Extension of MC-net-based Coalition Structure Generation: Handling Negative Rules and Externalities

(Extended Abstract)

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ABSTRACT

Forming effective coalitions is a major research challenge in AI and multi-agent systems. A Coalition Structure Generation (CSG) problem involves partitioning a set of agents into coalitions so that the social surplus is maximized. Ohta *et al.* introduce an innovative direction for solving CSG, i.e., by representing a characteristic function as a set of rules, a CSG problem can be formalized as the problem of finding a subset of rules that maximizes the sum of rule values under certain constraints. This paper considers two significant extensions of the formalization/algorithm of Ohta *et al.*, i.e., (i) handling negative value rules and (ii) handling externalities among coalitions.

Categories and Subject Descriptors

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General Terms

Algorithms, Theory

Keywords

coalition structure generation, constraint optimization, cooperative games

1. INTRODUCTION

Coalition formation is an important capability in automated negotiation among self-interested agents. Coalition Structure Generation (CSG) involves partitioning a set of agents so that social surplus is maximized. This problem has become a popular research topic in AI and multi-agent systems. Possible applications of CSG include distributed vehicle routing [7], multi-sensor networks [3], etc. and various algorithms for solving CSG have been developed.

Almost all existing works on CSG assume that the characteristic function is represented implicitly and we have oracle access to the function, that is, the value of a coalition (or a coalition structure as a whole) can be obtained using a certain procedure. This is because representing an arbitrary characteristic function explicitly requires $\Theta(2^n)$ numbers, which is prohibitive for large n.

However, characteristic functions that appear in practice often display a significant structure, and such characteristic functions can be represented much more concisely. Indeed, recently, several new methods for representing characteristic functions have been developed [1, 2, 4]. These representation schemes capture characteristics of interactions among agents in a natural and concise manner, and they can reduce the representation size significantly.

Recently, Ohta *et al.* [6] introduce an innovative direction for solving CSG. They assume that a characteristic function is represented using three compact representation schemes. Consequently, they show that a CSG problem can be formalized as a problem of finding the subset of rules that maximizes the sum of rule values under certain constraints. They also develop mixed integer programming (MIP) formulations of the above optimization problem and show that an off-the-shelf optimization package could perform reasonably well.

This paper considers two significant extensions of the work of Ohta *et al.* [6] on CSG when a characteristic function is represented by marginal contribution nets (MC-nets) [4]. Our extensions are introducing (i) negative value rules and (ii) rules that represent externalities among coalitions. Ohta *et al.* [6] consider other compact representation schemes. In this work, we choose MC-nets because its representation is more compact and natural than other representation schemes.

2. MODEL

Let $A = \{1, 2, \ldots, n\}$ be the set of agents. We assume a characteristic function game, i.e., the value of a coalition S is given by a characteristic function $v : 2^A \to \mathbb{R}$.

CSG involves partitioning a set of agents into coalitions so that social surplus is maximized. A coalition structure CS is a partition of A, divided into disjoint, exhaustive coalitions. To be more precise, $CS = \{S_1, S_2, \ldots\}$ satisfies the following conditions: $\forall i, j (i \neq j), S_i \cap S_j = \emptyset, \bigcup_{S_i \in CS} S_i = A$. In other words, in CS, each agent belongs to exactly one coalition, and some agents may be alone in their coalition. We denote by $\Pi(A)$ the space of all coalition structures over A. The value of a coalition structure CS, denoted as $V(CS)$, is given by: $V(CS) = \sum_{S_i \in CS} v(S_i)$. An optimal coalition structure CS^* is a coalition structure that satisfies the following condition: $\forall CS \in \Pi(A), V(CS^*) \geq V(CS)$.

An *embedded coalition* is a pair (S, CS), where $S \in CS \in \Pi(A)$. We let M denote the set of all embedded coalitions, that is, $M := \{(S, CS) : CS \in \Pi(A), S \in CS\}$. A partition function is a mapping $w : M \to \mathbb{R}$.

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3. EXISTING WORKS

This section briefly describes the *marginal contribution networks (MC-nets)* proposed by Ieong and Shoham [4] and the formalization/algorithm of Ohta *et al.* [6] for CSG problems based on MC-nets. Furthermore, we describe an extension of MC-nets for partition function games, *embedded MC-nets* proposed by Michalak *et al.* [5].

Definition 1 (MC-nets)**.** *An* MC-net *consists of a set of* rules R. *Each rule $r \in R$ is of the form: $(P_r, N_r) \rightarrow v_r$, where $P_r \subseteq A$, $N_r \subseteq A$, $P_r \cap N_r = \emptyset$, $v_r \in \mathbb{R}$. We say that rule r is* applicable *to coalition S if $P_r \subseteq S$ and $N_r \cap S = \emptyset$, i.e., S contains all agents in P_r (positive literals), and it contains no agent in N_r (negative literals). For a coalition S, $v(S)$ is given as $\sum_{r \in R_S} v_r$, where R_S is the set of rules applicable to S. We assume each rule has at least one positive literal.*

Ohta *et al.* [6] present a MIP formulation that finds a *feasible* rule set that maximizes the sum of rule values. A rule set R' is feasible if there exists a coalition structure CS such that each rule $r \in R'$ is applicable to coalition $S \in CS$.

The limitation of the method presented by Ohta *et al.* [6] is that it cannot handle negative value rules and externalities among coalitions. Quite recently, Michalak *et al.* proposed a concise representation of a partition function called *embedded MC-nets*, which is an extension of MC-nets.

Definition 2 (Embedded MC-nets)**.** *An* embedded MC-net *consists of a set of embedded rules ER. Each embedded rule $er \in ER$ is of the form: $r_0 | r_1, \ldots, r_k \rightarrow v_{er}$, where r_0 is satisfied in the coalition that receives the value and $r_1, \ldots, r_k$ are satisfied in other coalitions. We say that an embedded rule er is* applicable *to coalition S in CS if r_0 is applicable to S and that each rule of $r_1, \ldots, r_k$ is applicable to some coalition $S' \in CS \setminus \{S\}$. For a coalition S, $w(S, CS)$ is given as $\sum_{er \in ER_{(S,CS)}} v_{er}$, where $ER_{(S,CS)}$ is the set of embedded rules applicable to S in CS.*

4. CSG USING MC-NETS WITH NEGATIVE VALUES AND EXTERNALITIES

In this section, we generalize the work of Ohta *et al.* [6] on CSG problems to handle negative value rules and externalities. We assume R is divided into two groups, i.e., a set of positive value rules R_+ and a set of negative value rules R_-.

Handling negative value rules is a challenging task. If we simply add negative value rules, the MIP formulation in Ohta *et al.* [6] cannot properly find an optimal coalition structure. In this paper, we develop a concise and efficient way to handle negative value rules, i.e., adding dummy rules as follows.

Definition 3 (Dummy rules)**.** *Assume there exists a negative value rule $r_- : (P_{r_-}, N_{r_-}) \rightarrow -c\ (c > 0)$, where $P_{r_-} = \{p_1, \ldots, p_k\}$, $N_{r_-} = \{n_1, \ldots, n_l\}$. Dummy rules generated by this negative value rule are following two types:*

(i) $(\{p_1\}, \{p_i\}) \rightarrow 0$, where $2 \le i \le k$,

(ii) $(\{p_1, n_j\}, \{\}) \rightarrow 0$, where $1 \le j \le l$.

We denote the set of all dummy rules as R_d.

We extend the concept of a *feasible rule set* by Ohta *et al.* to handle negative value rules.

Definition 4 (Properly feasible rule set)**.** *We say a set of rules $R' \subseteq R \cup R_d$ is properly feasible if there exists CS, where each rule $r \in R'$ is applicable to some $S \in CS$ and $\forall r_- \in R_- \setminus R'$, r_- is not applicable to any $S \in CS$.*

CSG using MC-nets with negative values can be modeled as finding a properly feasible rule set that maximizes the sum of the values. We develop a MIP formulation to solve this optimization problem.

Next, we introduce a method to find the optimal coalition structure when a partition function is represented as an embedded MC-net. We extend the definition of properly feasible rule set (Definition 4) for an MC-net with negative values to handle an embedded MC-nets. Then, CSG using embedded MC-nets can be modeled as finding a properly feasible embedded rule set that maximizes the sum of the values. We also develop a MIP formulation to solve this optimization problem.

Furthermore, We experimentally evaluate the performance of our proposed methods and confirmed that the overhead of our extensions is reasonably small and our approach is scalable, i.e., an off-the-shelf optimization package (CPLEX) can solve problem instances with 100 agents and 100 rules within 10 seconds.

In this paper, we considered the formalization/algorithm of Ohta *et al.* on CSG when a characteristic function is represented by MC-nets and extended it in two directions: (i) handling a negative value rule, and (ii) handling the embedded rule proposed by Michalak *et al.*, which represents positive/negative externalities among coalitions. These two extensions are essential for dealing with a wider range of application domains of CSG. For either extension, we proved that the problem is NP-hard and inapproximable and developed a MIP formulation. Experimental results showed that the overhead of our extensions is reasonably small and our approach is scalable.

5. REFERENCES

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