How Credible is the Prediction of a Party-Based Election?

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ABSTRACT

In a party-based election, the voters are grouped into parties and all voters of a party are assumed to vote according to the party preferences over the candidates. Hence, once the party preferences are declared the outcome of the election can be predicted. However, in the actual election, the members of some "instable" parties often leave their own parties to join other parties. We introduce two parameters to measure the credibility of the prediction based on party preferences: MIN is the minimum number of voters leaving the instable parties such that the prediction is no longer true, while MAX is the maximum number of voters leaving the instable parties such that the prediction remains valid. Concerning the complexity of computing MIN and MAX, we consider both positional scoring rules (Plurality, Veto, r-Approval and Borda) and Condorcet-consistent rules (Copeland and Maximin). We show that for all considered scoring rules, MIN is polynomial-time computable, while it is NP-hard to compute MIN for Copeland and Maximin. With the only exception of Borda, MAX can be computed in polynomial time for other scoring rules. We have NP-hardness results for the computation of MAX under Borda, Maximin and Copeland.

Categories and Subject Descriptors

F.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity; G.2.1 [**Combinatorics**]: Combinatorial algorithms; J.4 [**Computer Applications**]: Social Choice and Behavioral Sciences

General Terms

Algorithms

Keywords

bribery; voting system; complexity; party-based election

1. INTRODUCTION

Voting has been recognized as a common approach for preference aggregation and collective decision making whenever there exists more than one alternative for a community to choose from. In particular, it has been widely used in multiagent systems [3, 4, 12]. Based on the conflicting preferences over the alternatives of different voters, some voting rules are designed in an effort to reach the best possible joint decision. Since long, voting has been a part and parcel of the fields of preference handling, decision making and social choice. It comes with a wide variety of applications which ranges from multi-agent systems, political elections, recommendation systems, etc. [1, 10, 12, 13].

Generally, a voting system can be characterized by a set of voters, a set of alternatives, and a voting rule. A voter casts his vote solely according to his own preference over the alternatives. However, in many real-world applications, we can observe that the voters do not act in a completely individual manner, but often form some parties (or interest groups) prior casting their votes. Each party predetermines a party preference over the alternatives and requires all its members to follow the party discipline, that is, the voters of this party should vote according to the party preference. Parliament voting is a typical example of such "party-based" voting systems. Moreover, party-based voting scenarios can also be found in board elections of universities and computer game competitions, where students and players are grouped in departments and teams. Thus, a party-based election system consists of a set of alternatives, a set of parties, each party being characterized by the number of its members and its party preference, and a voting rule. In this setting, once the preferences of all parties are declared, the outcome of the voting can be determined prior to the final voting.

However, in practice, the final results of such elections are often much different from the "predictions" based on the party preferences, mainly caused by the "instability" of some participating parties. That is, some members of these "instable" parties refuse to follow the preferences of their own parties and join other "stable" parties, possibly persuaded by the stable parties. Thus, it could be of great importance for all participants of the voting to measure the influence of the instable parties to the predictability of the voting. Hereby, we consider the following two parameters: MIN represents the minimum number of voters from the instable parties, who can change the outcome of the voting by joining other stable parties, and MAX represents the maximum number of voters from the instable parties, whose revoting will not affect the outcome.

Based on these definitions the prediction of a party-based voting, where both MIN and MAX have high values, can be considered as credible. Note that MAX+1-MIN ≥ 0, for all voting systems. However, we consider both MIN and MAX, since they could be also critical for party leaders to design their strategies for manipulating\defending the outcome of the election. For example, for a party fearing an unfavorable prediction, the parameter MIN indicates the minimum

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"budget" that the party needs to invest, that is, to persuade how many voters from the instable parties, while the minimum goal for the parties favoring the prediction is to have $n-$MAX voters obeying their own party preferences, where n is number of voters. Therefore, it is of great desiring for all participants of a party-based voting to compute these parameters before the actual voting takes place.

This work aims to explore the computational complexity of computing MIN and MAX for various voting rules. To be more general, we do not assume the exact knowledge of instable\stable parties. That is, our input consists of a set of alternatives, a set of parties with their member numbers and party preferences, a voting rule, and the number of instable\stable parties, instead of concrete lists of these parties. We show that MIN is polynomial-time computable for all common positional scoring rules and for Condorcet, but NP-hard for Maximin and Copeland. Moreover, the computation of Max can be done in polynomial-time for r-Approval, Plurality and Veto, but is NP-hard for Borda, Condorcet, Maximin and Copeland. See Table 1 for an overview. The P vs NP-hard results could be useful for designers of such party-based voting. In the case that party manipulation is not possible and a quick estimation of the prediction credibility with frequently changing party preferences is more relevant, a voting rule with polynomial-time computable MIN and MAX might be more suitable, for example, in a multi-agent system where robots act in groups and collective decision should be made based on preferences of the groups. If one is aiming for a voting resistent to party manipulation behavior, then a voting rule, whose MIN\MAX are hard to compute should be applied.

For ease of exposition, we only show the results for a variant of the above mentioned party-based election, where there is only one stable party, that is, the members of the instable parties can only join this stable party. However, our results can be extended to the case with arbitrarily many stable and instable parties. This variant could be of particular interest for one participating party to determine how hard it is to manipulate or defend the outcome of the voting by persuading members of other parties to join it.

2. RELATED WORKS

We are not aware of other work studying the computation of MIN\MAX for party-based elections. A similar model of party-based elections has been introduced by Perek et al. [11], where, in addition to the set of alternatives and the set of parties, the input contains a specific "leading" party, which favors the prediction based on party preferences. This leading party is the only instable party, whose members can switch to other parties. The main goal is to calculate how safe is the leading party with respect to losing its members to other parties. Hereby, Perek et al. also compute two parameters, the minimum number of members to lose to change the outcome (PES) and the maximum number of members to lose without changing the outcome (OPT). Their model shares certain similarities with ours, distinguishing stable and instable parties and voters switching from instable to stable parties. However, our model differs from their model in two main aspects. First, our model allows more than one instable parties while there is only one instable party in their model, which is the leading party. Second, in our model only the numbers of the instable and stable parties are specified, while their model requires a fixed instable party, that is, the leading party. Besides this, our work puts strong emphasis on the stability of the election as a whole, and thus, could be interested for every participant of the voting, in particular, for the voting designer. However, their work only measures the stability related to a fixed party. Moreover, although our model seems to admit similar complexity behavior as the one in [11] with the considered voting rules, our results cannot be inferred from theirs. The main reason for this is that our model does not specify the concrete instable and stable parties while one fixed instable party is given as input in their model, which makes the two models incomparable from the complexity point of view, and in particular, requires technically completely different strategies for showing NP-hardness. On the one hand, with fixed stable\instable parties, it is easier to calculate switching voters in polynomial time for some rules. On the other hand, for some rules, it becomes much more complicated to prove the NP-hardness without fixed stable\instable parties; most our reductions spend a major part to discuss which parties should be stable and instable.

Our study has clear connection to the bribery problem [5], where voters may be bribed to change their preferences in any possible way to influence the voting outcome. In contrast, we consider party-based elections, where voters can only switch from party to party and follow the party preferences, which could be more realistic in many settings. With limited possibilities to modify the preference of a voter in the party-based model, one might need for the same set of voters much more voter switches than voter briberies to change the outcome of the voting.

Recently, Yang et al. [15] studied a variant of the bribery problem in party-based elections, where voters may be bribed to switch from their own parties to other parties that have similar preferences. In particular, they adopted the Hamming-distance and the Kendall-Tau distance to measure the similarity between preferences. The question is to switch minimum number of voters so that a given distinguished candidate wins or does not win the election.

In addition, our parameter MIN is related to the concept "margin of victory", which is defined as the smallest number k such that changing k votes can change the winners of the election [9, 14]. However, under the framework of "margin of victory", the votes can be changed in any way. In our study, votes can only switch to an existing party.

Furthermore, our work is related to a possible winner problem in weighted elections studied by Baumeister et al. [2]. In their framework, there is an election with two sets of voters, where the weights of voters in one set are not specified yet, and the question is to assign weights to these voters such that a given distinguished candidate wins the election. In our framework, each party with n members and party preference $\succ$ can be considered as a single voter with weight n and preference $\succ$ in their model. Switching voters from a party to another party can be then regarded as reassigning weights of the two voters corresponding to these two parties. Therefore, our model of calculating the parameter MIN can be regarded as a variant of theirs, where every voter is associated with an integer weight, and the question is to reassign the weights of the voters to change the result of the election. Moreover, the total weight of all voters after the reassigning should remain the same as before.

3. PRELIMINARIES

A *party-based election* is defined as a tuple $\mathcal{E} = (\mathcal{C}, \mathcal{P})$, where $\mathcal{C} = \{c_1, \dots, c_m\}$ is a set of *alternatives\candidates* and $\mathcal{P} = \{P_1, \dots, P_l\}$ is a set of *parties*. Each party is characterized by the number n_i of its members (voters) and a party preference $\succ_i$, denoted as $P_i = (n_i, \succ_i)$. A *preference* is a linear order that ranks the candidates from the most preferred one to the least preferred one. For example, if $\mathcal{C} = \{a, b, c\}$ and some party likes a best, then b, and then c, then its preference is represented as $a \succ b \succ c$. The *position* of a candidate c in a preference $\succ$ is defined as $|\{c' \in \mathcal{C} \mid c' \succ c\}| + 1$. The *final voting* of a party-based election, denoted by $\mathcal{V}$, is a list of preferences, which one-to-one correspond to the voters in $\mathcal{E}$ and can be partitioned into l subsets $V_1, \cdots, V_l$

such that $|V_i| = n_i$ for all i's and all preferences in V_i are identical to $\succ_i$. For two distinct candidates c and d, we define $N_\mathcal{E}(c, d)$ as the number of preferences in $\mathcal{V}$ with $c \succ d$. We omit the index $\mathcal{E}$ if it is clear from the context. We say a candidate c beats (resp. ties) another candidate c' if $N(c, c') > N(c', c)$ (resp. $N(c, c') = N(c', c)$).

A *voting rule* is a function R that given an election $\mathcal{E} = (\mathcal{C}, \mathcal{P})$ returns a subset $R(\mathcal{E}) \subseteq \mathcal{C}$ of the candidates that are said to win the election. If $|R(\mathcal{E})| = 1$ is required, then we call it a *unique-winner model*; otherwise, it is a *nonunique-winner model*.

In this paper, we consider the following voting rules. An m-candidate *positional scoring rule* is defined through a non-increasing vector $\alpha = (\alpha_1, \ldots, \alpha_m)$ of non-negative integers. A candidate $c \in \mathcal{C}$ is assigned α_i points from each preference in $\mathcal{V}$ that ranks c in the i^{th} position. The score of a candidate is the sum of points he gets from all preferences. The candidate(s) with the maximum score are the winner(s). Many voting rules can be considered as positional scoring rules. We study the following scoring rules (for m candidates) in this paper: Plurality (scoring vector $(1, 0, \ldots, 0)$), Veto (scoring vector $(1, 1, \ldots, 1, 0)$), r-Approval (scoring vector with r ones followed by $m - r$ zeroes), and Borda (scoring vector $(m - 1, m - 2, \ldots, 0)$).

A *Condorcet-consistent rule* always elects the Condorcet winner, if it exists. The Condorcet winner is the candidate who beats all other candidates in $\mathcal{C}$. Examples of Condorcet-consistent rules, that will be considered in this paper, are Maximin and Copeland. For a candidate c in an election, let $B(c)$ be the set of candidates which are beaten by c and let $T(c)$ be the set of candidates which tie with c. Then, the Copeland$^\alpha$ score of c is $|B(c)| + \alpha \cdot |T(c)|$, for $0 \leq \alpha \leq 1$. A candidate is a Copeland$^\alpha$ winner if it has the highest score. On the other hand, the maximin score of a candidate c is given by $\min_{d \in \mathcal{C} \setminus \{c\}} N_\mathcal{E}(c, d)$, and the winner in a maximin election is a candidate with the highest score.

When we say that a voter *switches* from its original party to another party, we mean that the respective voter casts his vote according to the preference of the destination party. More formally, given $(\mathcal{C}, \mathcal{P})$ with $\mathcal{P} = \{P_1, \cdots, P_l\}$, a voter in $P_i \in \mathcal{P}$ switching to $P_j \in \mathcal{P}$ creates a new election $(\mathcal{C}, \mathcal{P}')$ where

$$\mathcal{P}' = \{P_1, \cdots, P_i', \cdots P_j', \cdots P_l\}$$

with $P_i' = (n_i - 1, \succ_i)$ and $P_j' = (n_j + 1, \succ_j)$. In our model, voters switch can only happen from an instable party to a stable one. Thus, members of stable parties cannot switch. The two problems considered in this paper are defined as follows. We use p to denote the winner of the election, if all voters follow their party preferences.

Now we formally define the two problems that are studied in this paper.

MIN

Input: A party-based election $\mathcal{E} = (\mathcal{C}, \mathcal{P})$ with the unique winner p, a voting rule, and two integers k and s.

Question: Is there a subset $S \subseteq \mathcal{P}$ with $|S| = s$ such that p is no longer the winner after switching at most k voters not in the parties of S to the parties of S?

MAX

Input: A party-based election $\mathcal{E} = (\mathcal{C}, \mathcal{P})$ with the unique winner p, a voting rule, and two integers k and s.

Question: Is there a subset $S \subseteq \mathcal{P}$ with $|S| = s$ such that p can remain the winner after switching at least k voters not in the parties of S to the parties of S?

In the following, we mainly study the following variant where there is only one stable set, that is, $s = 1$. This stable party is also called the destination party and the variant is called the one-destination model. We consider this model not only because it has applications as discussed in Section 1, but also because the results achieved here also apply to the general case with $s > 1$. The NP-hardness reductions for $s = 1$ can be extended to the general case by copying the constructed elections $s - 1$ times. Moreover, the basic idea behind the algorithms for $s = 1$ also leads to polynomial-time greedy algorithms for the case of $s > 1$. Furthermore, our results hold for both the unique-winner model and the nonunique-winner model, but we give here only the proofs for the unique-winner model.

Our hardness proofs are reduced from the following NP-hard problems [7]:

EXACT THREE SET COVER (X3C)

Input: A set $X = \{x_1, \ldots, x_m\}$ with $m \equiv 0 \mod 3$, and a collection $\mathcal{S} = \{S_1, \ldots, S_n\}$ of 3-element subsets of X.

Question: Does $\mathcal{S}$ have an exact cover S for X, i.e., a subcollection $S \subseteq \mathcal{S}$ such that every element of X occurs in exactly one subset of S ?

Throughout this paper, we assume that each element x_i occurs in exactly three subsets of $\mathcal{S}$. This assumption does not change the NP-hardness of X3C [8]. Notice that in this case we have $n = m$.

An independent set of a graph is a subset of vertices where no edge exists between any pair of vertices in this subset. A vertex cover of a graph is a subset of vertices whose removal results in an independent set.

INDEPENDENT SET (IS)

Input: A graph G and an integer $t \geq 0$.

Question: Is there an independent set of G of size at least t?

VERTEX COVER (VC)

Input: A graph G and an integer $t \geq 0$.

Question: Is there a vertex cover of G of size at most t?

4. COMPLEXITY OF COMPUTING MIN

In this section, we study the ONE-DESTINATION-MIN problem. In particular, we prove that this problem is polynomial-time solvable under all positional scoring rules. As for the Condorcet-consistent rules, we prove that the Condorcet rule behaves in the same way as the positional scoring rules, whereas both the Maximin voting and the Copeland$^\alpha$ voting for all $0 \leq \alpha \leq 1$ lead to NP-hardness of ONE-DESTINATION-MIN. Our results are summarized in the following theorems.

THEOREM 1. ONE-DESTINATION-MIN *for all positional scoring rules is polynomial-time solvable.*

PROOF. We prove the theorem by proposing a polynomial-time algorithm. Without loss of generality, let $\vec{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_m)$ be the scoring vector where $\alpha_1 \geq \alpha_2 \geq, ..., \geq \alpha_m$. In the first step of our algorithm we guess the replacing candidate p' which will have a score at least that of p after some voters switch their parties. Clearly, our guess involves candidates only from $\mathcal{C} \setminus \{p\}$ whose number is bounded by $m - 1$. Then, for each such guess

	NP-hard	P
MAX	Borda Condorcet Maximin Copeland	Plurality Veto 4-Approval
MIN	Maximin Copeland	Plurality Veto r-Approval Borda Condorcet

Table 1: Summary of Our Results. Here, 'P' stands for polynomial-time solvable.

p', we need to check if it is possible to make p' have a score at least that of p by switching at most k voters possibly from several parties to a certain destination party in $\mathcal{P}$. The best possible way of decreasing the gap between the scores of p and p' with switching the minimum number of voters is to fix a party, whose preference achieves the maximum value of $s_{\succ}(p') - s_{\succ}(p)$, as the destination. Here, $\succ$ denotes the preference of the party and $s_{\succ}(c)$ denotes the score of the candidate c from $\succ$. Now, we sort the party preferences of the remaining parties according to the non-increasing order of $s_{\succ}(p) - s_{\succ}(p')$. Finally, we switch the voters of the party ordered at the first place, then the one at the second place, and so on to the destination party, until k voters are switched or the score of p' is at least that of p. If the latter case applies, we return "yes"; otherwise, we return "no". The correctness and running time of the algorithm are easy to prove. □

Now we study the problem for Condorcet. Strictly speaking, Condorcet is not a voting rule since the Condorcet winner does not always exist. Nevertheless, due to the importance of the concept of the Condorcet winner in social choice, complexity of voting problems under Condorcet is also studied in the literature, see, e.g., [6]. In this setting, the ONE-DESTINATION-MIN problem has objective to prevent the current Condorcet winner from being the Condorcet winner after switching k voters, while the ONE-DESTINATION-MAX problem aims to remain the current Condorcet winner after switching k votes. In both problems, we assume that the given election has a Condorcet winner.

THEOREM 2. ONE-DESTINATION-MIN *for Condorcet is solvable in polynomial time.*

PROOF. The algorithm first guesses the candidate p' which beats p in the final election. Next, it fixes one party, whose party preference prefers p' to p, as the destination party. Then, it switches arbitrary k voters, which prefer p to p', to the destination party and checks the final winning status of p and p'. For each guessed candidate, the switch of voters and the calculation of scores can be done in polynomial time, and with at most $m-1$ such guesses we have an overall polynomial-time algorithm. □

It is well-known that Copeland$^{\alpha}$ is a Condorcet consistent voting correspondence, that is, the Copeland$^{\alpha}$ winner is the Condorcet winner whenever the Condorcet winner exists. However, the complexity of calculating the value MIN for Copeland$^{\alpha}$ is different from the Condorcet, as showed in the following theorem.

THEOREM 3. ONE-DESTINATION-MIN *for the Copeland$^{\alpha}$ voting rule is NP-hard, for every $0 \leq \alpha \leq 1$.*

PROOF. We reduce an instance $G = (V, E)$ of VERTEX COVER with $|V| = n$ to an instance $\mathcal{E} = (\mathcal{C}, \mathcal{P})$ of ONE-DESTINATION-MIN. Clearly, VERTEX COVER remains NP-hard with $t < \frac{n-5}{2}$. Without loss of generality, assume n being even. We further assume that there exist two vertices v' and v'' in V, which do not belong to some solution set (e.g., both v and v' have degree-1). Both assumptions do not change the NP-hardness of VERTEX COVER.

For each edge $e_i \in E$, we create a corresponding candidate in $\mathcal{C}$. With slight abuse of terminology, we use the same notation to denote the candidate as its corresponding edge in E. Let $E(v)$ be the set of candidates corresponding to the edges containing the vertex v and E^* be the set of all candidates corresponding to the edges in E. In addition, we have six candidates $p, A = \{a_1, a_2\}$ and $B = \{b_1, b_2, b_3\}$. In all the following preferences, we assume an order $(a_1 \succ a_2)$ for A and an order $(b_1 \succ b_2 \succ b_3)$ for B. In the following preferences, the elements of some subset $E' \subseteq E^*$ are ordered consecutively. Hereby, we use $\cdots \succ E' \succ \ldots$ to denote the suborder formed by E' and the elements in E' are assumed to be ordered in this suborder according to their indices. We first create the following two preferences: $E(v') \succ A \succ p \succ B \succ E^* \setminus E(v')$ and $E(v'') \succ A \succ p \succ B \succ E^* \setminus E(v'')$.

In addition, for every other vertex $v \in V \setminus \{v', v''\}$, we create a preference defined as follows: $E(v) \succ p \succ B \succ A \succ E^* \setminus E(v)$. Each of the above preferences represents a party. We denote by P_v the party corresponding to the vertex v. Furthermore, we have a party P containing one voter with the following preference: $B \succ E^* \succ p \succ A$. Finally, we have $n-2$ voters, out of which the first $\frac{n-2}{2}$ voters form a party denoted by P_1 with the preference: $a_1 \succ p \succ a_2 \succ E^* \succ B$. The other $\frac{n-2}{2}$ voters form a party denoted by P_2 with the following preference: $E^* \succ B \succ a_1 \succ p \succ a_2$. See Table 2 for the comparison between every two candidates.

Finally, set $k = t$. Before discussing the correctness, consider the score of each candidate first.

	p	a_i	e_i	b_i
p	0	-1	$n-5$	$n-1$
a_i	1	0 or 1	$n-5$	$-(n-3)$
e_i	$-(n-5)$	$-(n-5)$	$0, ..., \lvert E\rvert-1$	1
b_i	$-(n-1)$	$(n+3)$	-1	0 or 1 or 2

Table 2: Comparisons between candidates. The entry with row index c and column index c' is $N(c, c') - N(c', c)$, that is, the number of voters who prefer c to c' minus the number of voters who prefer c' to c. Here, the entry with both row and column indexed by e_i is $0, 1, ..., |E|-1$ since each e_i gets different value from $\{0, 1, \ldots, |E|-1\}$. More specifically, assume the order $e_1, e_2, ..., e_m$, e_1 gets $|E|-1$, e_2 gets $|E|-2$, and so on. Note that $n > 5$.

For a candidate c, let $s(c)$ be the Copeland$^{\alpha}$ score of c. Then we have $s(p) = |E|+4$, $s(a_1) = |E|+2$, $s(a_2) = |E|$, $s(b_i) = 5-i$, where $i = 1, 2, 3$, and $s(e_i) = |E| - i + 3$.

It is clear that p is the current winner. Some useful observations are as follows:

CLAIM 1. *The following claims hold: (1) $s(p)$ cannot be decreased by switching at most k voters. (2) $s(a_i)$, $s(e_i)$ cannot be increased by switching at most k voters. (3) Switching of at most k voters can increase $s(b_1)$ to at most $|E|+4$.*

We prove the claim as follows.

(1) Since for every $e_i \in E^*$ we have $N(p, e_i) - N(e_i, p) = n - 5$, and for every $b_i \in B$, we have $N(p, b_i) - N(b_i, p) = n - 1$, with the assumption that $t < \frac{n-5}{2}$, switching arbitrary $k = t$ voters can never decrease the score of p.

(2) Similar to (1).

(3) Observe that $N(e_i, b_1) - N(b_1, e_i) = 1$ for every $e_i \in E^*$. Therefore, b_1 has the potential to beat every e_i: just switch one voter of a party with $e_i \succ b_1$ to a party with $b_1 \succ e_i$. Therefore, b_1 has the potential to have a score of $|E| + 4$. However, since $N(p, b_1) - N(b_1, p) = n - 1$, b_1 has no chance to increase its score furthermore. This finish the proof of the above claim.

Now we prove that G has a vertex cover of size at most t, if and only if ONE-DESTINATION-MIN on $\mathcal{E} = \{\mathcal{C}, \mathcal{P}\}$ has a "yes" answer.

($\Rightarrow$:) Let G have a vertex cover C of size t. Consider the election after all voters corresponding to C switch to the party P. Since C is a vertex cover, for every edge e_i, there is at least one vertex $v \in C$ with $e_i \in E(v)$. Therefore, for every edge e_i, at least one voter of a party with preference $e_i \succ b_1$ is switched to the party P, where $b_1 \succ e_i$. Due to the analysis of the third claim in Claim 1, b_1 beats every e_i and thus p is not the unique winner anymore.

($\Leftarrow$:) Assume that ONE-DESTINATION-MIN on $\mathcal{E} = \{\mathcal{C}, \mathcal{P}\}$ has a "yes" answer and C is the set of voters which switch to the destination party. Due to Claim 1, the only candidate which could have a score at least that of p is b_1. This can only happen if b_1 beats every $e_i \in E^*$. Based on this claim, we observe that the parties P_1 and P_2 cannot be the destination party. Among the remaining parties, it is obvious that P is the best possible destination party since b_1 beats every e_i in this party (in other words, if there is a solution in which the destination party is not P, we can always construct another solution with P being the destination party). Now consider which parties could be the instable parties. We claim the following.

CLAIM 2. *The voters of P_1 and P_2 cannot be switched.*

To verify the above claim, observe that switching one arbitrary voter from $P_1 \cup P_2$ to P would make b_1 reach its highest possible score $|E| + 4$. However, this also increases the score of p by one (from beating a_1); thus p remains the winner.

Now we show that the vertices corresponding to the voters in C must be a vertex cover in G. Due to the above analysis, b_1 has a score at least that of p, only if b_1 beats every $e_i \in E^*$. Therefore, for every edge e_i, there must be at least one voter in C corresponding to a vertex v with $e_i \in E(v)$, implying the vertices corresponding to C form a vertex cover of G. □

Now we investigate Maximin, which is another Condorcet-consistent voting systems. The following theorem shows our result.

THEOREM 4. ONE-DESTINATION-MIN *for Maximin is NP-hard.*

PROOF. We give a reduction from an X3C instance $(X, \mathcal{S})$ to an instance $\mathcal{E} = \{\mathcal{C}, \mathcal{P}\}$ of ONE-DESTINATION-MIN.

For each $x \in X$, we create a candidate. For convenience, we still use x to denote the candidate. In addition, we have four candidates p, z, α and β. For each set $S_t \in \mathcal{S}$ with $S_t = \{x_i, x_j, x_k\}$ and $i < j < k$, create the following preference:

$x_i \succ x_j \succ x_k \succ z \succ p \succ X \setminus \{x_i, x_j, x_k\} \succ \beta \succ \alpha.$

This preference represents a party with only one voter.

Next, create $n - \frac{m}{3}$ new voters; the half of them forms one party with the following preference:

$\beta \succ \alpha \succ p \succ \overrightarrow{X} \succ z.$

The other half forms a party with the preference:

$\alpha \succ p \succ \overrightarrow{X} \succ z \succ \beta.$

Let $B_1, B_2, ..., B_{m/3}$ be subsets of X, where

$$B_i = \{x_{3i-2}, x_{3i-1}, x_{3i}\}.$$

For each B_i, create two voters forming two parties with the following two preferences, respectively:

$\beta \succ \alpha \succ p \succ X \setminus B_i \succ z \succ B_i,$

$\alpha \succ p \succ X \setminus B_i \succ z \succ B_i \succ \beta.$

Finally, we create a party with one voter and the preference: $z \succ \overrightarrow{X} \succ p \succ \alpha \succ \beta$. This party is denoted by P, which we later prove to be the destination party. Overall, $|\mathcal{C}| = m + 4$ and totally $2n + \frac{m}{3} + 1$ many voters. Next, we calculate the score of each candidate. In the following, $s(c)$ denotes the maximin score of the candidate c, and $\min(c)$ is the set of candidates c', which reach the minimum value of $N(c, c')$:

$s(\beta) = (n + \frac{m}{3})/2$ and $\min(\beta) = \{p, z, x_i\}$,

$s(\alpha) = (n + \frac{m}{3})/2 + 1$ and $\min(\alpha) = \{\beta\}$,

$s(p) = n + 1$ and $\min(p) = \{\alpha\}$,

$s(z) = n$ and $\min(z) = \{x_i\}$,

$s(x_i) = 4$ and $\min(x_i) = \{p\}$.

Clearly, p is the current winner. Suppose that there is an exact 3-set cover S. If all the voters corresponding to S switch to the party P, z would beat every x_i by $n + 1$; and thus p is not the unique winner anymore. It remains to show the other direction. Observe that the only candidate, which could have a score at least that of p is z. Therefore, if $n/3$ voters switch to some party to make p not the unique winner, the destination party can only be the party P. The instable parties can only be the ones corresponding to $\mathcal{S}$, since with other parties being instable, the score of p would increase. However, the set corresponding to the voters which are switched to the destination party must be an exact 3-set cover, since otherwise, there would exist an x_i such that at most n voters prefer z to x_i, resulting in p still being the winner. □

5. COMPLEXITY OF COMPUTING MAX

In this section, we study the ONE-DESTINATION-MAX problems under the same voting systems that have been studied in the previous section. In particular, we prove the polynomial-time solvability of ONE-DESTINATION-MAX for Plurality, r-Approval and Veto rules and present NP-hardness results of the same problem for Borda, Condorcet, Maximin and Copeland rules.

THEOREM 5. ONE-DESTINATION-MAX *for Plurality, r-Approval with constant r and Veto voting rules are polynomial-time solvable.*

PROOF. We consider here only r-Approval. The cases with Plurality and Veto can be handled similarly. Our polynomial-time algorithm first guesses the destination party among all the parties in the given instance $\mathcal{E} = (\mathcal{C}, \mathcal{P})$. Among the total of $|\mathcal{P}|$ such guesses, we discard those with preferences which do not approve p. For each remaining guessed destination party we do the following.

Let $C \subseteq \mathcal{C}$ be the set of r candidates which are approved by the preference of the destination party. Since p is the unique winner, for each candidate $c \in C \setminus \{p\}$, there must exist at least one voter disapproving c in the original election. To maintain p as the unique winner, for each candidate $c \in C \setminus \{p\}$, there must be at least one voter disapproving c in the original election and this voter cannot be switched to the destination party. Therefore, we need to find out a minimum set of voters together disapproving $C \setminus \{p\}$. Since $|C| \leq r$ and r is a constant, this set can be found in polynomial time. □

The above theorem heavily depends on that every preference gives only points to constantly many candidates in $\mathcal{C} \setminus \{p\}$. Then, a minimum set of preferences distinguishing p from $\mathcal{C} \setminus \{p\}$ can be found in polynomial time. However, this does not hold for Borda. Indeed, the computation of MAX under Borda is NP-hard.

THEOREM 6. ONE-DESTINATION-MAX *for the Borda rule is NP-hard.*

PROOF. Let $(X = \{x_1, x_2, ..., x_m\}, \mathcal{S} = \{S_1, S_2, ..., S_n\})$ be an instance of X3C.

We have in total $m+6$ candidates. More precisely, for each $x_i \in X$, we have a corresponding candidate. For simplicity, we will use the same notation x_i to denote the corresponding candidate. In addition, we have six other candidates p, d_1, d_2, d_3, y, z. We create party preferences as follows.

Let $\overrightarrow{X}$ be the order of the elements in X according to the increasing order of their indices. For each subset $\{x_i, x_j, x_k\} \in \mathcal{S}$ with $i < j < k$, we create one preference $z \succ \overrightarrow{X}[x_i \to d_1, x_j \to d_2, x_k \to d_3] \succ p \succ x_i \succ x_j \succ x_k \succ y$. Here $\overrightarrow{X}[x_i \to d_1, x_j \to d_2, x_k \to d_3]$ is the linear order obtained from $\overrightarrow{X}$ with replacing x_i, x_j, x_k by d_1, d_2, d_3, respectively. The corresponding party is denoted by $P_{(i,j,k)}$ and has only one voter.

Next, we create one party P' with n voters and the preference: $y \succ p \succ \overleftarrow{X} \succ z \succ d_1 \succ d_2 \succ d_3$, where $\overleftarrow{X}$ is the reverse order of $\overrightarrow{X}$. Additionally, we create a party P with one voter and the following preference:

$\overrightarrow{X} \succ p \succ d_1 \succ d_2 \succ d_3 \succ y \succ z$.

It is clear that p is the current winner. More precisely, $s(p) - s(z) = 5$, $s(p) - s(y) = 3n+4$ and $s(p) - s(x_i) > 0$. Here, $s(c)$ denotes the Borda score of c. We claim that an exact 3-set cover exists if and only if $n - m/3$ voters can switch their parties to a party such that p is still the winner.

Suppose that there is an exact 3-set cover S for $(X, \mathcal{S})$. Then leave all the parties corresponding to S and the party P' unchanged, and switch all the other voters into the party P. It is easy to check that p is still the winner.

For the reverse direction, we first claim that only P can be the destination party if the constructed instance is a true-instance. P' cannot be the destination party, since otherwise, y would become the winner. $P_{(i,j,k)}$ cannot be the destination party, since otherwise, either z or some x_i would become the winner. This completes the proof of claim. We further claim that the party P' cannot be instable. This is true, since otherwise, some x_i would have a higher score than that of p. Now suppose that there is no exact 3-set cover. Then there must be an x_i such that after switching $n - m/3$ voters from $\cup_{i,j,k} P_{(i,j,k)}$ to the party P, all the remaining voters in the parties $P_{(i,j,k)}$ prefer x_i to p. This results in x_i having a greater score than that of p, and thus p cannot be the unique winner anymore. □

Now we come to the Condorcet. In the previous section, we have shown that ONE-DESTINATION-MIN is polynomial time solvable under Condorcet. In the following, we show that ONE-DESTINATION-MAX under Condorcet is NP-hard.

THEOREM 7. ONE-DESTINATION-MAX *for the Condorcet voting rule is NP-hard.*

PROOF. We give a reduction from an X3C-instance $(X, \mathcal{S})$ to an instance $\mathcal{E} = \{\mathcal{C}, \mathcal{P}\}$ of ONE-DESTINATION-MAX. The candidate set is $\mathcal{C} = X \cup A \cup B \cup C \cup D \cup \{p\}$, where $A = \{a_1, a_2, a_3, a_4\}$, $B = \{b_1, b_2, b_3, b_4\}, C = \{c_1, c_2, ..., c_n\}$ and $D = \{d_1, d_2, ..., d_n\}$. We also use X to denote the set of candidates, one-to-one corresponding to the elements in the X3C-instance. Hence, there are totally $2n + m + 9$ candidates. For a set U with elements $u_1, u_2, ..., u_t$, we denote by $\overrightarrow{U}$ the order $(u_1 \succ u_2 \succ \cdots \succ u_t)$ and by $\overleftarrow{U}$ the reverse order of $\overrightarrow{U}$. Moreover, for two elements u_i, u_j with $i < j$, we use $\overrightarrow{U}[u_i, u_j]$ to denote the suborder $(u_i \succ u_{i+1} \succ \cdots \succ u_j)$. We assume a fixed order $(S_1 \succ S_2 \succ \cdots \succ S_n)$ for the subsets in $\mathcal{S}$. We create two preferences for each $S_t \in \mathcal{S}$ with $S_t = \{x_i, x_j, x_k\}$:

	p	x	a	b	c	d
p	-	$n+4$	$n+4$	$n+4$	$n+5$	$n+5$
x	$n+1$	-	$n+4$	$n+4$	$n+5$	$n+5$
a	$n+1$	$n+1$	-	$n+$	$n+5$	$n+5$
b	$n+1$	$n+1$	$n+$	-	$n+5$	$n+5$
c	n	n	n	n	-	$n+5$
d	n	n	n	n	n	-

Table 3: Comparisons between candidates in the NP-hardness proof for ONE-DESTINATION-MAX for Condorcet in Theorem 7. The entry with row index c and column index c' is $N(c, c')$, that is, the number of voters who prefer c to c'.

1. $\overrightarrow{D}[d_{t+1}, d_n] \succ \overrightarrow{C}[c_1, c_t] \succ \overrightarrow{A} \succ x_i \succ x_j \succ x_k \succ p \succ \overrightarrow{X} \setminus S_t \succ \overleftarrow{B} \succ \overrightarrow{C}[c_{t+1}, c_n] \succ \overrightarrow{D}[d_1, d_t]$. The corresponding party denoted by P_t, has only one voter.

2. $\overrightarrow{D}[d_1, d_t] \succ \overrightarrow{C}[c_{t+1}, c_n] \succ \overrightarrow{B} \succ \overleftarrow{X} \setminus S_t \succ p \succ x_k \succ x_j \succ x_i \succ \overleftarrow{A} \succ \overleftarrow{C}[c_1, c_t] \succ \overrightarrow{D}[d_{t+1}, d_n]$. The corresponding party, denoted by P'_t, has only one voter.

Next, we construct additional preferences as follows, each representing a party of its own. Each of the parties has only one voter:

- $a_1 \succ b_1 \succ p \succ \overrightarrow{X} \succ \overrightarrow{A} \setminus \{a_1\} \succ \overrightarrow{B} \setminus \{b_1\} \succ \overrightarrow{C} \succ \overrightarrow{D}$; the corresponding party is denoted by $\bar{P}_1$
- $a_2 \succ b_2 \succ p \succ \overleftarrow{X} \succ \overrightarrow{A} \setminus \{a_2\} \succ \overrightarrow{B} \setminus \{b_2\} \succ \overleftarrow{C} \succ \overleftarrow{D}$; the corresponding party is denoted by $\bar{P}_2$
- $a_3 \succ b_3 \succ p \succ \overrightarrow{X} \succ \overrightarrow{A} \setminus \{a_3\} \succ \overrightarrow{B} \setminus \{b_3\} \succ \overrightarrow{C} \succ \overrightarrow{D}$; the corresponding party is denoted by $\bar{P}_3$.
- $a_4 \succ b_4 \succ p \succ \overleftarrow{X} \succ \overrightarrow{A} \setminus \{a_4\} \succ \overrightarrow{B} \setminus \{b_4\} \succ \overleftarrow{C} \succ \overleftarrow{D}$; the corresponding party is denoted by $\bar{P}_4$.
- $\overrightarrow{X} \succ p \succ \overrightarrow{A} \cup \overrightarrow{B} \succ \overrightarrow{C} \succ \overrightarrow{D}$; the corresponding party is denoted by P.

In total, we have $2n+5$ voters. Now we arrive at the correctness proof of the reduction. See Table 3.

($\Rightarrow$:) Clearly, p beats every other candidate and thus is the current winner. Suppose that $(X, \mathcal{S})$ has an exact 3-set cover S. We claim that after switching all the voters in the parties P'_t, which correspond to the subsets in S, to the party P, p will still be the winner. Observe that p beats every candidate in $C \cup D \cup A \cup B$ in the final election. Since S is an exact 3-set cover, for each $x \in X$ there is exactly one party P'_t in the solution preferring p to x. Even though the party P prefers x to p, p still beats x by $n+3$. The claim follows.

($\Leftarrow$:) Suppose that we switch a set S' of $m/3$ voters to a particular party in $\mathcal{P}$ such that p remains the Condorcet winner. We claim the following:

CLAIM 3. *No $\bar{P}_i$, where $i = 1, 2, 3, 4$, can be the destination party.*

We prove the above claim as follows. Due to the symmetry, we only need to give the proof for the party $\bar{P}_1$. All other cases are similar. Observe that all parties other than $\bar{P}_1$ prefer p to either a_1 or b_1, or both. Since there are $n+1$ parties preferring a to p in the original election, switching any arbitrary five voters to the party $\bar{P}_1$ will make a_1 or b_1 beat p, contradicting with the fact that p is the Condorcet winner in the final election. This finish the proof.

CLAIM 4. *None of P_t and P'_t can be the destination party, for all $t \in \{1, 2, ..., n\}$.*

We prove the above claim as follows. Due to the symmetry, we only need to give the proof for P_t for a certain t. Suppose that this is not true and we have switched a set S' of $m/3$ voters to the party P_t, where $S_t = \{x_i, x_j, x_k\}$, without changing the winning candidate. There is at most one voter in S' which is not from the parties $\bigcup_{z\in\{1,...,n\}} P_z$, since otherwise, some a_i would beat p, contradicting that p is the Condorcet winner. Therefore, at least $m/3 - 1$ voters of S' are from $\bigcup_{z\in\{1,...,n\}} P_z$. Moreover, at most two voters of S' are from $\bigcup_{z>t} P_z$, since otherwise, d_t would beat p. Symmetrically, at most two voters of S are from $\bigcup_{z<t} P_z$ (otherwise, c_t would beat p), implying that $|S'| \leq 5$, a contradiction. This finish the proof.

Due to the above two claims, the only possible destination party is P. We further claim the following.

CLAIM 5. *None of the parties $\bar{P}_i$ can be instable, where $i = 1, 2, 3, 4$.*

We prove the above claim as follows. Observe that p beats every x_i by $n+4$. Therefore, if we switch some voter in $\bar{P}_i$ to the party P, then no other voter can be switched to P, since every voter not in the party P prefers p to some x_i. The claim follows.

CLAIM 6. *For $t \in \{1, 2, ..., n\}$, one of the parties P_t can be instable.*

We prove the claim as follows. Suppose that we switch some voter in a certain party P_t to P, where $S_t = \{x_i, x_j, x_k\}$. Due to Claim 5, no voter is switched from $\bigcup_z \bar{P}_z$ to P. Besides, at most one voter in $\bigcup_z P'_z \cup P_z \setminus \{P_t\}$ can be switched to P, since otherwise, some x_i would beat p, contradicting that $|S'| = m/3$. This finish the proof.

According to the above claims, the instable parties can only be from $S' \subseteq \{P'_z \mid z \in \{1, \cdots, n\}\}$. Since p beats every $x \in X$ by $n+4$, at most one voter in S' prefers p to x, implying that the subsets corresponding to S', that is $S = \{\{x_i, x_j, x_k\} \mid \exists t, P'_t \in S' \text{ and } S_t = \{x_i, x_j, x_k\}\}$, form an exact 3-set cover. □

Now we study the Maximin voting. We have proved that ONE-DESTINATION-MIN is NP-hard under Maximin in the previous section. In the following, we show that the NP-hardness still holds in the ONE-DESTINATION-MAX problem.

THEOREM 8. ONE-DESTINATION-MAX *for the Maximin rule is NP-hard.*

PROOF. We give a reduction from INDEPENDENT SET. Given an instance $I = (G, t)$ of INDEPENDENT SET, where $G = (V, E)$, $E = \{e_1, \ldots, e_m\}$, $n = |V|$, we create an instance $\mathcal{E} = (\mathcal{C}, \mathcal{P})$ of ONE-DESTINATION-MAX as follows.

The candidate set is $\mathcal{C} = \{a, b, p, e_1, \ldots, e_m\}$. Let $\overrightarrow{E} = (e_1 \succ e_2 \succ \cdots \succ e_m)$ be an order of E. We first create a set Z of n parties corresponding to the vertices in the graph. More specifically, for each vertex $v \in V$, we create a preference defined as follows:

$a \succ \overrightarrow{E \setminus E(v)} \succ p \succ \overrightarrow{E(v)} \succ b.$

Each preference represents a party with one voter, denoted by P_v. Then, we create a party P' containing n voters with the preference:

$b \succ p \succ \overleftarrow{E} \succ a.$

Finally, we create a party P containing only one voter with the preference:

$\overrightarrow{E} \succ p \succ b \succ a.$

Finally, set $k = t$. Now we prove the correctness of the reduction. We refer to Table 4 for the comparison of scores of candidates. It is clear from the table, that p is the current unique winner as p is preferred to all other candidates by at least $n+1$ voters.

	p	b	$e_j(i>j)$	$e_j(i<j)$	a
p	-	$n+1$	$n+2$	$n+2$	$n+1$
b	n	-	n	n	$n+1$
e_i	$n-1$	$n+1$	$\geq n$	$\geq n-1$	$n+1$
a	n	n	n	n	-

Table 4: Comparisons between candidates in the NP-hardness reduction for ONE-DESTINATION-MAX **for the Maximin in Theorem 8. The entry with row index c and column index c' is $N(c, c')$, that is, the number of voters who prefer c to c'.**

($\Rightarrow$:) Assume that I is a true-instance and S is an independent set of G of size t. Consider the election after $k = t$ voter corresponding to S switch to the party P. Let V_s be the set of these $k = t$ voters. It is clear that p beats a and b by $n+1+k$ and $n+1$, respectively. Since S is an independent set, for each edge e_i, there is at most one voter in V_s which prefers e_i to p. Hence, p beats every edge candidate e_i by at least $n+1$, implying the maximin score of p is $n+1$. Moreover, the scores of a and b do not increase. It remains to show that score of every e_i still remains less than that of p. To check this, consider the comparison of the scores of p and e_i. Since S is an independent set, the same reason discussed above implies that every e_i beats p by at most n. Thus, the maximin score of e_i cannot be greater than n, implying that p still remains the unique winner.

($\Leftarrow$:) Assume that it is possible to switch a set S' of k voters from their original parties to the destination party such that p still remains the winner in the overall election.

We first claim that P' cannot be the destination party. It is easy to see that if P' is the destination party, b will become the new winner replacing p. We then distinguish the following cases:

Case 1. P is the destination. In this case, we can assume that at most one voter in S' is from P', since otherwise, e_1 would replace p as a winner. Assume now that there is exactly one voter of S' which belongs to the party P'. Clearly, all other voters of S' come from the parties in the set Z. Since the party P prefers every e_i to p, and the parties in the set Z prefer p to some edge candidates, the score of p will be at most n. However, e_1 has a score at least n, contradicting that p is the unique winner. Based on the above fact, it is safe to assume that all the votes in S' belong to the parties in the set Z. We claim now that the vertices corresponding to the voters of S' form an independent set. If this is not true, there must be some edge candidates, each of which is preferred to p by two voters of S'. Let e_i be such an edge candidate with maximum index i. Consider the election after all the voters in S' are switched to the party P. It is clear that e_i beats p, a, b by at least $n+1$, and e_i beats e_j for all $j < i$ by at least n. Now consider the comparison between some e_j with $j > i$ with e_i. Since the voters in the set Z

	p	a_i	b_i	e_i	c_i
p	-	$n+1$	$n+1$	$n+2$	1
a_i	n	-	$n+1$	n	$n+1$
b_i	n	n	-	n	$n+1$
e_i	$n-1$	$n+1$	$n+1$	-	$n-1$
c_i	$2n$	n	n	$n+2$	-

Table 5: Comparisons between candidates in the NP-hardness proof for ONE-DESTINATION-MAX for Copeland$^\alpha$ in Theorem 9. The entry with row index c and column index c' is $N(c, c')$, that is, the number of voters who prefer c to c'.

which prefer e_j to e_i, are switched to the party P' where the voters prefer e_i to e_j, all the voters in the set Z and all the voters in the party P prefer e_i to e_j, implying that e_i beats e_j by $n+1$. Thus, we conclude that e_i has a final score of n. However, since p beats e_i by n in the final election, p is not the unique winner anymore.

Case 2. Now we consider the case that the destination party is some P_v. Again, we claim that the vertices corresponding to S' form an independent set, if p is still the unique winner. For the sake of contradiction, assume this is not true. Then there must be an edge $e_i = (u, w)$ with minimum index i, such that two voters in S' are switched to the party P_v. Note that e_i cannot be adjacent to v. Thus, p beats e_i by n, implying that the score of n is at most n. Now consider the score of e_i. It is easy to verify that e_i beats a, b, p by $n+1$, and beats e_j for all $j > i$ by n. Moreover, since the only two voters in the set Z which prefer e_j to e_i are switched to the party P_v, which prefers e_i to e_j, the score of e_i is at least n. Therefore, p no longer remains the unique winner, contradicting the assumption. □

Now we come to the Copeland voting. We have proved that ONE-DESTINATION-MIN is NP-hard under Copeland$^\alpha$ for every $0 \leq \alpha \leq 1$ in the previous section. In the following, we show that the NP-hardness still holds in the ONE-DESTINATION-MAX problem.

THEOREM 9. ONE-DESTINATION-MAX *for Copeland*$^\alpha$ *is NP-hard, for every* $0 \leq \alpha \leq 1$.

PROOF. We show the NP-hardness by a reduction from IS. Given an instance $I = (G = (V, E), t)$ of IS where $E = \{e'_1, \ldots, e'_m\}$, and $n = |V|$, we construct the instance $I' = (\mathcal{C}, \mathcal{P})$ of ONE-DESTINATION-MAX as follows: Our candidate set is $\mathcal{C} = A \cup B \cup C \cup \{p\} \cup E^*$ where $A = \{a_1, \ldots, a_m\}$, $B = \{b_1, \ldots, b_m\}$, $C = \{c_1, \ldots, c_m\}$ and E^* contains a candidate for each edge in E. We construct the following set of parties. Here, the elements in A, B, C and E^* are ordered according to the indices of the elements.
(1) For each $v \in V$ create a party P_v containing one voter with the preference $A \succ E^* \setminus E(v) \succ C \succ p \succ E(v) \succ B$. Here $E(v)$ denotes the set of the edges incident to the vertex v. Let Z denote the set of the voters in these parties.
(2) We have one party P' containing n voters with the preference $B \succ C \succ p \succ E^* \succ A$ and one party P containing one voter with the preference $E^* \succ p \succ A \succ B \succ C$.

Observe that we have $2n+1$ voters; thus there is no tie. The initial scores of the candidates, which follow directly from Table 5, are as follows:

$s(p) = |A| + |B| + |E^*|$
$s(a_i) = (|A| - 1) + |B| + |C|$
$s(b_i) = (|B| - 1) + |C|$
$s(e_i) \leq |A| + |B| + |E^*| - 1$
$s(c_i) \leq 1 + |E^*| + |C| - 1$

We are ready to prove the correctness.

($\Rightarrow$:) Let S be an independent set of size k in G. We switch all the voters corresponding to the vertices in S from Z to party P. Since S is an independent set, for every edge candidate e_i, there is at most one voter in S preferring p to e_i. Thus, even after the switching of these voters, p still beats every candidate $e_i \in E^*$ by at least $n+1$ voters. Thus, the score of p remains unchanged. The only candidates, whose score may increase after the switching of voters, are the candidates $e_i \in E^*$. A candidate e_i can have a score at least that of p only if e_i beats p or some candidate c_i. However, this is impossible, since S is an independent set and e_i beats p and every c_i by $n-2$ in the original election. Thus p still remains the unique winner.

($\Leftarrow$:) Suppose it is possible to switch a set S' of k voters to a destination party, such that p still remains the winner. First observe that P' cannot be the destination party, since otherwise, b_1 would replace p as the winner. We distinguish the following two cases:

Case 1. P is the destination party. Observe that irrespective of the composition of S', p still beats all the candidates in $A \cup B$ but none in C. Moreover, no candidate in $A \cup B \cup C$ can increase its score. Since p is the unique winner in the final election, no voters in S' come from the party P', since otherwise, some e_i would beat p and thus prevent p from being the unique winner. Therefore, all voters of S' must be from Z. More specifically, the vertices corresponding to S' form an independent set, since otherwise, some edge candidate would replace p as the winner.

Case 2. Some party P_v is the destination party. In this case, no voter of S' is from $P \cup P'$, since otherwise, since a_1 would prevent p from becoming the winner. Thus, all the votes of S' are from the set Z. We claim that the vertices corresponding to S' form an independent set. For contradiction, assume that this is not true. Then, there must be an edge e_i for which there are two voters in S' preferring p to e_i. Note that $e_i \notin E(v)$. Therefore, p cannot beat e_i, leading to that p's score is one less than that of e_i in the original election. Hence, a_1 would prevent p from becoming the unique winner, a contradiction. □

6. CONCLUSION

We examined the election systems with parties. Here, parties can be partitioned into stable and instable parties. Since members of instable parties may switch to stable parties, the outcome of the election could be different to the prediction made based on the preferences of the parties. We introduced two parameters MIN and MAX to measure the credibility of the prediction of such elections and presented a comprehensive study of the complexity for computing MIN and MAX under the most common positional scoring rules Plurality, r-Approval, Borda, Veto and three Condorcet-consistent rules (Condorcet, Maximin, Copeland). Our results are summarized in Table 1.

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REFERENCES

[1] A. Alba, V. Bhagwan, J. Grace, D. Gruhl, K. Haas, M. Nagarajan, J. Pieper, C. Robson, and N. Sahoo.

Applications of voting theory to information mashups. In *ICSC*, pages 10–17, 2008.

[2] D. Baumeister, M. Roos, J. Rothe, L. Schend, and L. Xia. The possible winner problem with uncertain weights. In *ECAI*, pages 133–138, 2012.

[3] V. Conitzer. Comparing multiagent systems research in combinatorial auctions and voting. *Ann. Math. Artif. Intell.*, 58(3-4):239–259, 2010.

[4] E. Ephrati and J. S. Rosenschein. A heuristic technique for multi-agent planning. *Ann. Math. Artif. Intell.*, 20(1-4):13–67, 1997.

[5] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra. How hard is bribery in elections? *J. Artif. Intell. Res. (JAIR)*, 35:485–532, 2009.

[6] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra. Multimode control attacks on elections. *J. Artif. Intell. Res. (JAIR)*, 40:305–351, 2011.

[7] M. R. Garey and D. S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman & Co., New York, NY, USA, 1979.

[8] T. F. Gonzalez. Clustering to minimize the maximum intercluster distance. *Theor. Comput. Sci.*, 38:293–306, 1985.

[9] T. R. Magrino, R. L. Rivest, and E. Shen. Computing the margin of victory in IRV elections. In *EVT/WOTE*, 2011.

[10] A. Mao, A. D. Procaccia, and Y. Chen. Better human computation through principled voting. In *AAAI*, pages 1142–1148, 2013.

[11] T. Perek, P. Faliszewski, M. S. Pini, and F. Rossi. The complexity of losing voters. In *AAMAS*, pages 407–414, 2013.

[12] J. Pitt, L. Kamara, M. J. Sergot, and A. Artikis. Voting in multi-agent systems. *Comput. J.*, 49(2):156–170, 2006.

[13] G. Popescu. Group recommender systems as a voting problem. In *HCI (26)*, pages 412–421, 2013.

[14] L. Xia. Computing the margin of victory for various voting rules. In *EC*, pages 982–999, 2012.

[15] Y. Yang, Y. R. Shrestha, and J. Guo. How hard is bribery in party based elections? In *AAMAS*, 2015. extended abstract.