How Hard is Bribery in Party Based Elections?

(Extended Abstract)

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ABSTRACT

In a party-based election voters are grouped into parties and the voters belonging to the same party are assumed to cast their votes according to the fixed party preference over the set of candidates. For such elections, we investigate the complexity of the following problem: can we make some distinguished candidate win (or lose) the election by bribing at most k voters to switch from their original parties to parties with similar preferences? Here, we adopt the Kendall-Tau distance and the Hamming distance to measure the similarity of the party preferences. We achieve a wide range of complexity results for this problem under a variety of voting rules, including Borda, r-Approval, Condorcet, Copeland$^{\alpha}$ for every $0 \leq \alpha \leq 1$ and Maximin.

Categories and Subject Descriptors

F.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity; J.4 [**Computer Applications**]: Social Choice and Behavioral Sciences

General Terms

Algorithms

Keywords

bribery, voting system, complexity, party-based election

1. INTRODUCTION

In this paper, we study the voting scenarios, where voters can be grouped into parties (or interest groups), each with a fixed preference over a set of candidates (or alternatives), and the party members are required to follow party discipline, that is, the voters of the same party should all vote according to the party preference. Moreover, an external agent (for example, the leader of a major party) attempts by bribing or persuading some voters to change their preferences to change the outcome of the voting in his own favor. This might be to make some distinguished candidate win the election or make some distinguished candidate lose the election. Often one can observe that, while a voter, bribed by an external agent, is willing to "join" other parties, that is, to vote according to the preference of other parties than her/his own party, she/he may prefer to join a party, whose preference deviates as little as possible from the preference of her/his own party. Indeed, if voting is public, a voter may be worried that changing her/his preference dramatically may harm her/his reputation; the opinion change may be considered as reasonable if her/his final vote is sufficiently similar to the preference of her/his own party. Thus, in our model of this party-based bribery, we assume that the party switch of the voters is constrained by some deviation between party preferences. To quantify the amount of deviation allowed in party switch, we use two *distance* measures, the arguably most prominent ones on votes, namely, the *Hamming distance* and *Kendall-Tau distance*.

2. PRELIMINARIES

Party-based election. A *party-based election* is defined as a tuple $\mathcal{E} = (\mathcal{C}, \mathcal{P})$, where $\mathcal{C} = \{c_1, \ldots, c_m\}$ is a set of *alternatives/candidates* and $\mathcal{P} = \{P_1, \ldots, P_l\}$ is a set of *parties*. Each party is characterized by the number of its members (voters) n_i and a party preference $\succ_i$, $P_i = (n_i, \succ_i)$. A *preference* is a linear order that ranks the candidates from the most preferred one to the least preferred one. For example, if $\mathcal{C} = \{a, b, c\}$ and some party likes a best, then b, and then c, then its preference is represented as $a \succ b \succ c$. The *position* of a candidate c in a preference $\succ$ is defined as $pos_{\succ}(c) = |\{c' \in \mathcal{C} \mid c' \succ c\}| + 1$. The final voting of a party-based election denoted by $\mathcal{V}$ is a list of preferences, which one-to-one corresponds to the voters in $\mathcal{E}$, and can be partitioned into l subsets $V_1, \cdots, V_l$ such that $|V_i| = n_i$ for all i's and all preferences in V_i are identical to $\succ_i$. For two distinct candidates c and d, we define $N_{\mathcal{E}}(c, d)$ as the number of preferences in $\mathcal{V}$ with $c \succ d$. We say a candidate c beats (resp. ties) another candidate c' if $N_{\mathcal{E}}(c, c') > N_{\mathcal{E}}(c', c)$ (resp. $N_{\mathcal{E}}(c, c') = N_{\mathcal{E}}(c', c)$).

Voting rule. A *voting rule* is a function R that given an election $\mathcal{E} = (\mathcal{C}, \mathcal{P})$ returns a non-empty subset $R(\mathcal{E}) \subseteq \mathcal{C}$ of the candidates that are said to win the election.

In this paper, we consider the following voting rules. An m-candidate *positional scoring rule* is defined through a non-increasing vector $\alpha = (\alpha_1, \ldots, \alpha_m)$ of non-negative integers. A candidate $c \in \mathcal{C}$ is assigned α_i points from each preference in $\mathcal{V}$ that ranks c in the i^{th} position. The score of a candidate is the sum of points he gets from all preferences. The candidate(s) with the maximum

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score are the winner(s). Many voting rules can be considered as positional scoring rules. We study the following scoring rules (for m candidates) in this paper: r-Approval (scoring vector with r ones followed by $m-r$ zeroes, and Borda (scoring vector $(m-1, m-2, \ldots, 0)$).

A *Condorcet-consistent rule* always elects the Condorcet winner, if it exists. The *Condorcet winner* is the candidate who beats all other candidates in $\mathcal{C}$. Examples of Condorcet-consistent rules, that will be considered in this paper, are Maximin and Copeland$^\alpha$. For a candidate c in an election, let $B(c)$ be the set of candidates which are beaten by c and let $T(c)$ be the set of candidates which tie with c. Then, the Copeland$^\alpha$ score of c is $|B(c)| + \alpha \cdot |T(c)|$, for $0 \leq \alpha \leq 1$. A candidate is a Copeland$^\alpha$ winner if it has the highest score. On the other hand, the maximin score of a candidate c is given by $\min_{d \in \mathcal{C} \setminus \{c\}} N_{\mathcal{E}}(c,d)$, and the winner in a maximin election is a candidate with the highest score.

Distance. The Hamming distance between two linear orders $\succ_v$ and $\succ_u$ is the total number of positions where they differ. For example, let $\succ_v$ be $a \succ_v b \succ_v c \succ_v d$ and $\succ_u$ be $c \succ_u b \succ_u a \succ_u d$, then the Hamming distance of $\succ_v$ and $\succ_u$ is two since they have two positions (the first and the third positions) which have different elements.

The Kendall-Tau distance between two linear orders is the total number of pairs of candidates who are ranked differently. The formal definition is as follows.

$$\mathrm{d}_{KT}(\succ_v, \succ_u) = |\{(c, c') | c \succ_v c' \text{ and } c' \succ_u c\}|$$

The Kendall-Tau distance between two linear orders $\succ_v$ and $\succ_u$ is also equal to the least number of swaps of adjacent candidates that transform $\succ_v$ into $\succ_u$.

Problem Definitions. We mainly study the following problem under different voting rules. When we say that a voter *switches* from its original party to another party, we mean that the respective voter casts his vote according to the preference of the destination party. More formally, given $(\mathcal{C}, \mathcal{P})$ with $\mathcal{P} = \{P_1, \cdots, P_l\}$, a voter in $P_i \in \mathcal{P}$ switching to $P_j \in \mathcal{P}$ creates a new election $(\mathcal{C}, \mathcal{P}')$ where $\mathcal{P}' = \{P_1, \cdots, P_i', \cdots, P_j', \cdots, P_l\}$ with $P_i' = (n_i - 1, \succ_i)$ and $P_j' = (n_j + 1, \succ_j)$. Two parties are *$d$-close* if the distance between them is at most d.

CONSTRUCTIVE/DESTRUCTIVE-$distance(d)$-φ
(C/D-$distance(d)$-φ for short)
Input: A party based election $(\mathcal{C} \cup \{p\}, \mathcal{P})$, a designed voting rule φ and two integers $k \geq 0$ and $d \geq 0$. Here, p is a distinguished candidate.
Question: Is it possible to make the distinguished candidate p winner (resp. not winner) by switching at most k voters from their own parties to parties which are d-close to their original parties, under the voting rule φ? Here, $distance$ can be either Hamming distance or Kendall-Tau distance.

See Tables 1 and 2 for a summary of our results.

	$d=1,2$		$3 \leq d \leq 6$		$d=7,8$		$d \geq 9$	
	C	D	C	D	C	D	C	D
4-approval		P		P		P	NP-h	P
Borda		P	NP-h	P	NP-h	P	NP-h	P
Condorcet	P		NP-h	P	NP-h	P	NP-h	P
Maximin		P	NP-h		NP-h			
Copeland$^\alpha$			NP-h		NP-h			

Table 1: A summary of results concerning Kendall-Tau distance. Here, 'C' stands for the constructive case and 'D' stands for the destructive case. Moreover, 'P' stands for polynomial-time solvable and 'NP-h' stands for NP-hard. Empty entries mean that the corresponding problems are open. The results for Copeland$^\alpha$ apply to every $0 \leq \alpha \leq 1$.

		Scoring	Condorcet	Maximin	Copeland$^\alpha$
$d \geq 2$	C		NP-h	NP-h	NP-h
	D	P	P	NP-h	NP-h

Table 2: A summary of results concerning Hamming distance. The results for Copeland$^\alpha$ apply to every $0 \leq \alpha \leq 1$.

3. RELATED WORK

Our paper is related to the work by Perek et al. [3]. They considered the party-based elections, where there is a "leading" party with a favorable prediction. The main goal is to calculate how safe is the leading party with respect to losing its members to other parties. To this end, they introduced two parameters, the minimum number of members to lose to change the outcome (PES) and the maximum number of members to lose without changing the outcome (OPT), and studied the complexity of calculating these two parameters.

Recently, Guo et al. [1] studied the complexity of calculating two further parameters MIN and MAX in party-based elections. The parameter MIN is defined as the minimum number of voters switching from their own parties to other existing parties such that the winner changes, while the parameter MAX is defined as the maximum number of voters switching from their parties such that the winner does not change. The model studied by Guo et al. differs from the one by Perek et al. in the way that the former model allows more than one party (not specified in advance) whose members can switch to other parties, while the latter model allows only the members in the leading party (given in advance) to switch.

With respect to the distance constraint of the preference, Obraztsova and Elkind [2] studied a variant of manipulation, where a manipulator aims to construct a manipulative vote, that achieves the manipulator's goal, but is as close as possible to his true preference order. They analyzed this problem for three natural notions of closeness, namely, Kendall-Tau distance, footrule-distance, and maximum displacement distance, for a variety of voting rules.

4. CONCLUSION

We have studied the bribery model, where voters in a party may switch to another party, which has a similar opinion. In particular, we adopted the Hamming distance and the Kendall-Tau distance to measure the similarities between two parties and achieved both NP-hardness and polynomial-time solvability results for Condorcet, Maximin, Copeland$^\alpha$ for every $0 \leq \alpha \leq 1$, Borda and 4-Approval.

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