VCG-equivalent in Expectation Mechanism: General Framework for Constructing Iterative Combinatorial Auction Mechanisms

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ABSTRACT

In this paper, we develop a new class of iterative mechanisms called a *VCG-equivalent in expectation* mechanism. Iterative auctions are preferred over their sealed-bid counterparts in practical settings, since they can avoid full revelation of type information. However, to guarantee that sincere strategies are an *ex post* equilibrium, the mechanism needs to achieve exactly the same outcome as the Vickrey-Clarke-Groves (VCG) mechanism. To guarantee that a mechanism is VCG-equivalent, it inevitably asks an irrelevant query, in which a participant has no incentive to answer the query sincerely. Such an irrelevant query causes unnecessary leakage of private information and a different incentive issue. In a VCG-equivalent in expectation mechanism, the mechanism achieves the same allocation as VCG, but the transfers are the same as VCG only in expectation. We show that in a VCG-equivalent in expectation mechanism, sincere strategies constitute a sequential equilibrium. Also, we develop a general procedure for constructing a VCG-equivalent in expectation mechanism that does not ask any irrelevant query. To demonstrate the applicability of this idea in a practical application, we develop a VCG-equivalent in expectation mechanism that can be applied to the Japanese 4G spectrum auction.

Categories and Subject Descriptors

I.2.11 [**ARTIFICIAL INTELLIGENCE**]: Distributed Artificial Intelligence – Multiagent systems

General Terms

Algorithms, Economics, Theory

Keywords

mechanism design, VCG mechanism, ascending auction, iterative auction

1. INTRODUCTION

Iterative auctions, which include ascending price auctions, are preferred over their sealed-bid counterparts in practical settings, since they provide a process to discover the type/valuation of each participant and can avoid full revelation of the types of some participants [14]. In designing an iterative auction mechanism, we need to consider participants' incentive issues. Ideally, sincere strategies should be an *ex post* equilibrium, i.e., for each participant, using a sincere strategy is best as long as other participants use sincere strategies. In a general setting where we do not put any specific assumptions on participants' types, such an ex post equilibrium can be achieved only when the iterative auction mechanism achieves exactly the same outcome as the well-known sealed-bid Vickrey-Clarke-Groves (VCG) mechanism [7, 11, 16], if the mechanism must achieve a Pareto efficient allocation [10, 13].

However, to achieve exactly the same outcome as VCG, a mechanism tends to ask too many irrelevant queries, i.e., although the mechanism has already gathered relevant information to identify a Pareto efficient allocation, it needs to obtain additional information to identify the VCG transfers [8, 15]. Asking irrelevant queries can be problematic to prevent unnecessary leakage of private information.

Furthermore, it can cause another incentive issue. A participant does not have a strong incentive to answer an irrelevant query sincerely. Thus, she might answer it randomly because of laziness. She can also have a slight incentive to pretend her true type/valuation differently. For example, in a standard auction, if the true valuation of a winner, as well as her VCG transfer, becomes public, her net utility also becomes public. When her net utility is very high, she might have a problem making subcontracts related to the auction, or she can be accused by the general public. Thus, it is reasonable to assume that as long as her utility (i.e., her allocation and transfer) is the same, she prefers to declare lower valuation. When only one good is sold, the English auction works fine since it does not ask any irrelevant query and can hide the exact valuation of the winner. However, in a combinatorial auction, a mechanism needs to know the exact valuation of the winner to calculate the transfers to other winners.

There can be a situation that a participant prefers to declare higher valuation. Let us assume two participants are joining a charity auction. Then, it is possible that a participant prefers to declare higher valuation as long as her utility is the same. The English auction does not work well in this situation. Here, the winner is proud to announce her higher valuation, while the loser awkwardly reveal her lower valuation public. We can use a modification of Dutch/descending auction as follows. If one participant says "stop" at a certain price, the auction does not close immediately. The price

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continues to fall, until the other participant says "stop". The first participant wins the good at the price where the second participant said "stop". In principle, sincere strategies constitute an ex post equilibrium in this mechanism, but they do not work in practice if the loser has slight incentive to pretend that her valuation is higher; she says "stop" immediately after the winner. This modified Dutch/descending auction might look ridiculous, but any iterative auction mechanism inevitably asks such a ridiculous query to calculate the VCG transfers.

One way to avoid such an incentive issue is to hide the bid information from participants or to obscure the auction procedure. If a participant lacks information about the previous queries/answers of other participants, or she does not know how queries are ordered in advance, she cannot tell whether the current query is irrelevant. Actually, Vickrey [16] presents a mechanism that is identical to the modified Dutch/descending auction, but in his mechanism, the information that the first participant said "stop" is hidden from other participants. However, if we are designing an auction mechanism for public property such as spectrum rights, the mechanism should be highly transparent; hiding bid information or obscuring the auction process is not desirable.

In summary, we are confronted with the following dilemma. If we want to avoid an incentive issue, a mechanism needs to ask an irrelevant query. Such an irrelevant query causes unnecessary leakage of private information and a different incentive issue. To avoid this dilemma, we use the following simple idea, namely, we give up achieving exactly the same outcome as VCG. Instead, we achieve an outcome that is equivalent to VCG only in expectation. To be more precise, our mechanism achieves exactly the same allocation as VCG (a Pareto efficient allocation). However, the transfers are expected values of the VCG transfers. For example, in the above Dutch/descending auction, we modify the mechanism so that the participant who said "stop" wins the good, but she pays the expected valuation of the other participant. If we assume the valuation is uniformly distributed, she pays half of the price where she said "stop".

Since we rely on expected values, we can no longer guarantee that sincere strategies constitute an ex post equilibrium. We switch to a weaker, but one of the most refined equilibrium concepts in dynamic games with imperfect information: a sequential equilibrium [12]. To make a rigorous examination, we first define a simple model of iterative auction mechanism called Binary Decision Tree (BDT) based mechanism. In this model, at one node of a binary decision tree, the mechanism chooses one participant and makes a "yes/no" type query regarding her type, e.g., "Does your valuation for this good exceeds 100 dollars?". When the mechanism obtains enough information about the types of participants, it decides the allocation of goods and the monetary transfers/payments. This model assumes *full bid information*, i.e., the complete history of all actions/bids by all participants are available for all participants.

We show that a mechanism that always achieves the same outcome as VCG (which we call a VCG-equivalent mechanism) will inevitably ask an irrelevant query. We then prove that in a *VCG-equivalent in expectation* mechanism, sincere strategies can constitute a sequential equilibrium. If it satisfies an additional condition on the query asked at each node, it will not ask any irrelevant query. Furthermore, we develop a general procedure for constructing a VCG-equivalent in expectation mechanism that does not ask any irrelevant query. This procedure is flexible so that it can produce a variety of mechanisms, including ascending/descending price auctions, or binary search based mechanisms. Furthermore, to demonstrate the applicability of our idea to a practical application, we develop a mechanism that can be applied to the Japanese 4G spectrum auction based on the idea of VCG-equivalent in expectation mechanisms.

2. RELATED WORKS

There exists a vast amount of works related to ascending price combinatorial auctions [1, 2, 3, 10, 13], as well as preference elicitation and communication costs in combinatorial auctions [5, 6, 8, 15]. Compared to more elaborated models used in the literature of preference elicitation such as [8, 15], our BDT-based model is rather abstract and would not be powerful enough for addressing research issues in the preference elicitation literature. However, it is simple and convenient for checking equilibria, and general enough to model various mechanisms including ascending price auctions.

In the mechanism design literature, there exist several works that utilize expected values. Our mechanism is inspired by the well-known expected-externality mechanism [9], which achieves Bayesian Nash incentive compatibility, Pareto efficiency, and budget balancedness. Also, instead of using a truthful deterministic mechanism, a mechanism can be randomized so that it becomes truthful in expectation, while the computational or communication complexity of the randomized mechanism can be lower compared to the original deterministic mechanism [4].

3. MODEL

We use the following model of combinatorial auctions. A set of indivisible goods $M = \{1, \ldots, m\}$ is allocated to a set of participants $N = \{1, \ldots, n\}$. Participant i privately observes her type θ_i, which is chosen independently from Θ_i. For simplicity, we assume Θ_i is a finite, discrete set. For $B \subseteq M$, $v(\theta_i, B)$ represents the gross utility of a participant, whose type is θ_i and she obtains B. We assume v is normalized by $v(\theta_i, \emptyset) = 0$, and satisfies free-disposal, i.e., for any $B \subseteq B'$, $v(\theta_i, B) \leq v(\theta_i, B')$ holds.

We assume quasi-linear utility, i.e., if a participant, whose type is θ_i, obtains goods B and a monetary transfer t_i, her utility is given as $v(\theta_i, B) + t_i$. Furthermore, we assume $p(\theta_i) > 0$ denotes the probability that the type of participant i is θ_i, and $\sum_{\theta_i \in \Theta_i} p(\theta_i) = 1$ holds. Also, for $\Theta'_i \subseteq \Theta_i$, we define $p(\Theta'_i)$ as $\sum_{\theta_i \in \Theta'_i} p(\theta_i)$. Θ denotes $\prod_{i \in N} \Theta_i$ and Θ_{-i} denotes $\prod_{j \neq i} \Theta_j$.

Furthermore, $\theta = (\theta_1, \ldots, \theta_n) \in \Theta$ denotes the profile of the types of all participants. Also, $\theta_{-i} \in \Theta_{-i}$ denotes the profile of the types of all other participants than i. (θ'_i, θ_{-i}) denotes the type profile here i's type is θ'_i and the type profile of the other participants is θ_{-i}. $p(\theta)$ is defined as $\prod_{i \in N} p(\theta_i)$, and $p(\theta_{-i})$ is defined as $\prod_{j \neq i} p(\theta_j)$.

Let $\mathcal{X}$ denote a set of feasible allocations. A feasible allocation $X \in \mathcal{X}$ is written as $X = (X_1, \ldots, X_n)$, where each $X_i \subseteq M$ denotes the allocation to participant i. For feasible allocation X, $X_i \cap X_j = \emptyset$ holds for any $i \neq j$. We say an allocation $X \in \mathcal{X}$ is Pareto efficient for θ if there does not exist another allocation $X' \in \mathcal{X}$ such that $\sum_{i \in N} v(\theta_i, X'_i) > \sum_{i \in N} v(\theta_i, X_i)$.

A direct revelation mechanism consists of an allocation function $a : \Theta \rightarrow \mathcal{X}$ and a transfer function $t : \Theta \rightarrow \mathcal{R}^n$. An allocation function $a(\theta) = (a_1(\theta), \ldots, a_n(\theta))$ maps the declared types to an allocation. a transfer function $t(\theta) = (t_1(\theta), \ldots, t_n(\theta))$ maps them to monetary transfers. The VCG mechanism is defined by the following allocation function a^* and transfer function t^*.

Definition 1 (VCG mechanism). *$a^*(\theta) = X$, where $X \in \mathcal{X}$ is a Pareto efficient allocation for θ.*
$t_i^*(\theta) = \sum_{j \neq i} v(\theta_j, X_j) - \sum_{j \neq i} v(\theta_j, X'_j)$, where $X' \in \mathcal{X}$ is an allocation that maximizes $\sum_{j \neq i} v(\theta_j, X'_j)$.

Let us consider a very simple example as follows.

Example 1. *There are two goods $M = \{1, 2\}$ and three participants $N = \{1, 2, 3\}$. Participant 1 requires good 1, participant 2 requires good 2, and participant 3 requires both good 1 and 2 (only having good 1 or good 2 is meaningless). Since these valuations are one-dimensional, we represent the type of each participant by her valuation. We assume $\Theta_1 = \{2, 4, 6, 8\}$, $\Theta_2 = \{2, 4, 6, 8\}$, and $\Theta_3 = \{9\}$. We assume each type is chosen independently and uniformly at random, i.e., the probability that each type is chosen (for participant 1 and 2) is 1/4. If $\theta = (2, 2, 9)$, $a^*(\theta) = (\emptyset, \emptyset, \{1, 2\})$ and $t^*(\theta) = (0, 0, -4)$. If $\theta = (8, 8, 9)$, $a^*(\theta) = (\{1\}, \{2\}, \emptyset)$ and $t^*(\theta) = (-1, -1, 0)$.*

4. BINARY DECISION TREE

Let us introduce a (full) Binary Decision Tree (BDT) based mechanism that represents an iterative auction mechanism.

Definition 2 (BDT-based mechanism). *Given N, M, and Θ, a binary decision tree (BDT) based mechanism is defined as $\langle d_{\rm r}, D_{\rm in}, D_{\rm l}, int, q, \bar{a}, \bar{t} \rangle$. $D_{\rm in}$ and $D_{\rm l}$ are a set of internal and leaf nodes, respectively. $d_{\rm r} \in D_{\rm in}$ indicates the root node. Each node $d \in D_{\rm in} \cup D_{\rm l}$ has its parent node $p(d) \in D_{\rm in}$. Also, each node $d \in D_{\rm in}$ has its left child $lc(d)$ and its right child $rc(d)$, where $lc(d), rc(d) \in D_{\rm in} \cup D_{\rm l}$. If $lc(d) = d'$ then $p(d') = d$ holds, while if $rc(d) = d'$ then $p(d') = d$ holds.*

For $d \in D_{\rm in}$, $int(d) \in N$ gives an interrogee, and $q(d) \subset \Theta_{int(d)}$ gives a query. $\bar{a}$ is an allocation function, and $\bar{t}$ is a transfer function.

At each node d, one participant $int(d) \in N$ is asked a query whether her type belongs to $q(d) \subset \Theta_{int(d)}$. The input of node d is a possibly degenerated n-dimension hypercube $\Theta^d = \prod_{i \in N} \Theta_i^d$, where each $\Theta_i^d \subseteq \Theta_i$, Here, we represent $\prod_{j \neq i} \Theta_j^d$ as Θ_{-i}^d.

Definition 3 (node input). *The input of node d is defined as follows:*

- *For $d = d_{\rm r}$, its input is Θ.*
- *For $d \neq d_{\rm r}$, where $p(d) = d'$, $int(d') = i$, $q(d') = \Theta'_i$, d's input Θ^d is $\Theta'_i \times \Theta_{-i}^{d'}$ if $d = lc(d')$, and $(\Theta_i^{d'} \setminus \Theta'_i) \times \Theta_{-i}^{d'}$ if $d = rc(d')$.*

Starting from $d_{\rm r}$, the mechanism moves to the left child if the answer of the query $q(d)$ is "yes". It moves to the right child if the answer of the query is "no". Input Θ^d is divided into two hypercubes by a hyperplane according to the query.

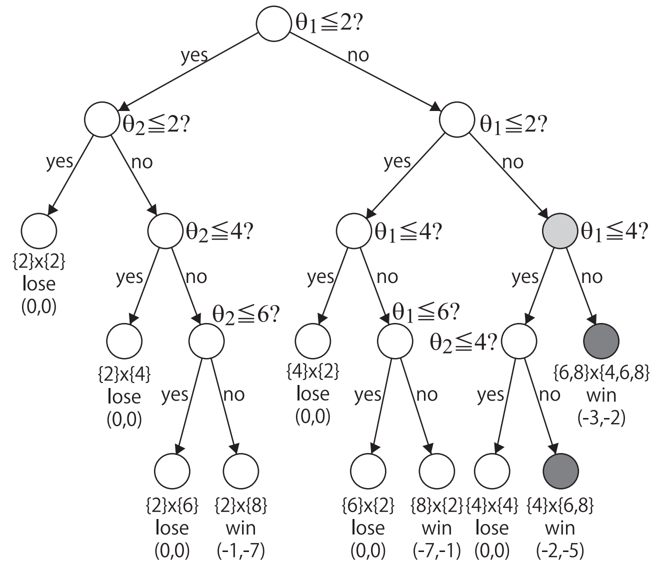

Figure 1: Example of Binary Decision Tree

We assume queries are defined so that the input of each node cannot be an empty set, i.e., for each node d, where $i = int(d)$, $q(d)$ must be a proper subset of Θ_i^d.

At leaf node $d \in D_{\rm l}$, the mechanism determines the outcome of the auction, which is defined as $\bar{a}(\Theta^d)$ and $\bar{t}(\Theta^d)$, where $\bar{a}(\cdot)$ is an allocation function and $\bar{t}(\cdot)$ is a transfer function, each of which takes $\Theta^d \subseteq \Theta$ as an argument.

Figure 1 shows an example of a BDT-based mechanism, which is applied to the combinatorial auction described in Example 1. Since the type of participant 3 is constant, we only describe possible types, allocations, and transfers for participant 1 and 2. In the leaf node, "win" means good 1 is allocated to participant 1, and good 2 is allocated to participant 2, respectively. On the other hand, "lose" means both goods are allocated to participant 3. We can assume this BDT-based mechanism corresponds to an ascending price auction, in which the price of good 1 increases first, then the price of good 2 increases, and so on. The input of the root node is $\{2, 4, 6, 8\} \times \{2, 4, 6, 8\}$. This hypercube (actually, a rectangle) is divided into two parts by the query $(\theta_1 \leq 2?)$, i.e., $\{2\} \times \{2, 4, 6, 8\}$ and $\{4, 6, 8\} \times \{2, 4, 6, 8\}$. The first part becomes the input of the left child, and the second part does the input of the right child. In the left child of the root node, again, this hypercube $\{2\} \times \{2, 4, 6, 8\}$, which is degenerated in the first dimension, is divided into two parts by the query $(\theta_2 \leq 2?)$: $\{2\} \times \{2\}$ and $\{2\} \times \{4, 6, 8\}$, and so on.

In this mechanism, it is possible to decide the allocation and transfer even when the type profile of the participants are not uniquely determined. For example, in the rightmost leaf node, the possible types of participant 1 are $\{6, 8\}$, and the possible types of participant 2 are $\{4, 6, 8\}$.

First, let us define a VCG-equivalent mechanism.

Definition 4 (VCG equivalence). *We say a BDT-based mechanism is VCG equivalent, if for each leaf node d, $\forall \theta, \theta' \in \Theta^d$, $a^*(\theta) = a^*(\theta') = \bar{a}(\Theta^d)$ and $t^*(\theta) = t^*(\theta') = \bar{t}(\Theta^d)$ hold. In other words, at each leaf node d, $\forall \theta \in \Theta^d$, the values of the VCG allocation/transfer function are the same, and the mechanism uses them.*

The mechanism described in Fig. 1 is almost VCG-equivalent, i.e., the allocation and transfers are identical to VCG in most of the leaf nodes, except the shaded nodes (i.e., the right-most and the second nodes from the right).

Next, let us introduce our newly developed framework called VCG-equivalent in expectation mechanism.

Definition 5 (VCG equivalence in expectation). *We say a BDT-based mechanism is VCG equivalent in expectation, if for each leaf node d, $\forall \theta, \theta' \in \Theta^d$, $a^*(\theta) = a^*(\theta') = \bar{a}(\Theta^d)$ holds. In other words, at each leaf node d, there exists an allocation that is Pareto efficient for all $\theta \in \Theta^d$, and the mechanism uses this allocation. The transfer function $\bar{t}(\Theta^d)$ is defined as follows:*

$$\bar{t}_i(\Theta^d) = \sum_{\theta_{-i} \in \Theta^d_{-i}} t^*_i((\theta^d_i, \theta_{-i})) \times \frac{p(\theta_{-i})}{p(\Theta^d_{-i})}$$

Here, θ^d_i can be an arbitrary type in Θ^d_i.

Note that in a VCG-equivalent in expectation mechanism, at each leaf node d, $\forall \theta_i, \theta'_i \in \Theta^d_i, \forall \theta_{-i} \in \Theta^d_{-i}, t^*_i((\theta_i, \theta_{-i})) = t^*_i((\theta'_i, \theta_{-i}))$ holds. $\bar{t}_i(\Theta^d)$ corresponds to the expected transfer of VCG, where the type of participant i is any $\theta_i \in \Theta^d_i$ and the type profile of other participants is within Θ^d_{-i}.

The mechanism described in Fig. 1 is an example of a VCG-equivalent in expectation mechanism. In the right-most node, the allocation is identical for any type profile in $\{6, 8\} \times \{4, 6, 8\}$. Here, if we fix the type of participant 2 to 4, the VCG transfer for participant 1 is determined independently from 1's type. The VCG transfer for participant 1 becomes -5, -3, or -1, according to the type of participant 2 (4, 6, or 8, respectively). For each type of $\theta_2 \in \{4, 6, 8\}$, $p(\theta_2)$ is equal to 1/4, and $p(\{4, 6, 8\}) = 3/4$. Thus, the transfer for participant 1 is given as: $\frac{-5}{3} + \frac{-3}{3} + \frac{-1}{3} = -3$. Similarly, in this node, the VCG transfer for participant 2 becomes -3, or -1, according to the type of participant 1 (6, or 8, respectively). For each type of $\theta_1 \in \{6, 8\}$, $p(\theta_1)$ is equal to 1/4, and $p(\{6, 8\}) = 2/4$. Thus, the transfer for participant 2 is given as: $\frac{-3}{2} + \frac{-1}{2} = -2$.

When executing the BDT-based mechanism, we assume the information of the BDT is announced to participants beforehand. Also, the answer of each query is observable for other participant, i.e., this model assumes full bid information. These assumptions are desirable to make a mechanism as transparent as possible. Also, finding a profitable strategic behavior becomes more difficult if a participant has less information. In this paper, we examine incentive issues in a challenging setting where all information gathered during the mechanism execution becomes public.

In the BDT-based mechanism, we assume at one node, only one query for one participant will be asked. This assumption is just to make the description of the mechanism simple. Extending this model so that it can handle multiple simultaneous queries is straightforward. Also, we assume that the set of possible types Θ_i is a discrete, finite set. This is another assumption for simplicity. We can easily extend the idea of a VCG-equivalent in expectation mechanism to the case where each type space is continuous. However, when the valuations of two competing participants are very close, to determine an efficient allocation, the mechanism needs to ask too many queries.

5. CHARACTERISTICS OF BDT-BASED MECHANISMS

In this section, we examine the characteristics of VCG-equivalent mechanisms and VCG-equivalent in expectation mechanisms. First, we are going to introduce some more terms and concepts. We say node d is targeted to participant i if $int(d) = i$. Furthermore, for participant i and type θ_i, we say node d is compatible with (i, θ_i), if d is targeted to i and $\theta_i \in \Theta^d_i$ holds.

Now, let us formally define a strategy in this model and properties related to strategies.

Definition 6 (Strategy). *A strategy of participant i, which is denoted as s_i, is a mapping from each node d that is targeted to i, to "yes/no".*

Definition 7 (Consistent report). *For participant i and type θ_i, we say a strategy s_i reports consistently with (i, θ_i) at node d that is compatible with (i, θ_i), if $\theta_i \in q(d)$, then $s_i(d)$ is "yes", otherwise $s_i(d)$ is "no".*

Definition 8 (Consistent strategy). *For participant i and type θ_i, we say a strategy s_i is consistent with respect to (i, θ_i), if for each node d, where d is compatible with (i, θ_i), s_i reports consistently with (i, θ_i).*

Definition 9 (Sincere strategy). *We say a strategy s_i is sincere for participant i, whose true type is θ_i, if it is consistent with respect to (i, θ_i).*

We assume for each participant i, her initial belief that the type of participant j is θ_j, is given as $p(\theta_j)$. Then, the initial belief is updated as follows.

Definition 10 (Updated belief). *Assume each participant j, where $j \neq i$, uses a sincere strategy s_j. For participant i and node d that are targeted to i, we assume her updated belief that the type of participant $j \neq i$ is θ_j at node d, is given as $p(\theta_j \mid d) = p(\theta_j)/p(\Theta^d_j)$.*

At node d, the possible types of participant j are reduced from Θ_j to Θ^d_j. Thus, the updated belief is given as $p(\theta_j)/p(\Theta^d_j)$ by Bayes' rule. For example, at the hatched node in Fig. 1, the updated belief of participant 1 for the type of participant 2 will be a discrete uniform distribution over three possible types $\{4, 6, 8\}$

Definition 11 (Sincere+ strategy). *We say a sincere strategy s_i for participant i, whose true type is θ_i, is sincere+, if for each node d, which is not compatible with (i, θ_i), s_i chooses its action $s_i(d)$ so that the expected utility is maximized under the updated belief at node d, assuming each participant j, where $j \neq i$, uses a sincere strategy s_j.*

Assuming other participants use sincere strategies, by using backward induction, for each node d, which is not compatible with (i, θ_i), we can decide an appropriate action $s_i(d)$ so that the expected utility is maximized. For example, for participant 1, whose type is 2, at the hatched node in Fig. 1, she should say "yes", since if she says "yes", with probability 1/3, she loses, and with probability 2/3, she wins good 1 with transfer -2. Thus, her expected utility is 0. If she says "no", she wins good 1 by transfer -3. Thus, her expected utility is -1. Therefore, her expected utility is larger when she says "yes".

Since the formal definition of a sequential equilibrium is quite involved [12], we show a rather abstract definition.

Definition 12 (sequential equilibrium). *We say a profile of strategies and beliefs at each decision point of participants constitutes a sequential equilibrium, if the following conditions are satisfied: (a) each strategy is sequentially rational, i.e., at each decision point, it maximizes her expected payoff, given her belief at the decision point and subsequent strategy combination, (b) the beliefs satisfy the condition called consistent assessment property, which means that beliefs are obtained by Bayes' rule if possible, and for an off-equilibrium path where Bayes' rule is not applicable, the beliefs are given by Bayes' rule applied to a slightly perturbed strategy profile (under which all nodes are reached with positive probabilities).*

Theorem 1. *In a VCG-equivalent mechanism, a profile of sincere strategies constitutes an ex post equilibrium, i.e., a sincere strategy is a best response assuming other participants also use sincere strategies. Also, a Pareto efficient allocation is achieved at the equilibrium and no participant ever suffers any loss as long as she uses a sincere strategy.*

We omit the proof due to space limitation.

Let $E(s_i, \theta_i \mid d)$ denote the expected utility of participant i, whose type is θ_i, assuming i uses strategy s_i, the current node is d, and each participant $j \neq i$ uses a sincere+ strategy. We use the following lemma in the proof of Theorem 2.

Lemma 1. *In a VCG-equivalent in expectation mechanism, $E(s_i, \theta_i \mid d)$, where d is compatible with respect to (i, θ_i), is identical to the expected utility of VCG, given the updated belief at node d, when all participants declare their true types. More specifically, it is given as follows:*

$$\sum_{\theta_{-i} \in \Theta^d_{-i}} \frac{u_i^*(\theta_i, (\theta_i, \theta_{-i})) \times p(\theta_{-i})}{p(\Theta^d_{-i})}.$$

Here, $u_i^*(\theta_i, (\theta_i', \theta_{-i}))$ denotes the utility of participant i in VCG, where i's true type is θ_i, and i's declared type is θ_i', and the declared type profile of other participants are θ_{-i}.

We omit the proof due to space limitation, but it is intuitively natural that the VCG-equivalent in expectation mechanism achieves the same expected utility as VCG.

Theorem 2. *In a VCG-equivalent in expectation mechanism, the combination of sincere+ strategies and beliefs updated for each node constitute a sequential equilibrium. At the sequential equilibrium, a Pareto efficient allocation is achieved.*

Proof. To show that the strategy profile and beliefs constitute a sequential equilibrium, we need to show that (a) each strategy is sequentially rational, and (b) the beliefs satisfy the consistent assessment property.

Regarding (b), for each node d, which is targeted to participant i, the belief given as Definition 10 satisfies the consistent assessment property. If node d is compatible with (i, θ_i), the belief is given by Bayes' rule. If node d is incompatible with (i, θ_i), the belief is identical to the belief given by Bayes' rule, assuming each participant chooses an wrong answer with a very small probability.

Next, we show that a sincere+ strategy is sequentially rational. To be more precise, assume s_i is a sincere+ strategy and s_i' is any strategy, we need to show $E(s_i, \theta_i \mid d) \geq E(s_i', \theta_i \mid d)$ holds for each node d that is targeted to i.

If d is not compatible with θ_i, from the definition of a sincere+ strategy, this condition is automatically satisfied. Thus, let us assume d is compatible with θ_i.

From Lemma 1, $E(s_i, \theta_i \mid d)$ is given as follows:

$$\sum_{\theta_{-i} \in \Theta^d_{-i}} \frac{u_i^*(\theta_i, (\theta_i, \theta_{-i})) \times p(\theta_{-i})}{p(\Theta^d_{-i})}. \quad (1)$$

Next, let us examine $E(s_i', \theta_i \mid d)$. Let $lv(d)$ denote a set of all the leaf nodes of a sub-tree where d is the root node. Depending on s_i', a subset of $lv(d)$ is reachable from d. Let us represent this subset as $D' \subset lv(d)$. $E(s_i', \theta_i \mid d)$ can be calculated as follows.

$$E(s_i', \theta_i \mid d) = \sum_{d' \in D'} (v(\theta_i, \bar{a}_i(\Theta^{d'})) + \bar{t}_i(\Theta^{d'})) \times \frac{p(\Theta^{d'}_{-i})}{\sum_{d' \in D'} p(\Theta^{d'}_{-i})}$$

We can rewrite $v(\theta_i, \bar{a}_i(\Theta^{d'})) + \bar{t}_i(\Theta^{d'})$ as:

$$\sum_{\theta_{-i} \in \Theta^{d'}_{-i}} u_i^*(\theta_i, (\theta_i^{d'}, \theta_{-i})) \times \frac{p(\theta_{-i})}{p(\Theta^{d'}_{-i})}.$$

For each $\theta_{-i} \in \Theta^d_{-i}$, there exists exactly one $d' \in D'$ such that $\theta_{-i} \in \Theta^{d'}_{-i}$. Thus, $\bigcup_{d' \in D'} \Theta^{d'}_{-i} = \Theta^d_{-i}$ holds. Thus, we can rewrite $E(s_i', \theta_i \mid d)$ as follows.

$$\sum_{d' \in D'} \sum_{\theta_{-i} \in \Theta^{d'}_{-i}} \frac{u_i^*(\theta_i, (\theta_i^{d'}, \theta_{-i})) \times p(\theta_{-i})}{p(\Theta^d_{-i})}. \quad (2)$$

Assume $E(s_i', \theta_i \mid d) > E(s_i, \theta_i \mid d)$ holds. Then, from Equations (1) and (2), each of which takes a summation over $\theta_{-i} \in \Theta^d_{-i}$, there must be at least one θ_{-i} such that the following equation holds.

$$u_i^*(\theta_i, (\theta_i^{d'}, \theta_{-i})) > u_i^*(\theta_i, (\theta_i, \theta_{-i})).$$

However, since VCG is strategy-proof, it is not possible. Thus, $E(s_i, \theta_i \mid d) \geq E(s_i', \theta_i \mid d)$ holds.

Also, it is clear that if all participants use sincere+ strategy, the obtained allocation is Pareto efficient. □

Theorem 3. *In a VCG-equivalent in expectation mechanism, when participant i uses a sincere strategy, she never suffers any loss.*

Proof. Assume that participant i uses a sincere strategy and that leaf node d_l is achieved. Using a similar argument to the proof of Theorem 2, the utility of participant i is given as follows:

$$\sum_{\theta_{-i} \in \Theta^{d_l}_{-i}} \frac{u_i^*(\theta_i, (\theta_i, \theta_{-i})) \times p(\theta_{-i})}{p(\Theta^{d_l}_{-i})}.$$

Since each $u_i^*(\theta_i, (\theta_i, \theta_{-i}))$ is non-negative, the summation is also non-negative. □

If a participant obtains additional information, she might have an incentive to deviate from a sincere strategy. For example, if participant 1, whose type is 8, learns that participant 2's type is 8 from some outside source, she can increase her transfer from -3 to -1 by saying "yes" at the root node.

Next, we examine whether a BDT-based mechanism makes an irrelevant query or not, by introducing a concept called irrelevant node.

Definition 13 (Irrelevant node). *For participant i, we say node d, where $int(d) = i$, is irrelevant to i, iff $\forall \theta_{-i} \in \Theta^d_{-i}, \forall \theta_i, \theta'_i \in \Theta^d_i$, $\bar{a}(\Theta^{d_l}) = \bar{a}(\Theta^{d'_l})$ and $\bar{t}_i(\Theta^{d_l}) = \bar{t}_i(\Theta^{d'_l})$ hold, where d_l and d'_l are leaf nodes such that $\Theta^{d_l}_i \ni (\theta_i, \theta_{-i})$ and $\Theta^{d'_l}_i \ni (\theta'_i, \theta_{-i})$ hold.*

If node d is irrelevant to i, i has no incentive to answer the query seriously, since her answer does not affect the total allocation and her own transfer (which are uniquely determined according to θ_{-i}, i.e., other participants' types). Thus, it is desirable that a binary decision tree does not contain any irrelevant node.

Theorem 4. *For some N, M, and Θ, there exists no VCG-equivalent mechanism without an irrelevant node.*

Proof. We show that for the combinatorial auction described in Example 1, such a mechanism does not exist. Let us assume such a mechanism exists. Since this mechanism is VCG-equivalent, there exists a leaf node d_l, such that $\Theta^{d_l}_1 = \{8\}$ and $\Theta^{d_l}_2 = \{8\}$ hold. This is because the VCG transfers in this situation, i.e., $(-1, -1, 0)$ is unique. There must be node d on the path from the root node to d_l, such that $ind(d) = 1$, and $q(d)$ or $\Theta^d_1 \setminus q(d)$ is equal to $\{8\}$. Also, there must be node d' on the path from the root node to d_l, such that $ind(d') = 2$, and $q(d')$ or $\Theta^{d'}_2 \setminus q(d')$ is equal to $\{8\}$. Let us assume that d is closer to d_l than d'. Then, d is irrelevant to participant 1. This is because at any leaf node $d'_l \in lv(d)$, the type of participant 2 is 8. Thus, the VCG allocation is the same (i.e., both participant 1 and 2 win), and the VCG transfers for participant 1 is the same (i.e., she pays 1). If d' is closer to d_l than d, then d' is irrelevant to participant 2. This is a contradiction. □

Now, we are going to show a general procedure for constructing a VCG-equivalent in expectation mechanism that has no irrelevant node. Let us introduce a concept called *indifferent set*.

Definition 14 (Indifferent set). *For a (possibly degenerated) n-dimension hypercube $\Theta' = \prod_{i \in N} \Theta'_i$, where each $\Theta'_i \subseteq \Theta_i$, we say $\Theta^{ind}_i \subseteq \Theta'_i$ is i's indifferent set for Θ', iff $\forall \theta_i, \theta'_i \in \Theta^{ind}_i$, $\forall \theta_{-i} \in \prod_{j \neq i} \Theta'_j$, $a^*((\theta_i, \theta_{-i})) = a^*((\theta'_i, \theta_{-i}))$ holds.*

Also, we say Θ^{ind}_i is maximal if there exists no indifferent set that is a strict superset of Θ^{ind}_i.

In Example 1, if $\Theta' = \Theta'_1 \times \Theta'_2 = \{4, 6, 8\} \times \{4, 6, 8\}$, for participant 1, $\{6, 8\}$ is an indifferent set, since for each possible type of participant 2, the Pareto efficient allocations and her transfers are the same.

Note that for Θ', it is possible that there exists no i's indifferent set, or there exist multiple maximal indifferent sets. However, if there exist multiple maximal indifferent sets, they must be disjoint (since "indifference" is an equivalence relation).

The following theorem shows a sufficient condition that a node is relevant to i.

Theorem 5. *In a VCG-equivalent in expectation mechanism, node d, where $i = int(d)$, is relevant to i, if $\forall \Theta^{ind}_i \subseteq \Theta^d_i$, which is i's indifferent set for Θ^d, either $\Theta^{ind}_i \subseteq q(d)$ or $\Theta^{ind}_i \subseteq \Theta^d_i \setminus q(d)$ holds.*

Proof. Assume d, where $i = int(d)$, $\forall \Theta^{ind}_i \subseteq \Theta^d_i$, which is i's indifferent set for Θ^d, either $\Theta^{ind}_i \subseteq q(d)$ or $\Theta^{ind}_i \subseteq \Theta^d_i \setminus q(d)$ holds, but d is irrelevant for i. Let us choose $\theta_i \in q(d)$ and $\theta'_i \in \Theta^d_i \setminus q(d)$. From the assumption, θ_i and θ'_i cannot be in the same indifferent set. Thus, there exists $\theta_{-i} \in \Theta^d_{-i}$ such that $a^*((\theta_i, \theta_{-i})) \neq a^*((\theta'_i, \theta_{-i}))$. Now, let us assume the type profile of other participants is θ_{-i} and other participants use sincere strategies. Let us assume d_l is the leaf node that is reached when i uses strategy s_i, which is consistent with respect to (i, θ_i), and d'_l is the leaf node that is reached when i uses strategy s'_i, which is consistent with respect to (i, θ'_i). Then, since the mechanism is a VCG-equivalent in expectation mechanism, $(\theta_i, \theta_{-i}) \in \Theta^{d_l}$ and $\bar{a}(\Theta^{d_l}) = a^*((\theta_i, \theta_{-i}))$ holds. Also, $(\theta'_i, \theta_{-i}) \in \Theta^{d'_l}$ and $\bar{a}(\Theta^{d'_l}) = a^*((\theta'_i, \theta_{-i}))$ holds. Since $a^*((\theta_i, \theta_{-i})) \neq a^*((\theta'_i, \theta_{-i}))$, $\bar{a}(\Theta^{d_l}) \neq \bar{a}(\Theta^{d'_l})$ holds. However, this fact violates the assumption that d is irrelevant for i. Thus, d must be relevant to i. □

Theorem 6. *For any N, M, and Θ, there exists a VCG-equivalent in expectation mechanism that has no irrelevant node.*

Proof. We show a method for constructing such a mechanism. From the root node, we recursively create a node. The input of the root node is Θ.

For node d with its input Θ^d, if $\forall i \in N$, either (i) $|\Theta^d_i| = 1$, or (ii) Θ^d_i is i's indifferent set is true, we make d a leaf node. From the definition of an indifferent set, it is clear that $\forall \theta \in \Theta^d$, the efficient allocation is the same. For example, in Figure 1, at the right most node, where its input is $\{6, 8\} \times \{4, 6, 8\}$, $\{6, 8\}$ for participant 1 and $\{4, 6, 8\}$ for participant 2 are indifferent sets. Thus, this leaf node satisfies the condition in Definition 5.

Otherwise, we choose participant i, such that $|\Theta^d_i| > 1$. If there exists i's indifferent set, we choose a maximal indifferent set Θ^{ind}_i (if there exist multiple maximal indifferent set, we choose an arbitrary one), and make $int(d)$ i and $q(d)$ Θ^{ind}_i. If i has no indifferent set, we simply choose some Θ'_i, which is a proper subset of Θ^d_i, and make $int(d)$ i and $q(d)$ Θ'_i. This procedure eventually terminates, since the input of a node decreases monotonically. Also, if there exist multiple indifferent sets, they must be disjoint. Thus, for each node d, the conditions in Theorem 5 are satisfied for each node. Therefore, this mechanism has no irrelevant node. □

This procedure is flexible so that it can produce a variety of mechanisms, by changing the way for choosing Θ'_i, which is a proper subset of Θ^d_i, when there exists no indifferent set for i. Assume types are one-dimensional, if we choose the smallest/largest element first, the obtained mechanism becomes similar to ascending/descending price auctions. If we try to divide Θ^d_i equally, the obtained mechanism becomes similar to a binary-search type mechanism, in which the height of the BDT is small.

6. APPLICATION TO JAPANESE 4G SPECTRUM AUCTION

In this section, to demonstrate the applicability of our idea to a practical application, we develop a mechanism that can be applied to the Japanese 4G spectrum auction based on the idea of VCG-equivalent in expectation mechanisms.

The Japanese government is planning to allocate 3.4 ∼ 3.6 GHz spectrum rights (i.e., the bandwidth of 200MHz) to wireless carriers. This will be the first experience of spectrum auction in Japan. The current plan is to divide the 200MHz into 10 lots, thus the bandwidth of each lot is 20MHz. There are two alternative technologies called Time Division Duplex (TDD) and Frequency Division Duplex (FDD). In FDD, two lots (one for uplink and the other for downlink) must be allocated together. Thus, for a carrier that uses FDD, having just one lot becomes useless, i.e., two lots are complementary. The government is required to maintain technical neutrality concerning the competing technology standards. Thus, dividing ten lots into two groups in an arbitrary way, and allocating one group for TDD and the other group to FDD is not desirable.

Although it is possible to apply the VCG mechanism, having a simple ascending price auction mechanism would be more desirable. However, the complementarity of two lots prohibits using simple ascending price auction mechanisms such as the Ausubel auction [1]. In this paper, we propose a simple ascending price auction mechanism based on the idea of a VCG-equivalent in expectation mechanism.

We use the following simplified model of Japanese 4G spectrum auctions. We assume m units of identical goods (lots) are allocated, where m is even. One license for FDD consists of two units, and one license for TDD consists of one unit. Let us denote (a, b) as an allocation of a FDD licenses and b TDD licenses. We assume for each participant i, the values for FDD and TDD are additive, i.e., $v(\theta_i, (a, b)) = v(\theta_i, (a, 0)) + v(\theta_i, (0, b))$ holds. Also, we assume the marginal value for one license is decreasing (note that for FDD, one license consists of two lots). For simplicity, we assume the valuations are discrete, i.e., each valuation is a multiple of δ. Also, we assume the valuations of participants are diverse enough so that we can ignore the possibility of ties. Thus, the number of aggregated demands decreases at most one or two units by increasing p or q. The purpose of this simplification is just to make the mechanism description for tie-breaking simple. The mechanism can handle ties appropriately, since it obtains enough information to determine a Pareto efficient allocation. Also, we assume if a participant is indifferent between two demands, she chooses a larger one.

The key point of the ascending price auction mechanism is that it has two different prices p and q, where the price of one TDD license is p, while the price of one FDD license (which consists of two units) is $p + q$.

Let us show how this mechanism works in a simple example. Assume $m = 2$. There are four participants, where participant 1 and 2 require one license for TDD, and participant 3 and 4 require one license for FDD. The valuation of each participant is chosen uniformly at random between 1 and 10. First, let us assume the valuations of the four participants are 2, 4, 8, and 10, respectively. We set δ to 1.

Initially, $p = q = 0$. The mechanism increments p and q one by one. When $p = q = 3$, participant 1 decreases her demand for TDD from 1 to 0. Then, the mechanism increments p one by one. When $p = 5$, participant 2 decreases her demand for TDD from 1 to 0. Then, flag is set to "q", and the mechanism increments q one by one to 5. When $q = 4$, participant 3 decreases her demand for FDD from 1 to 0. Now, $D = 2 \leq m$, thus the auction is closed. Participant 4, who declares one demand for FDD, obtains one license for FDD. In this case, for participants 1, 2 and 3, the mechanism knows that their types are 2, 4, and 8, respectively. Thus, the mechanism can calculate the VCG transfer for participant 4, which is equal to -8.

```
1. set p to 0, q to 0, and flag to "p".
2. each participant i declare her demand
      FDD(i, p + q) and TDD(i, p).
3. if the number of units in the aggregated demands
      D = ∑_{i∈N} 2 × FDD(i, p + q) + TDD(i, p),
      is less than or equal to m,
   then each participant i obtains her declared demand,
      in addition, if D = m − 1,
         the participant who reduced her demand for TDD
         most recently obtains one license for TDD,
      close the auction.
4. if ∑_{i∈N} TDD(i, p) ≥ m,
      then set both p and q to p + δ, goto 2.
5. when flag="p" and some participant reduce her demand
      for TDD, set flag to "q".
6. when flag="q" and p = q, set flag to "p"
7. if flag="p",
      then set p to p + δ, otherwise set q to q + δ, goto 2.
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Figure 2: Japanese 4G spectrum auction mechanism

Next, let us assume the valuations of the four participants are 3, 7, 5 and 9 respectively. The mechanism increments p and q one by one. When $p = q = 3$, participant 3 decreases her demand for FDD from 1 to 0. Next, when $p = q = 4$, participant 1 decreases her demand for TDD from 1 to 0 Then, the mechanism increments p. When $p = 6$, participant 4 decreases her demand for FDD from 1 to 0. Now, $D = 1 < m$, thus the auction is closed. In this case, as well as participant 2, who declares one demand for TDD, the participant 1, who reduced her demand for TDD most recently, obtains one license for TDD. The expected VCG transfer are calculated as follows. For participants 1 and 4, the mechanism knows that their types are 3 and 9, respectively. For particpant 3, the mechanism knows her type is one of $\{5, 6\}$. Thus, the mechanism can calculate the VCG transfer for participant 2, which is equal to -6. For participant 2, the mechanism knows that her type is one of $\{7, 8, 9, 10\}$, each of which can happen with probability 1/4. Thus, the VCG transfer for participant 1 becomes $(7-9)/4+(8-9)/4+(9-9)/4+(10-10)/4 = -3/4$.

THEOREM 7. *The ascending price auction mechanism achieves a Pareto efficient allocation.*

PROOF. Let us call a non-zero demand when the auction is closed as a satisfied demand, and other demands as unsatisfied demands. T_S/F_S denotes a set of satisfied demands for TDD/FDD, and T_U/F_U denotes a set of unsatisfied demands for TDD/FDD. When the auction is closed, the smallest marginal valuation within T_S is more than or equal to p, and the smallest marginal valuation within F_S is more than or equal to $p + q$.

Also, the largest marginal valuation within T_U is at most $p - \delta$, and the largest marginal valuation within F_U is at most $p + q - \delta$. Furthermore, the second largest marginal valuation within T_U is at most $q - \delta$.

If the number of aggregated demands is equal to m, the mechanism allocates m units to all satisfied demands. We can see this allocation is Pareto efficient, since the smallest marginal valuation within F_S is more than or equal to $p+q$, while the sum of the largest and the second largest marginal valuations within T_U is at most $p + q - 2\delta$.

If the number of aggregated demands is equal to $m - 1$, the mechanism allocates m units to all satisfied demands and the largest marginal valuation within T_U. Let us examine whether this allocation is actually Pareto efficient.

First, let us assume flag="p" when the auction is closed. Then, since we assume the number of aggregated demands is equal to $m-1$ when the auction is closed, one demand for FDD is decreased at the current price p and q. Thus, the largest marginal valuation within F_U is equal to $p+q-\delta$. Also, the largest marginal valuation within T_U is equal to $q-\delta$. Since the smallest marginal valuation within T_S is more than or equal to p, adding the largest marginal valuation within T_U (which is equal to $q-\delta$) is more than (or at least equal to) removing the smallest marginal valuation within T_S (which is more than or equal to p) and adding the largest marginal valuation within F_U (which is equal to $p+q-\delta$).

Next, let us assume flag="q" when the auction is closed. In this case, it is clear that one demand for FDD is decreased at the current price p and q. Thus, the largest marginal valuation within F_U is equal to $p+q-\delta$. Also, the largest marginal valuation within T_U is equal to $p-\delta$. Since the smallest marginal valuation within T_S is more than or equal to p, adding the largest marginal valuation within T_U (which is equal to $p-\delta$) is better than (or at least equal to) removing the smallest marginal valuation within T_S (which is more than or equal to p) and adding the largest marginal valuation within F_U (which is equal to $p+q-\delta$), since $q \leq p$. □

THEOREM 8. *The ascending price auction mechanism has no irrelevant node.*

We omit the proof due to space limitation. Intuitively, all queries are relevant since there always exists a chance that declaring any non-truthful demand decreases her utility.

There are several limitations in using this mechanism in practice. First, it relies on a prior knowledge on the distribution of types, which must be common knowledge. Also, although the way for determining the allocation is simple and transparent, the way for determining the transfers (i.e., the expected VCG transfers) is very complicated and difficult to understand. In practice, we need to introduce a simple and transparent method to calculate an approximate values of the expected VCG transfers. If these approximate values are not very sensitive to the change in the actual distribution of types, even though sincere strategies do not constitute a sequential equilibrium, the gain of deviating from a sincere strategy can be bounded. Thus, each participant does not have a strong incentive to deviate.

7. CONCLUSIONS AND FUTURE WORKS

One reason that iterative auctions are preferred over their sealed-bid counterparts is that they can avoid full revelation of type information. However, there is a dilemma that if we want to avoid an incentive issue, a mechanism needs to ask an irrelevant query, which causes unnecessary leakage of private information and a different incentive issue. A VCG-equivalent in expectation mechanism avoids this dilemma by achieving an outcome that is equivalent to VCG only in expectation. Based on a simple model called a BDT-based mechanism, we showed that a VCG-equivalent mechanism needs to ask an irrelevant query. Also, we proved that in a VCG-equivalent in expectation mechanism, sincere strategies constitute a sequential equilibrium. Also, we developed a general procedure for constructing a VCG-equivalent in expectation mechanism that does not ask any irrelevant query. To demonstrate the applicability of this idea in a practical application, we developed a VCG-equivalent in expectation mechanism for the Japanese 4G spectrum auction.

Our future works include elaborating the Japanese 4G spectrum auction mechanism, so that the transfers in it are calculated in a simple and transparent way, while they are close enough to the transfers in a VCG-equivalent in expectation mechanism.

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