Minimal Concession Strategy for Reaching Fair, Optimal and Stable Marriages

(Extended Abstract)

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Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multi-agent systems

Keywords

Negotiation, Social choice theory, Matching mechanism

1. INTRODUCTION

We are interested in a well-known problem in Computer Science and Economics, the *Stable Marriage Problem* (SM). Considering two communities in which each member has some preferences on the potential partners, the goal is to make pairs taking into account their preferences. This abstract problem has many applications. From a multiagent approach, the seminal Gale-Shapley algorithm [2] solves the SM problem by distinguishing two agent behaviors: a community of proposers and a community of responders [1]. The negotiations between agents lead to a stable solution which is unfair: the community of proposers is favored. In fact, even if the solution given by the Gale-Shapley algorithm is stable, it is the best one for the community of proposers, but the worst for the community of responders. Therefore, we think that this solution is not socially acceptable for a part of the users. In this paper, we propose the $\mathbb{S}$wing method where agents alternatively play the two roles in many bilateral negotiations. Our approach may lead to the emergence of some stable matchings which cannot be reached by the Gale-Shapley algorithm. These matchings are more fair, because they do not favor one community and more optimal for the whole society viewpoint. Then, it is suitable for real-world applications because the solutions are socially acceptable for the users involved in the process.

2. $\mathbb{S}$wing

$\mathbb{S}$wing realizes the minimal concession strategy [5]. Based on this strategy, an agent goes first to its preferred partner. If that fails, the agent concedes, which consists of the withdrawal of its expectation, and so it sends a proposal to the following partners in its preference list. Meanwhile, the potential partners play the role of responder: these agents receive some proposals they can accept or reject depending on their concession levels. When all the agents are married, the $\mathbb{S}$wing method stops.

Each individual is represented by an agent with the following internal state.

Definition 1. *Let SM be a problem of size n. At every moment, the* ***agent*** *$a \in \mathcal{A}$ is represented by a tuple $\langle \sigma, \pi, \kappa, \mu \rangle$ where:*

- *$\sigma \in \{\top, \bot\}$ is the* ***marital status*** *($\top$ if married, $\bot$ if single);*
- *π is its* ***preference list****;*
- *$\kappa \in [0, n]$ is its* ***concession level****;*
- *$\mu \in \mathcal{A} \cup \{\theta\}$ is its current* ***partner*** *(eventually no one).*

We note $\pi(1)$ the most preferred partner, $\pi(2)$ the second most preferred partner, and so on. If $regret(\lambda) = k$, then $\pi(k) = \lambda$. We define the concession level as the maximum rank in the preference list that the agent considers as acceptable at a certain time. $\kappa = 1$ means that the agent focus on its most preferred partner and so the other potential partners are not acceptable. Initially, $\sigma = \bot$, $\kappa = 1$, $\mu = \theta$ for all the agents. The preference lists π are different from one agent to another.

In $\mathbb{S}$wing, men propose and women respond alternatively. In the odd steps, the men play the role of proposers and the women play the role of responders. In the even steps, the roles are swapped (cf Fig. 1). Each proposer sends a proposal to the acceptable partners from the preferred ones to the least preferred ones. As soon as a responder accepts this proposal:

1. if the proposer is married, then it will get divorced, i.e. its previous partner will become free and it will concede;
2. if the responder is married, then it will get divorced, i.e. its previous partner will become free and it will concede;
3. the proposer get married with the responder;

Appears in: *Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2013)*, Ito, Jonker, Gini, and Shehory (eds.), May, 6–10, 2013, Saint Paul, Minnesota, USA.

4. the concession levels of the proposer and the responder become equals to the level of their new partner.

It is worth noticing that, at each step, a proposer stops sending proposals as soon as he gets married. If all the responders reject its proposals, the proposer will concede.

Require: SM a problem with n men X and n women Y
  return M a matching
  $step \leftarrow 0$;
  while $|M| < n$ **do**
    if $step$ is even **then**
      $proposers \leftarrow X$;
      $responders \leftarrow Y$;
    else
      $proposers \leftarrow Y$;
      $responders \leftarrow X$;
    end if
    for all $p \in proposers$ **do**
      for $(i = 1\ ;\ i \leq p.\kappa\ ;\ \text{i++})$ **do**
        $r \leftarrow p.\pi(i)$; {p send a proposal to r}
        if $r.regret(p) \leq r.\kappa$ **then** {r accepts this proposal}
          if $p.\sigma = \top$ **then** {p divorce with $p.\mu$}
            $divorced \leftarrow p.\mu$
            $divorced.\sigma \leftarrow \perp$;
            $divorced.\kappa \leftarrow divorced.regret(p) + 1$
          end if
          if $r.\sigma = \top$ **then** {r divorce with $r.\mu$}
            $divorced \leftarrow r.\mu$
            $divorced.\sigma \leftarrow \perp$;
            $divorced.\kappa \leftarrow divorced.regret(r) + 1$
          end if
          $p.\mu \leftarrow r$
          $p.\sigma \leftarrow \top$;
          $p.\kappa \leftarrow p.regret(r) - 1$;
          $r.\mu \leftarrow p$
          $r.\sigma \leftarrow \top$;
          $r.\kappa \leftarrow r.regret(p) - 1$;
          break;
        else
          {r rejects this proposal}
        end if
      end for
      if $p.\sigma = \perp$ **then**
        $p.\kappa \leftarrow min(p.\kappa + 1, n)$;
      end if
    end for
    $step + +$;
  end while

Figure 1: $\mathbb{S}$wing method

When $\mathbb{S}$wing stops, it reaches a solution for the SM problem: a stable matching.

THEOREM 1. *If the $\mathbb{S}$wing stops, then the matching reached by $\mathbb{S}$wing is stable.*

PROOF 1. *If M is unstable, then it will contain a blocking pair (x_k, y_k). We can deduce that x_k has made a proposal to y_k which has rejected this proposal or y_k has divorced. If it is the case, either y_k is married with a partner which is preferred to x_k, or y_k has divorced and y_k is single. Contradiction.*

3. EVALUATION

We have implemented $\mathbb{S}$wing in order to compare our approach to the existing methods: Gale-Shapley [2], Zig-Zag [6], SML2 [3]. We generate (pseudo-) random instances of SM problems of sizes between $n = 2$ and $n = 100$. We consider 20 different instances for each n. As shown in Table 1, $\mathbb{S}$wing is the only method which reaches stable, fair and optimal outcomes. Additionally, the method $\mathbb{S}$wing is scalable and it can be implemented in a decentralized way with any MAS platform. To our best knowledge, it is not the case for SML2 and Zig-Zag.

Method	Stable	Fair	Optimal	Scalable	MAS
GS [2]	✓	✗	✗	✓	✓
SML2 [3]	✗	✓	✓	✗	✗
Zig-Zag [6]	✗	✓	✓	✓	✗
$\mathbb{S}$wing	✓	✓	✓	✓	✓

Table 1: Methods for the SM problem

4. CONCLUSIONS

We have proposed in [4] a fully decentralized method which describes the behavior of agents alternatively playing the role of proposers and responders. In this paper, our proposal differs from our previous paper since we introduce here a synchronized version of our algorithm. This improvement is a step toward an algorithm which is proved to terminate. In this paper, we have proposed a method which leads to a stable solution which is more equitable and more optimal for the whole society viewpoint. Furthermore, this method can be implemented over any MAS platform.

5. ACKNOWLEDGMENTS

We thank the reviewers for their helpful comments.

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