False-name bidding in first-price combinatorial auctions with incomplete information

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ABSTRACT

False-name bids are bids submitted by a single agent under multiple fictitious names such as multiple e-mail addresses. False-name bidding can be a serious fraud in Internet auctions since identifying each participant is virtually impossible. It is shown that even the theoretically well-founded Vickrey-Clarke-Groves auction (VCG) is vulnerable to false-name bidding. Thus, several auction mechanisms that cannot be manipulated by false-name bids, i.e., *false-name-proof* mechanisms, have been developed.

This paper investigates a slightly different question, i.e., how do they affect (perfect) Bayesian Nash equilibria of first-price combinatorial auctions? The importance of this question is that first-price combinatorial auctions are by far widely used in practice than VCG, and can be used as a benchmark for evaluating alternate mechanisms. In an environment where false-name bidding are possible, analytically investigating bidders' behaviors is very complicated, since nobody knows the number of real bidders. As a first step, we consider a kind of minimal settings where false-name bids become effective, i.e., an auction with two goods where one naive bidder competes with one shill bidder who may pretend to be two distinct bidders. We model this auction as a simple dynamic game and examine approximate Bayesian Nash equilibria by utilizing a numerical technique. Our analysis revealed that false-name bidding significantly affects the first-price auctions. Furthermore, the shill bidder has a clear advantage against the naive bidder.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems*; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Algorithms, Economics, Theory

Keywords

Auction theory, mechanism design, first-price auctions, and false-name bidding

Cite as: False-name bidding in first-price combinatorial auctions with incomplete information, Atsushi Iwasaki, Atsushi Katsuragi and Makoto Yokoo, *Proc. of 10th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2011)*, Tumer, Yolum, Sonenberg and Stone (eds.), May, 2–6, 2011, Taipei, Taiwan, pp. 541-548.

1. INTRODUCTION

In a *combinatorial auction*, also called *package auction*, multiple goods are simultaneously for sale, and, in general, bidders can express arbitrary valuation functions over subsets of the goods. This allows bidders to express substitutability and complementarity of the goods in their valuations. A recent book by Cramton *et al.* [3] gives a thorough survey of the theory and practice of combinatorial auctions. False-name bids [16] are bids submitted by a single agent under multiple fictitious names such as multiple e-mail addresses. False-name bidding can be a serious fraud in combinatorial auctions on the Internet, since identifying each participant is virtually impossible.

The Vickrey-Clarke-Groves (VCG) auction is best motivated by its dominant strategy property under incomplete information, that is, truth-telling by all bidders in the auction leads outcomes (allocation of goods) to be efficient. However, VCG has several limitations in environments with complementarities among goods. One is vulnerability to false-name bidding. As mentioned above, since such dishonest actions are very difficult to detect, they can cause even more serious problems in auctions on the Internet. Several auction mechanisms that cannot be manipulated by false-name bids (i.e., *false-name-proof* mechanisms) have been developed [15, 7, 6]. We say a mechanism is *false-name-proof* if, for each bidder, declaring his true valuation function using a single identifier is a dominant strategy, even though the bidder can choose to use multiple identifiers.

In this paper, we investigate a slightly different question. We know false-name manipulations can affect a dominant-strategy equilibrium of strategy-proof mechanisms, i.e., VCG is not false-name-proof. How do they affect (perfect) Bayesian Nash equilibria of other non-direct-revelation mechanisms, in particular, the first-price combinatorial auction mechanism? The importance of such analysis is that first-price combinatorial auctions are by far widely used in practice than VCG, and the obtained results can be used as a benchmark for evaluating other auctions/mechanisms. In first-price auctions, bidders simply submit sealed bids, they are allocated the goods so that the combination of bids maximizes the seller's revenue, and each winning bidder pays the amount of the associated bid. However, it is not so far investigated how false-name bidding affects first-price combinatorial auctions.

In an environment where false-name bids are possible, analytically investigating bidders' behaviors is very complicated, since bidders are asymmetric and nobody knows the number of real bidders. Bidders are asymmetric if their values are drawn from asymmetric distributions. Much of the motivation in investigating false-name bidding arises from environments where bidders have complementarities among goods. The equilibria in first-price auctions do not have a well-known closed-form solution. Accordingly, many approaches by computer scientists and economists have been developed to approximate an equilibrium strategy. Seminal works by Wellman and his colleagues have developed techniques to obtain an analytically intractable Nash equilibrium in *empirical mechanism design* [14, 12, 8]. Those have recently been used to design and evaluate alternate mechanisms [13, 10]. Armantier *et al.* advocated a similar technique called a *constrained strategic equilibrium* approach [1].

False-name bidding affects first-price auctions in a different way than VCG auctions. At first glance, false-name bidding seems not effective in first-price auctions. In a first-price combinatorial auction, if a bidder wins, he pays the amount of his bid. Assume a (potential shill) bidder can win two goods X and Y with bid $b^{\{X,Y\}}$. Assume he uses false-names, splits his bid, and obtains X and Y separately by bid $b^{\{X\}}$ and $b^{\{Y\}}$, respectively. As far as $b^{\{X,Y\}} = b^{\{X\}} + b^{\{Y\}}$, his payment does not change. However, the behaviors of other bidders might be influenced by false-name bidding. Let us assume there exists a competing bidder (denoted as bidder 1) who also wants X and Y. For bidder 1, his bidding strategy changes if his belief about his opponents changes. In short, his bid decreases when he thinks he is facing two opponents, each of whom wants either X or Y, compared to the case where he thinks he is facing one opponent who wants both X and Y. This is because, when there exist two (real) bidders, each tries to *free-ride* the other bidder's effort; neither raises his bid in the hopes that the other raises his bids high enough to beat bidder 1 [11]. Thus, the total of these two bidders' bids tends to be small. Then, bidder 1 can safely decrease his bid. Consequently, when the shill bidder pretends to be two bidders, bidder 1 decreases his bid. The shill bidder can take advantage of this fact.

It is very complicated to construct a game of auctions with false-name bidding. In the analysis of auctions with incomplete information, it is assumed that each bidder knows how many bidders are participating before an auction begins. This is because we require the cumulative distribution function of each bidder as common knowledge to solve such auctions. Therefore, we must properly model how many identifiers a shill bidder uses and when bidders know the number of bidders.

As a first step, we consider a very simple and stylized model where false-name bids become effective, i.e., an auction with two goods where one naive bidder competes with one shill bidder who may pretend to be two distinct bidders. We model this auction as a dynamic game with incomplete information. We then examine approximate Bayesian Nash equilibria when bidders' preferences are drawn from asymmetric distributions, by utilizing the CSE approach [1].

This paper provides novel insights into the properties of first-price auctions in environments where false-name bidding is possible. The numerical results suggest that false-name bidding in first-price auctions can dramatically reduce the revenue and does not reduce the surplus so much. Furthermore, a shill bidder can highly increase his profit using two identifiers, while a naive bidder can keep his profit, though he is less likely to defeat the shill bidder.

Let us briefly describe the organization of this paper. Section 2 formalizes the first-price combinatorial auctions and the solution concepts. Section 3 constructs a dynamic game of auctions with false-name bidding. Section 4 shows the numerical results of equilibrium bidding strategies. Section 5 examines the effect of false-name bidding in terms of the major properties of auctions. Section 6 concludes this paper.

2. PRELIMINARIES

2.1 First-price combinatorial auctions

In a first-price single-item auction, each agent i submits sealed bid b_i for a good valued by agent i at v_i. Among all agents, the agent with the highest bid wins the good (ties are broken randomly). In a combinatorial auction setting, the auction is also called a *menu auction*. Bernheim and Whinston developed a theory of sealed-bid, first-price combinatorial auctions [2]. Let us consider a first-price combinatorial auction with two goods X and Y.

1. Each bidder i submits sealed bids $b_i = (b_i^{\{X\}}, b_i^{\{Y\}}, b_i^{\{X,Y\}})$ on $\{X\}$ only, $\{Y\}$ only, and the set/bundle of $\{X, Y\}$.
2. The auctioneer chooses an allocation, so that the combination of bids maximizes the seller's revenue.
3. Each winning bidder pays the amount of the associated bid.

We also assume a *quasi-linear, private value* model with *no allocative externality.* The utility (profit) of bidder i, if he wins either X or Y with $b_i^{\{X\}}$ or $b_i^{\{Y\}}$, is $v_i^{\{X\}} - b_i^{\{X\}}$ or $v_i^{\{Y\}} - b_i^{\{Y\}}$; and the utility of bidder i, if he wins both goods with $b_i^{\{X,Y\}}$, is $v_i^{\{X,Y\}} - b_i^{\{X,Y\}}$.

A losing bidder obtains nothing, pays zero, and thus, his utility is zero, since we assume *normalization.* The seller revenue, i.e., the utility of the auctioneer, is the sum of the payments of the winning bidders.

2.2 Equilibrium concepts

We use two of the most prevalent solution concepts from game theory: *Bayesian Nash equilibrium* (BNE) and *perfect Bayesian equilibrium* (PBE). BNE are used to analyze games with incomplete information, or Bayesian games, e.g., analysis of non-direct-revelation mechanisms, such as first-price auctions. A bidder's (expected) profit depends not only on the bids of other bidders but also on information that is only partly known to the bidder, i.e., a distribution function on the values of other bidders. Furthermore, PBE is a refinement of BNE for dynamic games, which is required to describe environments where false-name bidding is possible: a shill bidder can use multiple identifiers.

Let us define BNE in an auction with bidder 1 and 2 in environments where false-name bidding is not possible. Bidder i assigns a value of v_i for each combination of goods on sale drawn from a cumulative probability distribution with function F_{v_i} and associated probability density function f_{v_i}. Bidder i knows his own value v_i and only that any other bidder $j(\neq i)$'s value is independently distributed based on F_{v_j}. Thus, F_{v_j} for all other bidder j and the number of them are common knowledge. In general, the distribution of valuations is assumed to be the same for all bidders, i.e., symmetric. However, since it must be asymmetric in auctions with false-name bidding, we do not specify the distribution here.

A bidding strategy for bidder i is defined as a function s_i. For example, bidder i with v_i submits $b_i = s_i(v_i)$. The inverse function of s_i is denoted as s_i^{-1}. Any other bidder j's strategy s_j is assumed to be increasing and differentiable. To draw bidder i's bid b_i, we can obtain the cumulative probability distribution function F_{b_i} and the associated density function f_{b_i} for an arbitrary value of b:

$$F_{b_i}(b) = F_{v_i}(s_i^{-1}(b)) \text{ and } f_{b_i}(b) = \frac{f_{v_i}(s_i^{-1}(b))}{\frac{d}{db}s_i(s_i^{-1}(b))}.$$

The expected profits of bidder $i \in \{1, 2\}$ for given v_i and s_i are calculated as follows:

$$U_i(b_i, s_j; v_i) = (v_i - b_i)F_{b_j}(b_i) \text{ for all } i.$$

From these expected profit, we define a BNE in the auction with bidder 1 and 2.

Definition 1 (Bayesian Nash equilibrium) *A profile of bidding strategies (s_1^*, s_2^*) consists of a Bayesian Nash equilibrium in an auction with bidder 1 and 2 if*

$$\begin{aligned}&\forall v_1, \forall v_2, \forall s_1, \forall s_2,\\ &U_1(s_1^*(v_1), s_2^*; v_1) \geq U_1(s_1(v_1), s_2^*; v_1), \text{ and}\\ &U_2(s_2^*(v_2), s_1^*; v_2) \geq U_2(s_2(v_2), s_1^*; v_2).\end{aligned}$$

The profile of the strategies maximizes the expected profit of each bidder when the probabilistic distribution of values and the number of bidders are common knowledge.

If a shill bidder can use multiple identifiers, the bidders' equilibrium strategies become significantly more intricate. In the analysis of auctions with incomplete information, it is assumed that each bidder knows the number of participating bidders before an auction begins, as common knowledge. However, if the shill bidder may pretend to be multiple distinct bidders, it is essential for a naive bidder to consider the number of real bidders. For example, when the naive bidder faces two bids, he may think that those come from a shill bidder using two false identifiers or he may think they come from two distinct bidders. As a result, a bidder's (expected) profit comes to depend on the prior distribution of others' values and the partial information about the number of real bidders. To model this, we construct a dynamic game and focus on the PBE analysis in the later section.

PBE is the most commonly used for analyzing sequential (dynamic) games with observed actions and private types (values) [4]. Each bidder has a strategy s_i and *beliefs* that are represented as a cumulative probability distribution function about values of other bidders. A strategy profile s_i is a PBE if each bidder updates his beliefs using Bayes rule whenever possible (*consistency*) and, whenever it is bidder i's turn to move, s_i prescribes an action that maximizes i's expected payoff from then on, given i's beliefs (*sequential rationality*).

As a first step, we consider a very simple and stylized model where we restrict the number of false identifiers each bidder can use and each bidder's observable information. This is because computing a PBE is intractable in environments where false-name bidding becomes effective. Thus, in subgames of the restricted dynamic game, we can compute a BNE strategies by utilizing a numerical technique that enables one to approximate an analytically intractable Nash equilibrium in a broad class of games with incomplete information.

2.3 Constrained strategic equilibrium

This section briefly describes a solution concept for games with incomplete information, called *constrained strategic equilibrium* (CSE) [1]. The sequence of CSEs approximates an equilibrium and CSE provides a useful way to numerically compute BNE for games whose solutions cannot be analytically derived.

We consider a single play of an two-person simultaneous-move game. Let $N = \{1, 2\}$ denote a set of bidders (players). The subscript i denotes a specific player $i \in N$, and the subscript j refers to the player except i. CSE is defined as a Nash equilibrium of a modified game in which strategies are constrained to belong to an appropriate subset typically indexed by an auxiliary parameter vector. Let us denote S as a subset of all feasible strategy profiles and S^k as a set of constrained strategy profiles for parameter k. Formally,

Definition 2 (Constrained strategic equilibrium) *Let $S^k = \{S_1^k, S_2^k\}$ for a parameter k denote a set of constrained strategy. $S^{k*} \subset S^k$ is the set of CSEs if $\forall s_i^k \in S_i^k$ and $\forall i \in N$, $\tilde{U}_i(s_i^{k*}, s_j^{k*}) \geq \tilde{U}_i(s_i^k, s_j^{k*})$ where $\tilde{U}_i$ is the expected utility of player i.*

Armantier *et al.* [1] identified a compacity condition under which a sequence of CSEs converges toward a Nash equilibrium.

Proposition 1 ([1]) *If an expected utility $\tilde{U}$ is continuous and if a sequence of CSEs $\{s^{k*}\}_{k=1\to\infty}$ has a subsequence with limit $\bar{s} \in S$, then $\bar{s}$ is a Nash equilibrium.*

Corollary 1 ([1]) *If a set of strategy profiles S is compact, $\tilde{U}$ is continuous and there exists a CSE strategy s^{k*} for all $k > 0$, then there exists a Nash equilibrium in S, and any sequence of CSEs $\{s^{k*}\}_{k=1\to\infty}$ has a subsequence that converges toward a Nash equilibrium.*

The compacity of the strategy space is standard in games with incomplete information and it applies to a large class of games including several auction models, such as asymmetric first-price auctions. The numerical technique enables one to approximate an analytically intractable Nash equilibrium in such a class. CSE also has an approximation algorithm that can be applied for asymmetric games with incomplete information. Let us briefly show the algorithm:

1. Consider a family of parameterized constrained strategies: $s_i^k(v_i) = s_i(d_i^k, v_i) \in S_i^k$, with $d_i^k \in D_i^k \subset \mathbb{R}^{\gamma(k)}$. Note that $\gamma(k)$ is a function of the final dimension and is set to 2^{k-1}.

2. Maximize player i's expected utility after fixing parameter d_j^k. The approximation of the expected utility for d_j^k is defined as

$$\tilde{U}_i^M(s_i(d_i^k, v_i)) = \frac{1}{M}\sum_{m=1}^{M} V_i(d^k, \tilde{v}^m)G_i(s_i(d_i^k, \tilde{v}_i^m)).$$

   where V_i is the utility function of player i with value $\tilde{v}^m$, which denotes a vector of two values drawn randomly ($N = 2$). M is the Monte Carlo size and G_i is the cumulative probability distribution function that player i wins the game (auction) when he takes $s_i(d_i^k, \tilde{v}_i^m)$.

3. Step 2 is repeatedly applied for each player. If d^k for all i is not updated, this algorithm stops.

In most applications, G_i cannot be calculated analytically and needs to be approximated by a kernel density estimation. The kernel density estimation is a non-parametric way of estimating G_i by L sample drawn from the distribution of values F. Let $\{v^1, \ldots, v^L\}$ denote L sample drawn from F. The kernel density estimation $\hat{f}_h(v)$ is shown by the following equation:

$$\hat{f}_h(v) = \frac{1}{Lh}\sum_{l=1}^{L} K(\frac{v - v^l}{h}),$$

where $K(\cdot)$ is the Gaussian distribution as a kernel function and h is a smoothing parameter called a bandwidth

Notice that the accuracy of kernel density estimation depends on bandwidth h. For example, if you enlarge h more than an appropriate value, $\hat{f}$ loses its feature – and vice versa. However, it is difficult to find the optimal bandwidth because it heavily depends on the structure of problems. In this paper, we use the following equation that achieves empirically good accuracy [5].

$$h = (\int K(t)^2 dt)^{1/5} (\frac{3}{8\sqrt{\pi}}\sigma^{-5})^{-1/5} L^{-1/5}.$$

where σ is the sample variance of L.

3. A DYNAMIC GAME WITH FALSE-NAME BIDDING

This section illustrates the PBE analysis through a 2- or 3-bidder combinatorial auction with two different goods, X and Y. Consider a dynamic game shown in Figure 1 with two stages: identifier-choice and bidding. First, with probability p, bidder 1 and 2 participate in an auction ($N = 2$), and with probability $1 - p$, bidder 1, 3, and 4 participate ($N = 3$). Assume that bidders have no knowledge about probability p. Second, at the identifier-choice stage, each bidder chooses how many identifiers he uses, and, in practice, only bidder 2 can choose one or two identifiers. Last, at the bidding stage, each bidder bids after observing the number of participating bidders, which may include false identifiers.

We also need to define a type that each bidder receives in games with incomplete information to provide each bidder with strategy space and information. We assume that the type determines the value for each combination of auctioned goods and the number of identifiers he can use.

Let us define types of bidder 1-4 and the observable information in the following. Bidder 1 values only the set of two goods drawn from the sum of two uniform distributions on interval $[0, 1]$, $Uni(0, 1)$:

$$(v_1^{\{X\}}, v_1^{\{Y\}}, v_1^{\{X,Y\}}) = (0, 0, Uni(0, 1) + Uni(0, 1)),$$

each of which is drawn independently. At the identifier-choice stage, he does nothing, since he can use only a single identifier. He also has a belief about how many bidders are participating as probability p^1, with which he is competing with bidder 2. Here, p^1 does not always be true, i.e., p^1 may not be equal to the true probability p. Before the bidding stage, he observes the number of bidders. When he observes one other bidder, he realizes that his opponent is bidder 2 and bids to maximize his profit conditional on his belief about bidder 2's value. On the other hand, when he observes two other bidders, he realizes that his opponents are false identifiers of bidder 2 with probability p^1, or that they are distinct bidders of bidder 3 and 4, with $1-p^1$. Consequently, he bids to maximize his profit conditional on his joint belief about bidder 3 and 4s' value.

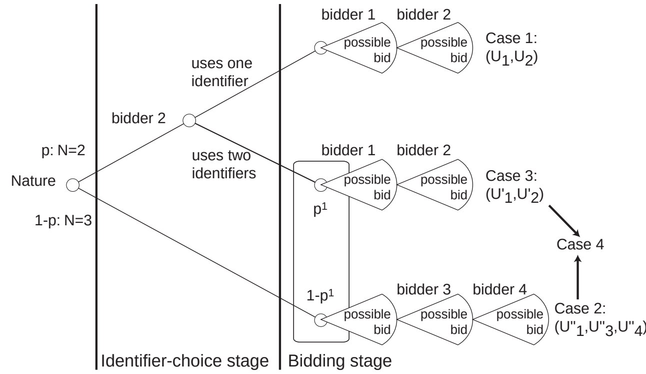

Figure 1: A dynamic game of an auction with false-name bidding

Bidder 2 positively values both $\{X\}$, $\{Y\}$, and $\{X, Y\}$. Each value on $\{X\}$ and $\{Y\}$ is independently drawn from $Uni(0, 1)$ and value on $\{X, Y\}$ is their sum:

$$(v_2^{\{X\}}, v_2^{\{Y\}}, v_2^{\{X,Y\}}) = (Uni(0, 1), Uni(0, 1), v_2^{\{X\}} + v_2^{\{Y\}}).$$

Thus, bidder 1 and 2 have a symmetric distribution on $\{X, Y\}$. At the identifier-choice stage, bidder 2 can use one or two identifiers and knows that $N = 2$ was chosen because he himself participated. He also exactly knows what information bidder 1 observes. When he uses one identifier, he knows that bidder 1 realizes that no bidder uses false identifiers. When he uses two identifiers, he knows that bidder 1 has that belief p^1 about bidder 2's presence. At the bidding stage, bidder 2 bids based on that information and his belief about bidder 1's value.

Bidder 3 and 4 can use only a single identifier. We only explain bidder 3's case only, since bidder 3 and 4 have almost identical information and value except the good he desires. Bidder 3 values only $\{X\}$ and $v_3^{\{X\}}$ is drawn from $Uni(0, 1)$. After the identifier-choice stage, he knows that $N = 3$ was chosen because he himself participated. At the bidding stage, bidder 3 bids based on that information and his joint belief about bidder 1 and 3s' values.

We explore the strategies in four specific subgames for some p and p^1 to effectively show how bidders' behaviors change. Then, we calculate bidders' expected profits in a subgame of the dynamic game by utilizing the CSE approximation algorithm.

Case 1: 2 bidders - 2 identifiers ($p = 1$).

> With probability $p = 1$, bidder 1 and 2 participate ($N = 2$) and bidder 2 always chooses to use a single identifier. Since bidder 1 and 2 use a single identifier and submit their bids, no false-name bidding occurs. They obtain profits of U_1 and U_2.

Case 2: 3 bidders - 3 identifiers ($p = 0$ and $p^1 = 0$).

With probability $p = 0$, bidder 1, 3 and 4 participate ($N = 3$), always use a single identifier, and submit their bids, knowing that bidder 1 believes that no false-name bidding occurs ($p^1 = 0$). They obtain profits of U_1'', U_3'' and U_4''.

Case 3: 2 bidders - 3 identifiers ($p = 1$ and $p^1 = 0$).

With probability $p = 1$, bidder 1 and 2 participate ($N = 2$), and bidder 2 always chooses to use two identifiers. Thus, three identifiers submit their bids. Since bidder 1 believes that no false-name bidding occurs ($p^1 = 0$), he chooses the same bidding strategy as in Case 2. Bidder 2 takes the best response to the bidding strategy of bidder 1. Bidder 1 and 2 obtain profits of U_1' and U_2'. Note that, for bidder 2, the expected profit when he uses two identifiers is always better than when he uses a single identifier; for bidder 2, the strategy using two identifiers is PBE.

Case 4: 2 or 3 bidders - 3 identifiers ($p = 1/2$ and $p^1 = 1/2$).

Case 4 stochastically combines Case 2 and 3 where, with $p = 1/2$, Case 3 occurs, and with $1 - p = 1/2$, Case 2 occurs. No bidder knows exactly the probability, but every bidder knows bidder 1's belief about Case 2 or 3 occurs ($p^1 = 1/2$). Except bidder 1, all bidder take a best response to bidder 1's bidding strategy in which he considers false-name bidding.

4. NUMERICAL RESULTS

This section illustrates the PBE bidding strategies in Case 1-4, which are the consequences of the dynamic game described in Section 3. For comparison, we also note the corresponding strategies in VCG auctions in Appendix A.1. Since the values of the bidders on $\{X, Y\}$ in Case 1 are drawn from symmetric distributions, PBE has a well-known closed-form solution. On the other hand, those in Case 2-4 are drawn from asymmetric distributions. Thus, we theoretically show the PBE bidding strategies in Case 1 and numerically show them in Case 2-4 by utilizing the CSE approximation method.[1] The required parameters are set to $k = 5$, $M = 1000000$, and $L = 1000$.

4.1 Case 1: 2 bidders – 2 identifiers

Let $v \in [0, 2]$ be a value on the bundle of $\{X, Y\}$ for bidder 1 and 2 and let $s(v) : R^+ \rightarrow R^+$ be a mapping function of the value to the bid. Since PBE in Case 1 has a closed-form solution, we can theoretically derive the equilibrium bidding strategy $s(v)$ [9]:

$$s(v) = \begin{cases} \frac{2}{3}v & \text{if } 0 \leq v \leq 1, \\ \frac{2}{3}(v + 1 + \frac{2v-1}{v^2-4v+2}) & \text{if } 1 < v \leq 2. \end{cases}$$

The red line in Figures 2-5 shows this bidding strategy, which is labeled as "Case 1: symmetric bidder." Bidder 1 with low value ($v < 1$) shades his bid to two-thirds of his value, and bidder 1 with high value ($v > 1$), further shades his bid as his value increases.

[1] In addition to the CSE approximation method, we examined these cases by a similar algorithm to [12] and obtained almost identical results.

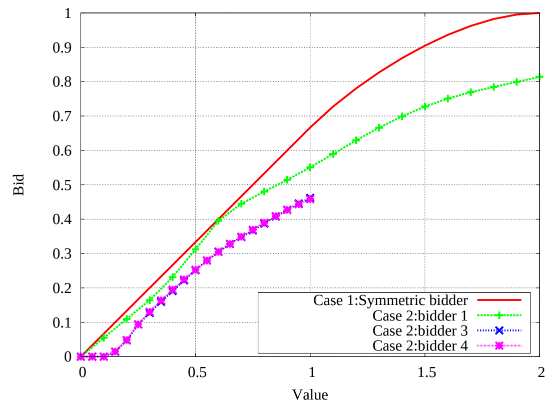

Figure 2: Bidding strategies in Case 2

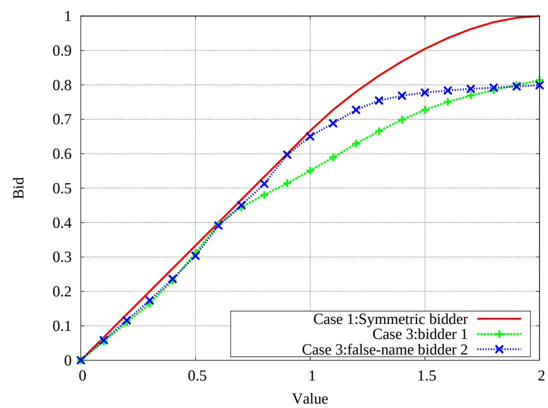

Figure 3: Bidding strategies in Case 3

4.2 Case 2: 3 bidders – 3 identifiers

Unlike Case 1, the values of all bidders are drawn from asymmetric distributions. In fact, bidder 1 values $[0, 2]$ only at $\{X, Y\}$, and bidder 3 and 4 values $[0, 1]$ at $\{X\}$ and $\{Y\}$, respectively. In general, there are no closed-form solutions for this case, but it can be easily solved by appropriate numerical methods. Figure 2 illustrates the bidding strategies of bidder 1, 3, and 4 with respect to their realizations of the values drawn from each distribution (blue and pink lines labeled as "Case 2: bidder 3" and "Case 2: bidder 4").

Bidder 1 with low value less than about 0.75, shades his bid to the same amount as in Case 1, and bidder 1 with high value reduces his bid more than in Case 1. The amount of reduction gradually increases as his value increases. Bidder 3 and 4 still shade their bids, in particular, with very low values, they prefer to bid zero.

This result is consistent with the *free-rider* problem in auctions [11]: A bidder does not raise his bid in the hopes that the other raises his bid high enough for that bidder to obtain a good. For example, bidder 3 and 4 value $\{X\}$ and $\{Y\}$, respectively. If the sum of their bids exceeds the amount of the bid on $\{X, Y\}$, bidder 3 and 4 win. Bidder 3 may expect bidder 4 to bid so high that bidder 1 loses and has an incentive to obtain $\{X\}$ with a low bid, and vice versa. Also, bidder 1 takes the best response to their shaded bids.

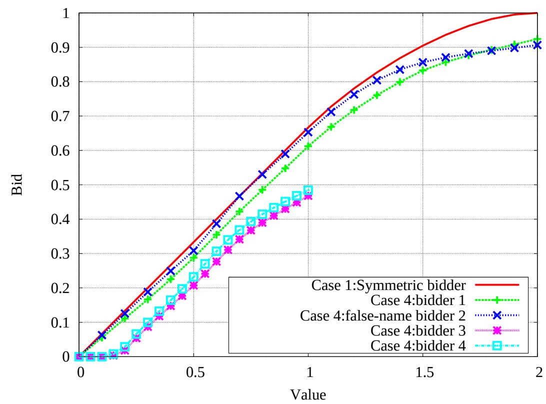

Figure 4: Bidding strategies in Case 4

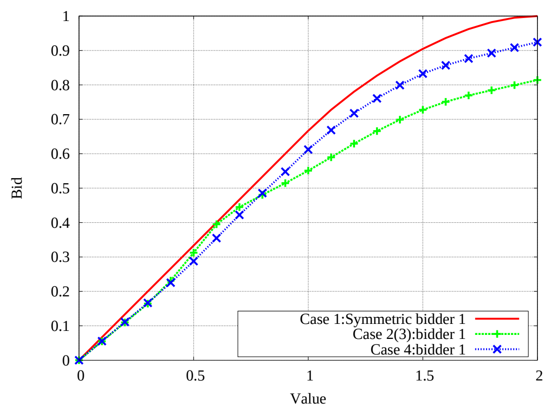

Figure 5: Bidding strategies of bidder 1

4.3 Case 3: 2 bidders – 3 identifiers

Case 3 has a more complicated strategy space, since bidder 2 can use two identifiers. Figure 3 illustrates the bidding strategies of bidder 1 and 2 (green and blue lines labeled as “Case 3: bidder 1” and “Case 3: false-name bidder 2”). Bidder 1’s strategy is equivalent to the one in Case 2, since he believes that he is competing with two distinct bidders. Bidder 2 splits his bid into two bids on $\{X\}$ and $\{Y\}$ by using two identifiers. The blue line in Figure 3 indicates the sum of those two bids in $v_2^{\{X,Y\}}$.

Bidder 2 with low value ($v_2^{\{X,Y\}} < 0.75$) or very high value ($1.8 < v_2^{\{X,Y\}}$) prefers to bid almost the same amount as bidder 1. On the other hand, bidder 2 with intermediate values first submits a slightly higher bid than bidder 1, raises his bid, and gradually reduces toward bidder 1, as his value increases.

This result suggests that bidder 2 can increase his profit as a result of taking the best response to the distribution of bidder 1’s value which is a joint distribution of two $Uni(0, 1)$. With such a distribution, bidder 1 is most likely to have his value of 1 and is least likely to have 0 or 2. Thus, bidder 2 raises his bids around 1 to maximize his profit. Therefore, bidder 2 has enough opportunities of false-name bidding to increase his expected profit.

4.4 Case 4: 2 or 3 bidders – 3 identifiers

In Case 4, bidder 1 considers the possibility that two of his competing bids come from one shill bidder (bidder 2) conditional on his belief about the actual number of participating bidders, i.e., $p^1 = 1/2$. Figure 4 illustrates the bidding strategies of bidder 1-4. Note again that bidder 2’s strategy is represented as the sum of two bids.

The doubt of bidder 1 that a shill bidder exists raises his bid, so it becomes much closer to that in Case 1. Bidder 1 averages his bidding strategies in Case 1 and 2 in Figure 5. Unlike bidder 1, bidder 2 can slightly raise his bid higher than bidder 1 because bidder 1 may not correctly suspect the number of real bidders. Thus, the opportunities of false-name bidding are reduced. As well as bidder 2, bidder 3 and 4 know bidder 1’s strategy. With lower value, they bid slightly lower than in Case 2, but, with higher value, they bid slightly higher.

5. DISCUSSION

This section discusses obtained properties from the numerical results: the social surplus, the auctioneer’s revenue, and the profits of bidders. We decide the values of bidders based on the settings in Case 1-4 and generate 10 million instances. Table 1 summarize the average properties when bidders take the equilibrium bidding strategies in first-price auctions. For comparison, we also note the corresponding results of VCG auctions in Appendix A.2.

From bidder 1’s perspective, Case 1 ($N = 2$) and Case 2 ($N = 3$) are seemingly the same, since the (aggregated) values of the opponents are the same. However, in Case 2, bidder 3 and 4 try to free-ride each other and decrease their bids. Thus, bidder 1 lowers his bids to maximize his profit. As a result, bidder 1 successfully increases his profit from 0.233 to 0.334 (+43%). This also significantly decreases the revenue from 0.767 to 0.620 (-19%); the decrease of the surplus from 1.23 and 1.22 (-1%) is relatively small. The fact that the surplus does not significantly change means that the obtained allocation is nearly efficient. All bidders decrease their bids. Occasionally, bidder 1 wins when the efficient allocation is allocating goods to bidder 3 and 4, but this happens only when bidder 1’s value is close to the sum of values of bidder 3 and 4.

Let us examine Case 3 where false-name bidding is possible, i.e., a naive bidder (bidder 1) and a shill bidder (bidder 2) participate. The naive bidder completely believes that he is competing with two bidders ($p^1 = 0$), and the shill bidder knows this fact and always uses two false identifiers. Recall that this behavior of bidder 2 consists of a PBE. The revenue of 0.681 is intermediate between Case 1 and 2 and it decreases from 0.767 in Case 1 to 0.681 (-11%). Here, bidder 1 believes that he is facing two small bidders. If they were real bidders, they would try to free-ride and lower their bids. Thus, bidder 1 also lowers his bid. However, bidder 2 optimizes his two bids against the wrong belief of bidder 1. Thus, false-name bidding by bidder 2 decreases the revenue. In contrast, the surplus hardly changes regardless of the existence of false-name bidding. This fact means that the obtained allocation is nearly efficient. All bidders decrease their bids. Occasionally, bidder 2 wins when the efficient allocation is allocating goods to bidder 1, but this happens only when bidder 2’s value is close to bidder 1’s value.

In addition, the existence of false-name bidding significantly affects the profits of bidders. Bidder 2 significantly increases his profit from 0.233 to 0.312 (+34%) by using

Table 1: Properties of Case 1-4 in first-price auctions

	Case 1	Case 2	Case 3	Case 4a	Case 4b
revenue	0.767	0.620	0.681	0.718	0.660
surplus (efficiency)	1.23 (100%)	1.22 (99%)	1.22 (99%)	1.23 (100%)	1.22 (99%)
profit (bidder 1)	0.233	0.334	0.226	0.244	0.319
profit (bidder 2)	0.233	-	0.312	0.269	-
profit (bidder 3)	-	0.134	-	-	0.122
profit (bidder 4)	-	0.134	-	-	0.122

false identifiers, but the profit of bidder 1 does not change much from Case 1, only from 0.233 to 0.226 (-3%). However, this is not what bidder 1 expected. If he were facing two real bidders, his profit would have been 0.334 (in Case 2). Bidder 2 steals a significant amount of bidder 1's profit by over-bidding bidder 1. Interestingly, the profit of bidder 2 in Case 3 (0.312) is relatively close to that of bidder 1 in Case 2 (0.334). Also, the profit of bidder 1 in Case 2 (0.334) is relatively close to the sum of profits of bidder 3 and 4 in Case 2 (0.384).

Let us examine Case 4 where a naive bidder (bidder 1) is suspicious of the number of real bidders. Bidder 1 is wondering if the two observed bids were submitted from two distinct bidders or one shill bidder. We categorize Case 4 as either Case 4a with false-name bidding or Case 4b without. These results are summarized in the last two columns of Table 1. In Case 4a, the revenue decreases from 0.767 in Case 1 to 0.718 (-6%), and it increases from 0.681 in Case 3 to 0.718 (+5%). Recall that bidder 1 in Case 4 takes an average bidding strategy of Case 1 and 2 under the suspicion of the actual number of participating bidders, i.e., $p^1 = 1/2$. Thus, bidder 1 raises his bids more than Case 3. By false-name bidding the profit of bidder 2 (0.269) is higher than in Case 1 (0.233). However, he cannot increase his profit (0.269) so much as in Case 3 (0.312). Accordingly bidder 1's suspicion effectively mitigates the decrease of revenue when a shill bidder may be present. The effect of false-name bidding is reduced by the fact that the naive bidder is aware of its possibility.

Let us turn to Case 4b where bidder 3 and 4 submit two distinct bids, considering the suspicion of bidder 1. Case 4b achieves the revenue of 0.660, and Case 2 does 0.620. The revenue increases by about +6%, but the profits of the bidders decrease, including bidder 3 and 4, who are the real bidders. In a contrast to Case 4a, the suspicion of bidder 1 increases the revenue and reduce the profits of all bidders.

It is worthy to note that, if the naive bidder can distinguish Case 4a and 4b for sure, false-name bidding is no longer profitable. This implies that, if your opponent is sure about your identity, it is useless that you pretend to be somebody else and there is no point using false-name bidding. However, since this is impossible on the Internet, a shill bidder can take advantage of false-name bidding. It is most effective when your opponent never imagines the possibility of disguise. Also, it is still effective if your opponent is aware of that possibility, but cannot distinguish a real person and a false identifier.

We have so far investigated situations where bidder 1's belief is correct ($p = p^1$), except Case 3 ($p = 1$ and $p^1 = 0$). Let us consider what happens if bidder 1's belief is incorrect ($p \neq p^1$). When p^1 increases in Case 3, bidder 1's belief gradually becomes correct for the probability of number of real bidders $p = 1$. Thus, the bidding strategies of bidder 1 and 2 change from Case 3 toward Case 1. If bidder 1 has $p^1 = 1$, the properties in Case 3 are identical to those in Case 1. On the other hand, when p^1 increases in Case 2 ($p = 0$ and $p^1 = 0$), bidder 1's belief gradually becomes incorrect. Then, if bidder 1's belief becomes $p^1 = 1/2$, the situation becomes identical to Case 4b. Bidder 1 increases his bid. As a result, the revenue increases and the profits of all bidders decrease.

6. CONCLUSION

This paper numerically analyzes how false-name bidding affects the outcomes in first-price combinatorial auctions. False-name bidding causes serious problems in the VCG auctions. However, to the best of the authors' knowledge, this is the first analysis about first-price combinatorial auctions. The game of first-price auctions is regarded as a game of incomplete information, which typically does not have a closed-form solution, except under such simplifying assumptions as symmetry among types of bidders. Thus, predicting the consequences of such games is often analytically intractable. In addition, the extension of games of auctions to games where false-name bidding is possible further complicates them. Therefore, we construct a dynamic game of auctions with false-name bidding and approximately solve the subgames in four specific settings.

We reveal how the existence of false-name bidding changes the equilibrium bidding strategies and its properties. The results suggest that false-name bidding in first-price auctions dramatically reduces the revenue and the profits of bidders who neither use nor are concerned about false-name bidding.

In future works, we will extend our analysis to a variety of empirical distributions and generalize the approximation algorithm to solve dynamic games of auctions with false-name bidding.

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Table 2: Properties of Case 1-4 in VCG auctions

	Case 1	Case 2	Case 3	Case 4a	Case 4b
revenue	0.767	0.617	0.00	0.00	0.617
surplus (efficiency)	1.23 (100%)	1.23 (100%)	1.00 (81%)	1.00 (81%)	1.23 (100%)
profit (bidder 1)	0.233	0.233	0.00	0.00	0.233
profit (bidder 2)	0.233	-	1.00	1.00	-
profit (bidder 3)	-	0.192	-	-	0.192
profit (bidder 4)	-	0.192	-	-	0.192

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APPENDIX

A. VCG COMBINATORIAL AUCTIONS

A.1 PBE bidding strategies

This subsection summarizes the consequences of Case 1-4 in VCG. Throughout Case 1 and 2, all bidders have a dominant strategy to bid their own values in VCG. In Case 3, bidder 1 believes that truth-telling is the dominant strategy, since he never considers the possibility of false-name bidding. In sharp contrast, bidder 2 uses two false identifiers and manipulates VCG so that he always obtain the goods and pays zero. This strategy is an equilibrium strategy as long as bidder 1 takes the truth-telling strategy.

For example, in our setting, the value on $\{X, Y\}$ of bidder 1 is drawn from a joint distribution on interval $[0, 2]$. Bidder 2 splits his bid and bids 2 on $\{X\}$ and $\{Y\}$, respectively. As long as bidder 1 keeps $s_1(v) = v$, bidder 2 has an equilibrium strategy to bid $b^{\{X\}}$ and $b^{\{Y\}}$, each of which is greater than or equal to $b_1^{\{X,Y\}}$, i.e., to bid 2 which is the maximum value of $b_1^{\{X,Y\}}$.

In Case 4, bidder 1 considers false-name bidding. When bidder 3 and 4 are present ($N = 3$), Truth-telling for them is clearly a dominant strategy as in Case 2. When bidder 2 is present ($N = 2$), as mentioned in Case 3, he has an equilibrium strategy so that he always win with zero payment as long as bidder 1 takes the truth-telling strategy. Against this strategy, we restrict our attention to a situation where bidder 1 takes the truth-telling strategy because bidder 1 has no chance to obtain any good. There exists no bidding strategy that outperforms the truth telling strategy, even if he is confident that his opponents are using false-name bidding. To be precise, we must consider situations so that bidder 1 over-bids to obtain the goods, which never increase his profit. Accordingly, for all bidders, an equilibrium bidding strategy in Case 4 is equivalent to Case 3.

A.2 Discussions

The results of VCG are much simpler than those of first-price auctions. The difference from Case 1 ($N = 2$) to Case 2 ($N = 3$) only depends on the payment rules. The revenue decreases from 0.767 to 0.617 (-20%), and the surplus remains unchange, since all bidders submit their own values in the equilibrium. The profit of bidder 1 also doesn't change, but the sum of the profits of bidder 3 and 4 in Case 2 exceeds the profit of bidder 2 in Case 1.

In Case 3 where false-name bidding is possible, the revenue is zero, and only the profit of bidder 2 is positive. As mentioned in Section 4, bidder 2 uses two false identifiers and manipulates the VCG outcome so that he always obtain the goods and pays zero. Also, the surplus drastically decreases from 1.23 to 1.00 (-19%). Note that the surplus of 1.00 equals the lowest achievable surplus in our setting. Furthermore, Case 4 inherits this result in Case 3. Case 4a corresponds to Case 3, and Case 4b corresponds to Case 2, even if bidder 1 is suspicious of the number of real bidders. These obtained results imply that false-name bidding is even more serious in VCG than in the first-price auctions, as existing theoretical considerations in mechanisms that have dominant-strategy equilibria. In other words, outcomes in VCG are significantly manipulated by false-name bidding, regardless whether bidder 1 considers the possibility of false-name bidding by bidder 2.