Iterative Game-theoretic Route Selection for Hostile Area Transit and Patrolling

(Extended Abstract)

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ABSTRACT

A number of real-world security scenarios can be cast as a problem of transiting an area patrolled by a mobile adversary, where the transiting agent aims to choose its route so as to minimize the probability of encountering the patrolling agent, and vice versa. We model this problem as a two-player zero-sum game on a graph, termed the *transit game*. In contrast to the existing models of area transit, where one of the players is stationary, we assume both players are mobile. We also explicitly model the limited endurance of the patroller and the notion of a base to which the patroller has to repeatedly return. Noting the prohibitive size of the strategy spaces of both players, we employ iterative oracle-based algorithms including a newly proposed accelerated scheme, to obtain optimum route selection strategies for both players. We evaluate the developed approach on a range of transit game instances inspired by real-world security problems in the urban and naval security domains.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence

General Terms

Algorithms, Economics, Security, Performance

Keywords

game theory, reasoning

1. INTRODUCTION

Hostile area transit and patrolling is an important problem relevant to many real-world security scenarios. For the transiting agent, the objective is to choose a route crossing the hostile area which minimizes the risk that it will be encountered and intercepted by the opponent patrolling agent, which moves strategically within the area; for the patrolling agent, the objective is the opposite.

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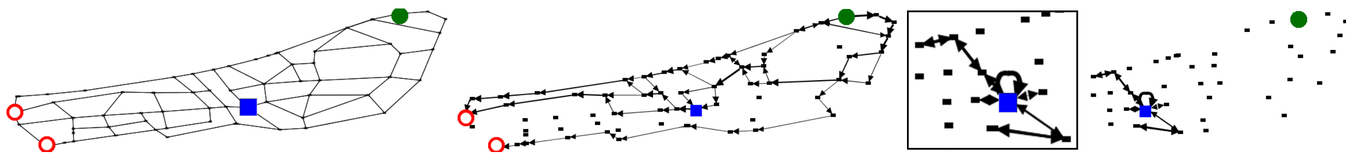
Game theory provides a principled way of computing optimum routes in such cases, taking into account strategic reasoning ability of the opponent. For these reasons, game theory has been successfully applied to various security problems in the past [1], resulting in a variety of games reflecting specific assumptions, domain restrictions and player capabilities. None of the existing games, however, allows to model the case where the patroller is mobile and has restrictions on its mobility – which is typical in real-world scenarios.

To provide a principled solution to the problem of hostile area transit and patrolling, we therefore present two main contributions. As the first contribution, we extend existing security games models by allowing *constrained mobility* of the patroller. Unfortunately, the simultaneous mobility of both players leads to combinatorial explosion in the number of possible strategies and makes standard methods for finding Nash equilibria inapplicable. We therefore employ iterative solution techniques known as *oracle-based algorithms* [3] which do not require explicit enumeration of strategies for one or both players. Although the oracle-based approach alleviates the scalability problem to some extent, it requires repeated best response calculation, which is hard in our case. As the second main contribution, building on our previous work [4], we therefore propose a novel *accelerated oracle* algorithm, which reduces the need for best response calculation, and thus speeds up the calculation of Nash equilibria.

2. PROBLEM DEFINITION

We assume the transit area is connected and we represent it as a simple directed graph, termed *transit graph*, with loops and with defined *entry* and *exit* nodes and a *base* node. The objective of the transiting player, termed *Evader*, is to get from any entry node to any exit node *without* encountering the patrolling player, termed *Patroller*. The Patroller's objective is to intercept the Evader's transit by strategically moving through the transit graph. In addition, because of its limited endurance, the Patroller has to repeatedly return to the base node. Both players move at the same speed and have full knowledge about the environment. However, they do not know the current location of the other player, unless they meet. Furthermore, the Patroller does not know if the Evader has already entered the area.

Movement of either player can be expressed as a walk on the transit graph. The transit game is then defined as a zero-sum game in a normal form. The set of all possible pure Evader's strategies is the set of all walks starting in an entry node and ending in an exit node, with the nodes



(a) Exemplar planar city graph. (b) Evader's optimal strategy. (c) Patroller's optimal strategy.

Figure 1: Example transit game on a planar city graph. The graph has one entry node in the eastern part of the graph (full green circle), two exit nodes in the western part of the graph (empty red circles) and the base (blue square) in the middle. The final game value is 0.146, i.e. giving the Patroller a chance of 14.6% to intercept the Evader.

in between not being an entry or exit node. The set of all possible pure Patroller's strategies is the set of all closed walks starting and ending in the base with a given bounded length. There is an *encounter* of two walks if they have a common node or an edge, or contain two contra-directional edges. To provide more expressivity to the transit game model, encounters are assumed to lead to interceptions only with a defined *interception probability* which is assigned to each node and edge of the transit graph.

The Patroller's utility for a pair of pure strategies (i.e. walks) is equal to the probability that the Evader will be intercepted when the strategies are enacted. The probability of interception is related to the number of encounters and the interception probability at encounter locations. The proper definition of the interception probability has to account for the dependencies introduced by the fact that the Evader can be intercepted *at most* once. The Evader's utility is the opposite to the Patroller's.

3. SOLUTION

We employ mixed-strategy Nash equilibrium as a solution concept for the transit game. Because of the enormous size of the strategy spaces of both players, we employ iterative techniques known as *oracle-based algorithms*, which search for a Nash equilibrium iteratively in a succession of increasingly larger *subgames* of the full game (see [3] for details). In each iteration the best response – in the form of a pure strategy – for the current subgame is provided by an *oracle* and added to the current strategy sets of respective player. The performance of the oracle plays a crucial role in the overall performance of the algorithm. Unfortunately, for the transit game, best responses calculation is an NP-hard problem.

We thus propose a modification of the single- and double oracle algorithms which does not require optimal best response calculation for each subgame yet still provides optimal solution. The core idea of the novel *accelerated* oracle algorithm is to use a special *subgame expansion oracle* for iterative construction of subgames and only use the best response oracle when checking the termination condition. The expansion oracle should either be significantly faster to compute and/or navigate the space of games more efficiently (or both). See [2] for more details.

4. EVALUATION

We have studied the properties of the transit game and its solution on two types of application-relevant graphs: (1) rectangular grid graphs, best representing open areas, and (2) irregular planar graphs, suitable for representing cities and/or other structured environments. Table 1 presents runtimes of the single oracle (ESO), double oracle (DO) and accelerated single oracle (ESO-A) algorithms on both types of graphs. Figure 1 shows an example solution on a transit graph representing the street network of northern Taipei. Practical application of the accelerated oracle algorithm depends on the size of Patroller's strategy space. We have been able to solve grids of size 12-by-4 where the Patroller has approximately 25 thousand strategies.

		ESO	ESO-A	DO
Regular grid	Iters	36	33	60
	Time [s]	7.2	6.2	91.1
Irregular plannar	Iters	13	14	16
	Total [s]	76.7	18.7	29.8

Table 1: Runtimes in seconds for different variants of the oracle algorithms and different types of transit graph.

5. CONCLUSION

Explicit modelling of constrained mobility of patrollers extends the range of scenarios to which the security game model can be applied. Due to the huge size of strategy spaces in such games, iterative solution techniques are necessary and practically useful, as shown in the evaluation. The newly introduced accelerated oracle algorithm further improves the scalability of the iterative oracle-based approach and is applicable to a wide range of games.

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7. REFERENCES

[1] M. Jain, J. Tsai, J. Pita, C. Kiekintveld, S. Rathi, M. Tambe, and F. Ordonez. Software Assistants for Randomized Patrol Planning for the LAX Airport Police and the Federal Air Marshals Service. *Interfaces*, 40:267–290, 2010.

[2] M. Jakob, O. Vaněk, B. Bošanský, O. Hrstka, and M. Pěchouček. Adversarial modeling and reasoning in the maritime domain year 2 report. Technical report, ATG, CTU, Prague, 2010.

[3] H. B. McMahan, G. J. Gordon, and A. Blum. Planning in the Presence of Cost Functions Controlled by an Adversary. In *ICML*, pages 536–543, 2003.

[4] O. Vaněk, B. Bošanský, M. Jakob, and M. Pěchouček. Transiting Areas Patrolled by a Mobile Adversary. In *IEEE CIG*, 2010.