $A$  itself. In other words,  $A$  can defend itself without relying on any other arguments.

*Definition 7.* A set  $S \subseteq \text{Args}$  of arguments is self-admissible to a set  $N \subseteq \text{Agents}$  of agents if  $S$  is conflict-free to  $N$  and each argument  $A \in S$  is acceptable to  $N$  with respect to  $\{A\}$ .

We call an element of a self-admissible set a self-admissible argument. Note that it is not always true that a self-admissible set has only one element. The credulous or preferred semantics of a practical argumentation framework is defined by the notion of preferred extension.

*Definition 8.* A set  $S \subseteq \text{Args}$  of arguments is a preferred extension to a set  $N \subseteq \text{Agents}$  of agents if  $S$  is a maximal admissible set to  $N$ .

The credulous semantics provides defeasible consequences of a practical argumentation framework. Another defeasible consequence of a practical argumentation framework is provided by a skeptical or grounded semantics. The semantics is defined by using the following operator.

*Definition 9.* Let  $S \subseteq \text{Args}$  and  $N \subseteq \text{Agents}$ . Then the operator  $F^N$  for  $N$  is defined as follows.

- $F^N(S) = \{A \in \text{Args} \mid A \text{ is acceptable to } N \text{ with respect to } S\}$

*Definition 10.* A set of  $S \subseteq \text{Args}$  of arguments is a grounded extension to a set  $N \subseteq \text{Agents}$  of agents if  $S$  is the least fixed point of  $F^N$ .

*Example 2.* Both  $\{A, D\}$  and  $\{C, D\}$  are preferred extensions to  $\{i, j\}$ , and  $\{D\}$  is a grounded extension of  $\{i, j\}$  in Example 1.

## 5. PROPERTIES OF PRACTICAL ARGUMENTATION SEMANTICS

In this section, we aim to show the relationships between our practical argumentation semantics and both Pareto optimality and Dungean semantics. For Pareto optimality, we show that evaluating Pareto optimal solutions can be translated to evaluating preferred extensions of a particular practical argumentation framework. The following lemma shows the relationship between preferred extensions and self-admissible arguments.

*Lemma 1.* Let  $\text{PRAF} = \langle \text{Args}, \text{Agents}, \text{Defeat} \rangle$  be a practical argumentation framework where the complement of  $\text{Defeat}(i)$  is transitive, for all  $i \in \text{Agents}$ . An argument  $A \in \text{Args}$  is a member of some preferred extension to  $\text{Agents}$  iff  $A$  is self-admissible to  $\text{Agents}$ .

**PROOF.** ( $\Leftarrow$ ) From Definition 8, a preferred extension is a conflict-free admissible set. Thus, if  $\{A\}$  is admissible set to  $\text{Agents}$  then there exists a preferred extension  $S$  to  $\text{Agents}$  such that  $\{A\} \subseteq S$ . ( $\Rightarrow$ ) We show that the contradiction is derived under the assumptions that  $A$  is a member of some preferred extension  $S$  to  $\text{Agents}$  and  $A$  is not self-admissible to  $\text{Agents}$ . Under the assumptions, there exists an argument  $B \in \text{Args}$  defeating  $A$ , under an agent  $i \in \text{Agents}$ , that is not defeated by  $A$  under any agent  $j \in \text{Agents}$  and is defeated by a third argument  $C \in S$  under an agent  $k \in \text{Agents}$ . Formally, the following formulas hold for  $S$ .

$$\begin{aligned} & \exists B \in \text{Args} (\exists i \in \text{Agents} ((B, A) \in \text{Defeat}(i)) \\ & \wedge \forall j \in \text{Agents} ((A, B) \notin \text{Defeat}(j)) \\ & \wedge \exists k \in \text{Agents} \exists C \in S ((C, B) \in \text{Defeat}(k))) \\ \Rightarrow & \exists B \in \text{Args} \exists i \in \text{Agents} ((B, A) \in \text{Defeat}(i)) \\ & \wedge \exists j \in \text{Agents} \exists C \in S ((A, B) \notin \text{Defeat}(j) \\ & \wedge (C, B) \in \text{Defeat}(j)) \quad (1) \\ \Rightarrow & \exists B \in \text{Args} \exists i \in \text{Agents} ((B, A) \in \text{Defeat}(i)) \\ & \wedge \exists j \in \text{Agents} \exists C \in S ((C, A) \in \text{Defeat}(j)) \quad (2) \end{aligned}$$

(2) can be derived from (1) under the following assumption that the complement of  $\text{Defeat}(i)$  is transitive.

$$\begin{aligned} & \forall A, B, C \in \text{Args} \forall i \in \text{Agents} ((A, B) \notin \text{Defeat}(i) \\ & \wedge (C, A) \notin \text{Defeat}(i) \rightarrow (C, B) \notin \text{Defeat}(i)) \\ \Leftrightarrow & \forall A, B, C \in \text{Args} \forall i \in \text{Agents} ((A, B) \notin \text{Defeat}(i) \\ & \wedge (C, B) \in \text{Defeat}(i) \rightarrow (C, A) \in \text{Defeat}(i)) \end{aligned}$$

$A, C \in S$  and there exists  $j \in \text{Agents}$  such that  $(C, A) \in \text{Defeat}(j)$  in (2). This contradicts the assumption that  $S$  is conflict-free to  $\text{Agents}$ .  $\square$

In Lemma 1,  $\text{Defeat}(i)$  is assumed to be transitive. In Theorem 1,  $\text{Defeat}(i)$  is substituted by the inverse complement of  $i$ 's preference expressed as quasi-order. The transitivity in Lemma 1 is a minimal assumption that makes Lemma 1 hold. The following lemma shows the relationship between self-admissible arguments and Pareto optimal solutions.

*Lemma 2.* Let  $\mathcal{O}$  be a set of outcomes,  $\text{Agents}$  be a set of agents, and  $\succsim_i$  ( $i \in \text{Agents}$ ) be a quasi-order on  $\mathcal{O}$ . An outcome  $o \in \mathcal{O}$  is Pareto optimal with respect to each agent  $i$ 's preference  $\succsim_i$  iff  $o$  is self-admissible of  $\text{PRAF} = \langle \mathcal{O}, \text{Agents}, \text{Defeat} \rangle$  to  $\text{Agents}$  where  $\text{Defeat}(i) = \prec_i$ , for all  $i \in \text{Agents}$ .

**PROOF.**  $o \in \mathcal{O}$  is self-admissible to  $\text{Agents}$  iff the following formula holds.

$$\begin{aligned} & \nexists i \in \text{Agents} (o \prec_i o) \wedge \forall o_1 \in \mathcal{O} (\exists i \in \text{Agents} \\ & (o_1 \prec_i o) \rightarrow \exists j \in \text{Agents} (o \prec_j o_1)) \quad (3) \end{aligned}$$

(3) can be transformed to the following formulas based on

the assumption that  $\succsim_i$  is a quasi-order.

$$\begin{aligned}
& \forall o_1 \in \mathcal{O} (\exists i \in \text{Agents}(o_1 \succsim_i o) \rightarrow \exists j \in \text{Agent} \\
& \quad (o \succsim_j o_1)) \\
\Leftrightarrow & \forall o_1 \in \mathcal{O} (\exists i \in \text{Agents}(o_1 \succ_i o \vee o \succsim_i o_1 \wedge o_1 \succsim_i o) \\
& \quad \rightarrow \exists j \in \text{Agents}(o \succsim_j o_1)) \\
\Leftrightarrow & \forall o_1 \in \mathcal{O} (\exists i \in \text{Agents}(o_1 \succ_i o) \vee \exists k \in \text{Agents} \\
& \quad (o \succsim_k o_1 \wedge o_1 \succsim_k o) \rightarrow \exists j \in \text{Agents}(o \succsim_j o_1)) \\
\Leftrightarrow & \forall o_1 \in \mathcal{O} ((\exists i \in \text{Agents}(o_1 \succ_i o) \rightarrow \exists j \in \text{Agents} \\
& \quad (o \succsim_j o_1)) \wedge (\exists k \in \text{Agents}(o \succsim_k o_1 \wedge o_1 \succsim_k o) \rightarrow \\
& \quad \exists l \in \text{Agents}(o \succsim_l o_1))) \\
\Leftrightarrow & \forall o_1 \in \mathcal{O} (\exists i \in \text{Agents}(o_1 \succ_i o) \rightarrow \exists j \in \text{Agents} \\
& \quad (o \succsim_j o_1)) \\
\Leftrightarrow & \forall o_1 \in \mathcal{O} (\#i \in \text{Agents}(o_1 \succ_i o) \vee \exists j \in \text{Agents} \\
& \quad (o \succsim_j o_1)) \\
\Leftrightarrow & \#o_1 \in \mathcal{O} (\exists i \in \text{Agents}(o_1 \succ_i o) \wedge \forall j \in \text{Agents} \\
& \quad (o \succsim_j o_1)) \tag{4}
\end{aligned}$$

(4) is equivalent to the definition of Pareto optimality, and therefore,  $o$  is Pareto optimal.  $\square$

From Lemma 1 and Lemma 2, we can reach the following theorem.

*Theorem 1.* Let  $\mathcal{O}$  be a set of outcomes,  $\text{Agents}$  be a set of agents, and  $\succsim_i$  ( $i \in \text{Agents}$ ) be a quasi-order on  $\mathcal{O}$ . An outcome  $o \in \mathcal{O}$  is Pareto optimal with respect to each agent  $i$ 's preference  $\succsim_i$  iff  $o$  is a member of some preferred extension of  $\text{PRAF} = \langle \mathcal{O}, \text{Agents}, \text{Defeat} \rangle$  to  $\text{Agents}$  where  $\text{Defeat}(i) = \succsim_i$ , for all  $i \in \text{Agents}$ .

Theorem 1 shows that evaluating Pareto optimal solutions can be translated to evaluating preferred extensions of a particular practical argumentation framework. This fact provides a theoretical basis for concluding that the practical argumentation semantics credulously justifies Pareto optimal solutions. Note that due to the particularity of the practical argumentation framework, it is generally the case that evaluating preferred extensions cannot be translated to evaluating Pareto optimal solutions.

For Dungean semantics, a link exists between our practical argumentation semantics and Dungean semantics.

*Proposition 1.* Let  $AF = \langle \text{Args}, \text{defeat} \rangle$  be an abstract argumentation framework. The preferred extensions and the grounded extension of  $AF$  are equivalent to the preferred extensions and the grounded extension of  $\text{PRAF} = \langle \text{Args}, \text{Agents}, \text{Defeat} \rangle$  to  $\text{Agents}$  where  $\text{Agents} = \{i\}$  and  $\text{Defeat}(i) = \text{defeat}$ .

Proposition 1 shows that our practical argumentation semantics justifies defeasible consequences instead of Dungean semantics. Furthermore, it provides a theoretical basis for concluding that our practical argumentation semantics is a natural extension of Dungean semantics in terms of handling subjective defeat relations. Note that due to the particularity of the practical argumentation framework, it is generally the case that evaluating extensions of a practical argumentation framework cannot be translated to evaluating extensions of an abstract argumentation framework.

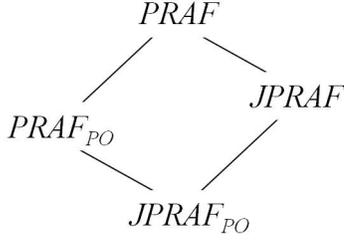
## 6. GENERALITY ORDER FOR PRACTICAL ARGUMENTATION FRAMEWORKS

This section gives a generality order of four practical argumentation frameworks specified by taking into account Dungean semantics and Pareto optimality. A practical argumentation framework and our practical argumentation semantics are insufficient to handle the practical argumentation shown in Section 2 because it takes no account of theoretical arguments that play a role of evaluating the truth of statements in practical arguments. Hence, we take into account theoretical arguments and the defeat relations that are unrelated to agents' subjective preferences, desires, values, morality, and ethics. A possible way to handle theoretical evaluation in practical argumentation is to unify our practical argumentation semantics and Dungean semantics into one semantics. However, it does not always work well. We sometimes take an attitude that reasoning about beliefs should be skeptical while reasoning about action should be credulous [12]. A unified semantics cannot evaluate these two types of reasoning in different ways, i.e., by preferred or grounded semantics. We take a different approach that stratifies a practical argumentation framework by taking into account an abstract argumentation framework evaluated by Dungean semantics. In addition, we further stratify the framework by considering Pareto optimality.

*Definition 11.* Let  $AF = \langle \text{Args}, \text{defeat} \rangle$  be an abstract argumentation framework where  $\text{Args} = \text{Targs} \cup \text{Pargs}$  and  $\text{defeat} \subseteq \text{Targs} \times \text{Args}$ , and  $\text{PRAF} = \langle S, \text{Agents}, \text{Defeat} \rangle$  be a practical argumentation framework where  $S \subseteq \text{Args}$ .

1.  $\text{PRAF}$  is a justified practical argumentation framework with respect to  $AF$ , denoted by  $\text{JPRAF}$ , if all arguments in  $S$  are members of the grounded extension of  $AF$ .
2.  $\text{PRAF}$  is a practical argumentation framework for Pareto optimality, denoted by  $\text{PRAF}_{PO}$ , if the complement of  $\text{Defeat}(i)$  is quasi-order, for all  $i \in \text{Agents}$ .
3.  $\text{PRAF}$  is a justified practical argumentation framework for Pareto optimality, denoted by  $\text{JPRAF}_{PO}$ , if  $\text{PRAF}$  is a justified practical argumentation framework with respect to  $AF$  and  $\text{PRAF}$  is a practical argumentation framework for Pareto optimality.

$AF$  does not allow practical arguments to defeat any arguments while it allows theoretical arguments to defeat theoretical and practical arguments. Figure 2 shows a generality order of practical argumentation frameworks in Definition 11. Top of the order is a general argumentation framework and bottom of the order is the most specialized practical argumentation framework, i.e.,  $\text{JPRAF}_{PO}$ . Note that it is generally the case that the intersection of the grounded extension of  $AF$  and the union of all preferred extensions of  $\text{PRAF}$  to a set of agents is not equal to the union of all preferred extensions of  $\text{JPRAF}$  to the set of agents. It means that we cannot obtain the same consequences with the preferred extensions of  $\text{JPRAF}$  to a set of agents by parallel evaluation of the grounded extension of  $AF$  and the preferred extensions of  $\text{PRAF}$  to the set of agents. From Theorem 1, an argument  $A \in \text{Args}$



**Figure 2: The generality order of practical argumentation frameworks**

is a member of some preferred extension of  $PRAF_{PO} = \langle Args, Agents, Defeat \rangle$  to  $Agents$  iff  $A$  is Pareto optimal with respect to each agent  $i$ 's preference defined by the inverse complement of  $Defeat(i)$ .  $JPRAF_{PO}$  is  $PRAF_{PO}$ . Therefore, it is noteworthy that a member of preferred extensions of  $JPRAF_{PO}$  is not just Pareto optimal but also it is theoretically justified with respect to  $AF$ .

## 7. ILLUSTRATIVE EXAMPLES

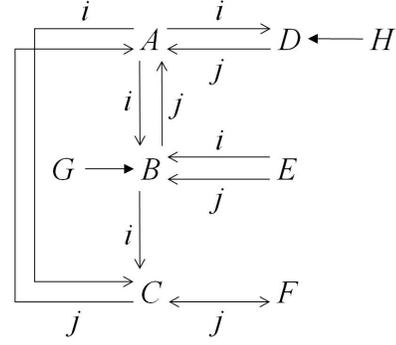
This section shows illustrative examples of specialized practical argumentation frameworks and consequences of the frameworks. We make the specialized frameworks by restricting a general practical argumentation framework. Restriction is defined as follows.

*Definition 12.* Let  $PRAF = \langle Args, Agents, Defeat \rangle$  be a practical argumentation framework. The restriction of  $PRAF$  to  $S \subseteq Args$  is the practical argumentation framework  $PRAF \downarrow_S = \langle S, Agents, Defeat' \rangle$  where  $Defeat'(i) = Defeat(i) \cap (S \times S)$  for all  $i \in Agents$ .

Consider the set  $Pargs = \{A, B, C, D, E, F\}$  of practical arguments and the set  $Targs = \{G, H\}$  of theoretical arguments. Each argument states that we ought to buy apartment 'a' because it is located in a safe area, denoted by an argument  $A$ , we ought to buy apartment 'b' because it is quiet and it has good access to transportation, by  $B$ , we ought to buy apartment 'c' because it has good access to transportation, by  $C$ , we ought not to buy 'a' because it is beyond the budget, by  $D$ , we ought not to buy 'b' because it is beyond the budget and located in a unsafe area, by  $E$ , we ought not to buy 'c' because it does not have sufficient sunlight, by  $F$ , 'b' is not located in a safe area because an airstrip is now under construction in that area, by  $G$ , and we can buy 'a' within the budget because the real estate gives us discount, by  $H$ . Furthermore, the objective defeat relation  $defeat = \{(G, B), (H, D)\}$  and the following subjective defeat relations are given.

$$\begin{aligned} Defeat(i) &= \{(A, B), (A, C), (A, D), (B, C), (E, B)\} \\ Defeat(j) &= \{(B, A), (C, A), (C, F), (D, A), (E, B), \\ &\quad (F, C)\} \end{aligned}$$

Figure 3 shows these arguments and the objective and subjective defeat relations where the filled arrows depict the objective defeat relations. Consider following abstract argumentation framework  $AF$  and practical argumentation



**Figure 3: The whole defeat relations between arguments**

framework  $PRAF$ .

$$\begin{aligned} AF &= \langle Targs \cup Pargs, defeat \rangle \\ PRAF &= \langle Pargs, Agents, Defeat \rangle \end{aligned}$$

The preferred extension, and the grounded extension as well, of  $AF$  is  $\{A, C, E, F, G, H\}$ . Moreover, the preferred extensions of  $PRAF$  to  $\{i, j\}$  are  $\{A, E, F\}$  and  $\{C, D, E\}$ , and the grounded extension of  $PRAF$  to  $\{i, j\}$  is  $\{E\}$ . The following is a justified practical argumentation framework with respect to  $AF$  obtained by restricting  $PRAF$  to  $\{A, C, E, F\}$ .

$$\begin{aligned} JPRAF &= \langle \{A, C, E, F\}, \{i, j\}, Defeat_{JPRAF} \rangle \\ Defeat_{JPRAF}(i) &= \{(A, C)\} \\ Defeat_{JPRAF}(j) &= \{(C, A), (C, F), (F, C)\} \end{aligned}$$

$\{C\}$  is the grounded extension of  $JPRAF$  to  $\{i, j\}$  and both  $\{A, E, F\}$  and  $\{C, E\}$  are the preferred extensions of  $JPRAF$  to  $\{i, j\}$ . Following  $PRAF_{PO}$  is a practical argumentation framework for Pareto optimality obtained by restricting  $PRAF$  to  $\{A, B, C\}$ .

$$\begin{aligned} PRAF_{PO} &= \langle \{A, B, C\}, \{i, j\}, Defeat_{PRAF_{PO}} \rangle \\ Defeat_{PRAF_{PO}}(i) &= \{(A, B), (A, C), (B, C)\} \\ Defeat_{PRAF_{PO}}(j) &= \{(B, A), (C, A)\} \end{aligned}$$

The grounded extension of  $PRAF_{PO}$  to  $\{i, j\}$  is the empty set and the preferred extensions of  $PRAF_{PO}$  to  $\{i, j\}$  are  $\{A\}$  and  $\{B\}$ . Therefore, both  $A$  and  $B$  are Pareto optimal arguments with respect to agents' preferences defined by the inverse complements of  $Defeat_{PRAF_{PO}}(x)$ , for  $x = i, j$ . Note that these inverse complements are quasi-order. Following  $JPRAF_{PO}$  is a justified practical argumentation framework with respect to  $AF$  for Pareto optimality obtained by restricting  $PRAF$  to  $\{A, C\}$ .

$$\begin{aligned} JPRAF_{PO} &= \langle \{A, C\}, \{i, j\}, Defeat_{JPRAF_{PO}} \rangle \\ Defeat_{JPRAF_{PO}}(i) &= \{(A, C)\} \\ Defeat_{JPRAF_{PO}}(j) &= \{(C, A)\} \end{aligned}$$

The grounded extension of  $JPRAF_{PO}$  to  $\{i, j\}$  is the empty set and the preferred extensions of  $JPRAF_{PO}$  to  $\{i, j\}$  are  $\{A\}$  and  $\{C\}$ . Therefore, both  $A$  and  $C$  are not just Pareto optimal but also they are theoretically justified with respect to  $AF$ .

## 8. RELATED WORK

Deliberation is a type of dialogue in which a group of agents or a single agent tries, through looking at a set of alternatives, to make a decision about which course of action among the possible alternatives to take [18]. Our practical argumentation semantics can be applied to the evaluation of argument-based deliberation. Many argument-based approaches for deliberation or practical reasoning, however, apply Dungian semantics as a fundamental principle for evaluating arguments. For instance, a decision of a single agent's course of action, who has more than one desire, is formalized by instances of an abstract argumentation framework [18, 12]. In [18], the authors propose two kinds of practical reasoning, positive and negative practical syllogisms, denoted by *PPS* and *NPS*. They are incorporated into arguments for drawing desirable and undesirable actions, respectively. Dungian semantics is used for evaluating arguments, and consequently decides what the best action is. In [12], the author gives a combined formalization for skeptical epistemic reasoning interleaved with credulous practical reasoning. He distinguishes practical arguments from theoretical arguments by informally dividing logical formulas into epistemic and practical ones. Epistemic and practical arguments are evaluated by skeptical semantics and credulous semantics defined by Dungian semantics, respectively. On the other hand, these approaches do not discuss the relationship to efficiency. We think that a decision of a course of action and the notion of efficiency are inseparable even when single agent's argumentation.

In [16], the authors introduce seven dialectical inference rules on dialectical logic DL and weaker dialectical logic DM [15] in order to realize concession or compromise from inconsistent theory. They apply the inferences into argument-based negotiation for reaching agreement. Similarly, in [9], the authors propose compromise reasoning on an abstract lattice, and illustrate that compromise arguments incorporating the reasoning realize compromise-based justification. Furthermore, in [1], the authors propose an abstract framework for argument-based negotiation, and introduce the notion of concession as an essential element of negotiation. We think that concessions and compromises should be chosen from Pareto optimal solutions. However, none of them discuss the relationship between Pareto optimality with the notions of concession and compromise.

Recently, in [14], the authors analyze Dungian semantics by means of Pareto optimality. Pareto optimal solutions are defined based on each agent's preferences on extensions of an abstract argumentation framework. However, it does not provide new argumentation semantics that is consistent with Pareto optimality. In [8], the authors introduce Pareto optimality into argument-based negotiation. The notion, however, is used in a process of negotiation, and it is not evaluated by argumentation semantics.

From the point of view of argumentation semantics, some authors introduce nonclassical semantics such as stage semantics [17], semi-stable semantics [4], ideal semantics [7], CF2 semantics [2], and prudent semantics [5] on Dung's abstract argumentation framework. All of them intend to overcome or improve some limitations or drawbacks of Dungian semantics. On the other hand, our practical argumentation semantics is defined on the different framework, i.e., practical argumentation framework consisting of minimal number of elements that our semantics can be defined. Further-

more, it specializes in evaluating practical argumentation, and it does not address the improvement of Dungian semantics. In order to evaluate practical argumentation involving agents' values, the author proposes value-based argumentation frameworks, denoted by *VAF*, and modifies Dungian semantics [3]. The modified semantics corresponds to applying Dungian semantics to each abstract argumentation framework constructed from an individual agent's defeat relation. The paper, however, does not explain the relationship between the modified semantics with another theory. We think that it is essential for establishing the correctness of the modified semantics.

## 9. CONCLUSIONS AND FUTURE WORK

We proposed a practical argumentation semantics specific to practical argumentation. This attempt was motivated by our hypothesis that extensions of practical argumentation are certain to be efficient in terms of Pareto optimality. We showed that an outcome is Pareto optimal iff the outcome is a member of some preferred extension of a particular practical argumentation framework. This fact established that our practical argumentation semantics is efficient in terms of Pareto optimality. We showed that our practical argumentation semantics is a natural extension of Dungian semantics in terms of handling more than one defeat relation. We defined four ordered practical argumentation frameworks and gave illustrative examples of these frameworks by restricting the most general one. We need to formalize dialectical proof theory for our semantics, i.e., procedures determining whether an argument is a member of some extension or not. In particular, we are interested in formalizing proof theory of *JPRAF* that need to be calculated based on two semantics, Dungian semantics and our semantics.

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