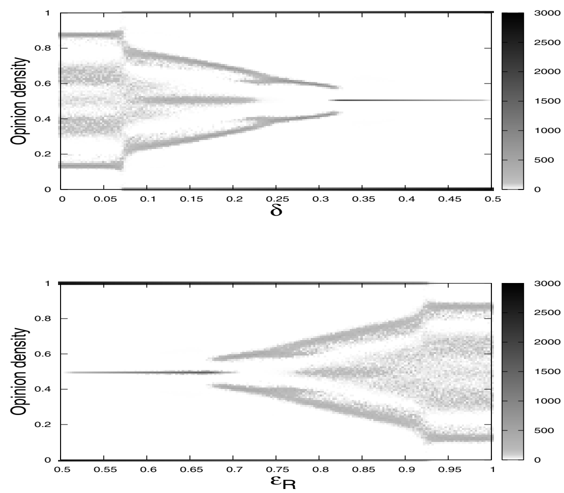

Figure 1: Opinion densities vs δ for M2 (top), ϵ_r for M1 (bottom), 100 runs, $n = 5000$, $\epsilon = 0.1$, $\mu = 0.5$

2.2 M2: Repulsion by Categorization

Repulsion in M2 is influenced by Self-categorization Theory [4]. Individuals in the model have a personal and group identity – the former differentiates an individual from other individuals, the latter from other groups. Repulsion for M2 represents of the increasing salience of the group identity. Two agents, a and b, repulse when:

$$(a^t \leq b^t) \wedge (a^t < \delta) \wedge (b^t > 1 - \delta), \quad (4)$$

assuming $2\delta + \epsilon < 1$; which prevents simultaneous attraction and repulsion. In other words, both agents must be in opposing groups to repulse. Repulsion is implemented through agents offsetting their opinion by:

$$(4) \Rightarrow a^{t+1} = a^t - \mu \left|0 - a^t\right|, b^{t+1} = b^t + \mu \left|1 - b^t\right|. \quad (5)$$

Unlike M1, agents in M2 do not need to "clip" their opinions since they will never go beyond the range [0,1].

3. RESULTS AND DISCUSSION

Figure 1 shows that the dynamics for M1 mirror those of M2. Regions of repulsion form in M1 when $\epsilon_r \geq 0.5$: agents only repulse from one extreme of the opinion spectrum. These "pseudo-groups" act like the groups in M2, yet, groups in M1 are not as rigid as those in M2: in M2, any member of a group may repulse from any member of an opposing group, but repulsion does not always occur between members of pseudo-groups. The interesting similarity between these two applied social theories shows that, when attitudes are split by only two groups, attitudinal judgment acts identically to group judgment.

3.1 Predicting extremist group formation

For a random interaction in model M2 between agents with opinions x and y, with $(x < 1 - \delta) \wedge (x < 1 - \epsilon)$, either $y \in [x, x + \epsilon]$, and x will increase by an average of $\frac{1}{2}\mu\epsilon$; $y \in [\max\{0, x - \epsilon\}, x]$, and x will decrease on average by $\frac{1}{2}\mu \min\{x, \epsilon\}$; $y \in [1 - \delta, 1]$, and x will decrease by μx; or x does not change. We predict that an extremist group will appear if there is a group of agents on the interval $[0, x_0]$, for some $x_0 > 0$, such that the average opinion change for the group is negative. When $x_0 < \delta < \epsilon$, the average opinion change is $\frac{\mu}{2} \int_0^{x_0} (\epsilon - x - 2x)\, \mathrm{d}x = \frac{\mu}{2}\left(\epsilon x_0 - \frac{3x_0^2}{2}\right)$, and the desired condition is satisfied for $x_0 > \frac{2}{3}\epsilon$. Since $x_0 < \delta$, the minimum associated δ value is $\frac{2}{3}\epsilon$. Since x_0 can be increased up to δ, we predict that the extremist group will capture all agents on the interval $[0, \delta]$. Analogous work predicts that extremist groups always form when $\delta > \epsilon$, and never when $x_0 > \delta$. Results for M1 are similar for typical configurations, despite a small dependency on the value of μ.

3.2 Conditions for reduced variability

Experiments showed that large n or small μ reduce variability in cluster position and formation. As $n \to \infty$, the distribution of opinion changes for agents with opinion x will approach (almost surely) the corresponding probability distribution. Large n population behavior can be closely approximated with a deterministic model using discrete opinion-space buckets. As $\mu \to 0^+$, continuous opinion change probability measures are more frequently sampled, and agents will almost surely approach the mean opinion adjustment across possible interactions.

3.3 Cluster formation in M2's middle region

Cluster formation in the middle region is very similar to that of the BC model. In fact, parameter values in M2 can be translated to an attraction threshold in the BC model such that cluster positions from M2 may be mapped to the BC model: $\epsilon_{BC} = \frac{\epsilon}{1-2\delta}$, with the limitation $\frac{\epsilon}{2} < \delta < 0.5 - \epsilon$ (where the middle region forms in M2). The result is that the middle region for select parameter values could be mapped to the BC model with ϵ_{BC}. This provides an interesting outlook on how categorization and lack thereof intermingle to reach a converged system.

3.4 Concluding Remarks

We investigate the similarities in emergent behavior of two distinctly motivated models of opinion adoption, M1 and M2; predict (a) the presence, size, and space consumed by extremist groups (b) the number of opinion clusters in the converged state, and (c) the bias distribution for large populations of agents; and explain decreased variability for large n and low μ. The dual-group setup of M2 is limited, but easily can be expanded to multiple groups.

4. REFERENCES

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