

# Incentive Compatible Influence Maximization in Social Networks and Application to Viral Marketing

## (Extended Abstract)

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### ABSTRACT

Information diffusion and influence maximization are important and extensively studied problems in social networks. Various models and algorithms have been proposed in the literature in the context of the influence maximization problem. A crucial assumption in all these studies is that the influence probabilities are known to the social planner. This assumption is unrealistic since the influence probabilities are usually private information of the individual agents and strategic agents may not reveal them truthfully. Moreover, the influence probabilities could vary significantly with the type of the information flowing in the network and the time at which the information is propagating in the network. In this paper, we use a mechanism design approach to elicit influence probabilities truthfully from the agents. Our main contribution is to design a scoring rule based mechanism in the context of the influencer-influencee model. In particular, we show the incentive compatibility of the mechanisms and propose a reverse weighted scoring rule based mechanism as an appropriate mechanism to use.

### Categories and Subject Descriptors

H.4 [Algorithms and Theory]: Social Networks, Scoring Rules, Mechanism Design

### General Terms

Algorithms

### Keywords

Social Networks, Information Diffusion, Influence Maximization, Mechanism Design, Incentive Compatibility, Scoring Rules, Viral Marketing

## 1. RELEVANT WORK

Kempe, Kleinberg, and Tardos in [5] considered the problem of influence maximization proposed by Domingos and Richardson in [3]. In [5] they proved that this problem is NP-hard even for simple models of information diffusion.

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There are a number of algorithms proposed in the context of influence maximization in the recent years [1].

In the work by Goyal, Bonchi, and Lakshmanan in [4], the approach is to use a machine learning for building the models to predict the influence probabilities in social networks. They validate the models they build on a real world data set.

A mechanism design based framework to extract the information from the agents has been proposed for query incentive networks [2].

## 2. INFLUENCER - INFLUENCEE MODEL

In a real world social network, given a social connection between two individuals, both the individuals will have information about different aspects and properties of the connection. We now present the influencer-influencee model which tries to model this scenario.

### 2.1 The Model

- Given a directed edge  $(i, j)$  in the social network, the social planner will ask:
  - agent  $i$  (the influencer) to report her influence probability  $\theta_{ij}$  on  $j$  and
  - agent  $j$  (the influencee) to report agent  $i$ 's influence on her.
- Consider an agent  $i$ . Let  $out(i) = \{j | (i, j) \in E\}$  and  $in(i) = \{j | (j, i) \in E\}$ . Thus agent  $i$  acts as influencer to nodes in the set  $out(i)$  and acts as the influencee for the nodes in set  $in(i)$ . In this model an assumption is that agent  $i$  knows the influence probabilities on the edges that are incident on  $i$  and that are emanating from  $i$ . Thus agent only knows about the influence probabilities in its neighborhood and nothing beyond that.
- Also no agent knows what influence probability is reported by the agents in its neighborhood. The only way an agent can predict the reported probability by its neighbor is by her own assessment of it. Thus we assume that for any given pair of nodes  $i$  and  $j$  having edge  $(i, j)$  between them, the conditional probability distribution function  $P(\theta_{ij}^j | \theta_{ij}^i)$  which has all the probability mass concentrated at  $\theta_{ij}^j = \theta_{ij}^i$ .
- Here we discretize the continuous interval  $[0, 1]$  into  $\frac{1}{1+\epsilon}$  equally spaced numbers and agents will have to report

the influence probability by quoting one of the  $\frac{1}{1+\epsilon}$  numbers. More concretely, given set  $T = \{1, 2, \dots, t\}$  we define  $z \in \{0, \epsilon, 2\epsilon, \dots, 1\}^t$  such that  $\sum_{i=1}^t z_i = 1$ . For the case of our problem,  $T = \{\text{active}, \text{inactive}\}$ , thus agents will only have to report one number  $\theta_{ij} \in \{0, \epsilon, 2\epsilon, \dots, 1\}$ .

Based on this model we will now design a scoring rule based payment schemes.

## 2.2 A Scoring Rule Based Mechanism

In this mechanism, the payment to an agent  $i$  depends on the truthfulness of the distribution she reveals on edges incident on  $i$  as well as on the edges emanating from  $i$ .

First we state a lemma without proof. The proof appears in the full version of the paper [6]

LEMMA 1. *If  $w, z \in \{0, \epsilon, 2\epsilon, \dots, 1\}^t$ ,  $0 < \epsilon \leq 1$  such that  $\sum_{i=1}^t w_i = 1$  and  $\sum_{i=1}^t z_i = 1$  and  $z_i = w_i \pm \epsilon$  for at least one integer  $1 \leq i \leq t$ , then*

- For quadratic scoring rule

$$V(z|w) \leq V(w|w) - 2\epsilon^2$$

We can derive similar result for the spherical and weighted scoring rule. We develop the mechanism assuming the quadratic scoring rule. A similar development will follow for other proper scoring rules. In the proposed mechanism, the payment received by an agent  $i$  is given by

$$\left( v_i(A, \theta) + \frac{d_i^2}{2\epsilon^2} \right) \left( \sum_{j \in \text{in}(i)} V_{ji}^i(\hat{\theta}_{ji}^j | \hat{\theta}_{ji}^i) + \sum_{j \in \text{out}(i)} V_{ij}^i(\hat{\theta}_{ij}^j | \hat{\theta}_{ij}^i) \right)$$

where  $d_i$  is the degree of agent  $i$ ,  $V_{ij}^i()$  is the expected score that agent  $i$  gets for reporting the distribution  $\hat{\theta}_{ij}^i$  on the edge  $(i, j)$ . We are now in a position to state the main result of this paper. The theorem specifically mentions quadratic scoring rule for the sake of convenience but will hold for any proper scoring rule except the logarithmic scoring rule. Here we only state the result, the full proof appears in [6]

THEOREM 1. Given the influencer-influencee model, reporting true probability distributions is a Nash equilibrium in a scoring rule based mechanism with quadratic scoring rule.

## 2.3 The Reverse Weighted Scoring Rule

Standard proper scoring rules such as quadratic, logarithmic, spherical, and weighted scoring rules have a serious limitation in the current context. If the influence probability on an edge is zero, all these scoring rules will give an expected score of 1. Thus, if the social network is the empty graph in which all the edges are inactive, these standard payment schemes will give maximum possible expected score. We now propose the following *reverse weighted scoring rule* to overcome the above limitation:

$$S_i(z) = 2z_i(t - i) - \sum_{j=1}^t z_j^2(t - j)$$

It can also be shown that the the reverse weighted scoring rule also satisfies the following desirable properties:

1. The expected score is proportional to the influence probability.

2. If  $\theta_{ij} = 0$  then the expected score for the edge  $(i, j)$  to both the agents  $u$  and  $v$  is zero. That is,  $V_{ij}^i(\theta_{ij}^j | \theta_{ij}^i) = V_{ij}^j(\theta_{ij}^i | \theta_{ij}^j) = 0$  if  $\theta_{ij} = 0$ .

Property 1 is desirable because the social planner would want to reward the agent which revealed the social connection through which the product can be sold with high probability. Property 2 ensures that an agent does not get anything for revealing a social connection through which the product cannot be sold.

## 3. SUMMARY AND FUTURE WORK

In this paper, we have proposed mechanisms for eliciting influence probabilities truthfully in a social network. Influence maximization in general and viral marketing in particular are the immediate applications. The work opens up several interesting questions:

- In this model we assumed that the influence probability is known exactly to the agents. We can relax this assumption and assume that agents know the belief probability rather than exact influence probability.
- In the influencer-influencee model, the payments depend on  $\epsilon$  which decides the accuracy of the probability distribution. The higher the accuracy is required, the higher is the payment to be made to the user. An interesting direction of future research would be to design incentive compatible mechanisms that are independent of this factor.

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