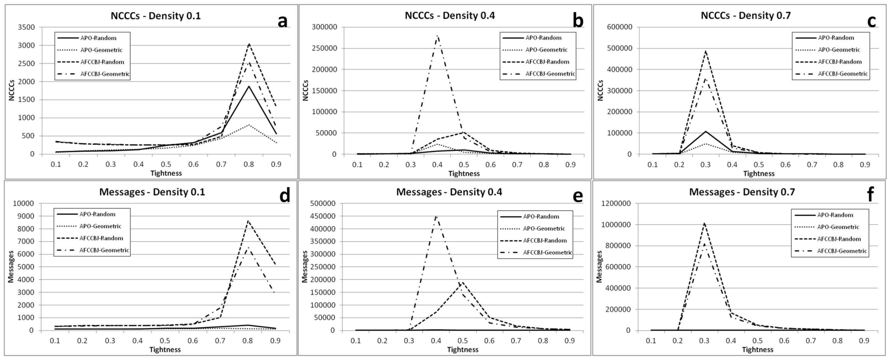

Figure 1: A comparison of AFC-CBJ and APO on 20 variable random and geometrically-structured DisCSPs.

values for p_1 of 0.1, 0.4, and 0.7 and values for p_2 that varied from 0.1 to 0.9. We conducted experiments on both random and geometrically-structured instances and collected 30 samples per data point. Both algorithms were given the exact same problem instances with the same initial variable assignments. We used a cycle based simulator where during each cycle, messages were delivered to the agents, they were allowed to process them, and then queue up messages for delivery at the beginning of the next cycle. During the runs, we counted the number of messages sent and the number of NCCCs used by each protocol.

The results of these experiments can be seen in Figure 1. We see that on low and high density problems, that distributed GS-CSPs on average are easier to solve than random DisCSP instances. This trend reverses for the medium density problems, where it is clear that for both APO and AFC-CBJ that a shifted phase transition has a meaningful effect. This has a particularly profound effect on AFC-CBJ.

Another interesting trend that is noteworthy is that the most recent implementation of APO outperforms AFC-CBJ on all instances for both metrics. In the best case, we found that it used 20X fewer NCCCs than AFC-CBJ. We should note that for time considerations, we were forced to stop some of the runs for AFC-CBJ at 250,000 cycles. This only affected the $p_1 = 0.4, p_2 = 0.4$ results by making them appear somewhat better than the actual values we would obtain if they were allowed to run to completion.

4. CONCLUSIONS AND FUTURE WORK

This paper introduces an important subset of the classical CSP formulation: the geometrically-structured CSP. The GS-CSP is based on the recognition that many real-world problems occur in n-dimensional space and that constraints in these domain are often based on distance. These problems can be represented as geometric graphs, which possess a unique set of properties. By exploring these problems, we have discovered that they are characteristically easier to solve when the density of their constraints are either fairly low or fairly high. However, for medium density problems, they become more difficult to solve than their random counterparts.

Many of these discoveries can be explained by examining the clustering properties of geometric graphs as the edge density increases. GS-CSPs form tightly coupled clusters at low densities, yet remain fairly disconnected overall when compared to random instances. There is considerable work that still remains to be done to fully understand the implications of geometric structure on constraint networks. For example, the experiments in this paper were done using a small number of variables with large domains. This choice was made in part to follow convention. However, we found that using a larger numbers of variables was impractical due to the run times of AFC-CBJ. We believe it is important to look at larger problems, potentially sacrificing the size of the variable's domains, in order to truly understand the consequences that structure has on problem solving complexity and solution optimality.

5. ACKNOWLEDGMENTS

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