Voter Dissatisfaction in Committee Elections

(Extended Abstract)

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ABSTRACT

The *minisum* and the *minimax* rules are two different rules for the election of a committee considered by Brams et al. [2]. As input they assume approval ballots from the voters. The first rule elects those committees which minimize the sum of the Hamming distances to the votes, the second one elects those committees with the smallest maximum Hamming distance to an individual vote. We extend this approach of measuring the dissatisfaction in committee elections to different forms of ballots, i.e., trichotomous votes, complete and incomplete linear orders. To measure the dissatisfaction we use a modified Hamming distance, ranksums, and a modified Kemeny distance.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent Systems*;
J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*

General Terms

Economics, Theory

Keywords

committee elections, social choice theory, computational social choice, winner determination

1. INTRODUCTION

Voting is a central task in diverse areas, and the study of axiomatic properties as well as algorithmic and computational aspects of problems related to voting is an active field of research in social choice (see e.g., [3]), where the focus is mostly on single-winner elections, and the winner is either one single candidate or has to be chosen from a set of winners by a tie-breaking rule. In addition to single-winner elections, voting is also employed to elect a set of candidates, like in the election of a council or of a committee. Besides the election of candidates, there are many other situations where a certain number of alternatives or objects must be chosen from a given set. Hence, committee elections may also be applied in artificial intelligence and multi agent systems, for example in the design of recommender systems, where the task is to choose a fixed number of products to recommend to a user (see [11]). The algorithmic aspects of several committee voting rules have been studied in [10, 1, 4], where the focus is mostly on approval-based rules.

A widely used rule for committee elections for approval votes is the *minisum* rule, where a winning committee minimizes the sum of the Hamming distances to the individual votes. This corresponds to an utilitarian approach. Brams, et al. [2] additionally proposed the *minimax* rule, which tries to minimize the voter dissatisfaction measured by the Hamming distance. Here, a committee with the smallest maximum Hamming distance to an individual vote will be elected, which corresponds to an egalitarian approach. We extend these approaches to trichotomous votes, complete and incomplete linear orders. Closely related to the minimization of voter dissatisfaction in committee elections are systems of proportional representation. The main difference is that in a proportional representation scheme each voter is represented by a candidate (i.e., the political party she voted for), and the dissatisfaction is computed for this candidate and not for the committee as a whole.

2. DEFINITIONS AND NOTATIONS

An election is a pair (C,V), where $C = \{c_1,\dots,c_m\}$ is the set of candidates and V is a list of voters represented by their vote. We study four different forms of votes: **approval votes** represented as $\{0,1\}^m$ vectors, where a 1 stands for approval and a 0 for disapproval of the corresponding candidate; **trichotomous votes** represented as $\{-1,0,1\}^m$ vectors, where a 1 stands for approval, a -1 for disapproval and a 0 for an abstention regarding the corresponding candidate; **complete linear orders** represented as a total, transitive, and asymmetric binary relation over the set of candidates; **incomplete linear orders** represented as a transitive and asymmetric (not necessarily total) binary relation over the set of candidates.

We study elections where the winner is a committee of $k \leq m$ candidates. Note that the size of the committee is fixed in advance. Since the winning set may consist of several committees, one has to use a tie-breaking rule to obtain a single winning committee, if necessary. In the case of approval votes, a committee may be represented as a $\{1,0\}^m$ vector having k ones, where the approved candidates are members of the committee. Analogously, committees for trichotomous votes may be denoted by $\{1,-1\}^m$ vectors having k ones. For complete and incomplete linear orders, we denote committees as a set $K \subseteq C$ of k candidates.

We study how the dissatisfaction of a single voter with a committee may be measured, for each of the different types of votes introduced above. In a single winner approval election, the candidates

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with the highest number of approvals are the winners. The most obvious way of measuring the dissatisfaction in case of committee elections for approval votes is the Hamming distance, as it is used by Brams et al. [2]. The disagreement between $v, w \in \{1,0\}^m$ is then formally defined by: $HD(v,w) = \sum_{1 \leq i \leq m} |v(i) - w(i)|$. Single winners for trichotomous votes may be elected by *Combined Approval voting*, see [6]. The combined approval score of a candidate c_i is the sum of the i-th entries of all trichotomous votes, and the candidates with the highest score wins. For committee elections, the Hamming distance is adapted to trichotomous votes such that a complete disagreement adds two points, and an abstention in one vote and an approval/disapproval in the other one adds one point. Formally, it is defined analogously to the Hamming distance, but to avoid confusion we denote it by δ.

Since there is no direct way to extent the Hamming distance to linear orders, we use other metrics in this case. For complete linear orders, we follow the approach of the Wilcoxon rank-sum test [12]. We take the sum of the ranks of the committee members in a vote to measure the dissatisfaction. Let $p(c,v)$ denote the position of candidate c in vote v, where the most liked candidate is on the first position. The normalized ranksum for voter v and committee K is $RS(K,v) = \sum_{c \in K} p(c,v) - \frac{k(k+1)}{2}$.

The last type of votes we consider are incomplete linear orders. Here, we use a modified version of the Kemeny distance [7]. The usual Kemeny distance is defined to compare two linear orders that may contain indifferences. The disagreement between two votes v and w is defined as $\sum_{a,b \in C} d'_{v,w}(a,b)$, where the distance $d'_{v,w}$ between the votes for an unordered pair of candidates a, b is 1, if in one vote a and b are considered equal and in the other vote one of them is preferred. The distance is 2, if in one vote it holds $a > b$ and in the other one $b > a$, in all other cases the distance is 0. The winners in a Kemeny election are those which are on a first position in a linear order for which the sum of the distances to the votes is minimum. Dwork et al. [5] consider a variant of Kemeny with incomplete linear orders, but they only define the case where a vote is a linear order over a subset of the candidates, whereas we consider arbitrary incomplete linear orders. We adapt the Kemeny distance in a slightly different way to measure the disagreement between incomplete linear orders and a committee. For two candidates a and b the distance $d_{K,v}(a,b)$ between a vote and a committee is 2, if only one of them is in the committee but in the vote the other one is preferred. The distance is 1, if only one of them is in the committee and in the vote the relation between both is unknown, in all other cases the distance is 0. This can be seen as a slightly more pessimistic variant, since in our model the distance also increases if the relation between both candidates is unknown in the vote. The disagreement between a vote v and a committee K is then $Dist(K,v) = \sum_{a,b \in C} d_{K,v}(a,b)$.

We follow the approach of Brams et al. [2] by defining *minisum* and *minimax* rules to elect a committee based on the different measures proposed above. They defined the winning committees in *minisum-approval* to be those which minimize the sum of the Hamming distances to all votes. Whereas in *minimax-approval* the committees with the smallest maximum Hamming distance to the votes are chosen as winning committees. Brams et al. [8] propose two possibilities for varying the minisum and minimax rules, namely the use of count weights and proximity weights. When determining the winning committee using count weights, the distance will always be multiplied by the number of times the ballot has been cast. Note that the use of count weights has only an effect for the minimax approach, since in the minisum approach in both definitions the sum of the distances is taken over all ballots. Let $F_k(C)$ denote all committees of size k. Now, we combine the minisum and minimax rules with the above defined measures of disagreement to define committee election systems.

Formally, the set of winning committees in the minisum rule are $\text{argmin}_{K \in F_k(C)} \sum_{v \in V} \triangle(K,v)$, and in the minimax rule $\text{argmin}_{K \in F_K(C)} \max_{v \in V} \triangle(K,v)$, where $\triangle \in \{HD, \delta, RS, Dist\}$ for the corresponding type of vote. The resulting voting systems are *minisum/minimax-CAV* for trichotomous votes, *minisum/minimax-ranksum* for complete linear orders, and *minisum/minimax-Kemeny* for incomplete linear orders.

THEOREM 1. *The set of winning committees in a minisum/minimax-ranksum election and in a minisum/minimax-Kemeny election with complete linear orders are always equal.*

This is true, since $Dist(K,v) = 2 \cdot RS(K,v)$, for any committee $K \in F_k(C)$ and any complete linear order v over C. It is known that winner determination for minisum-approval is in P (see [2]) and for minimax-approval it is NP-hard (see [9]). It is trivial that winner determination for minisum-CAV is also in P, the complexity of all other winner determination problems is open.

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