How Hard is Control in Multi-Peaked Elections: A Parameterized Study

(Extended Abstract)

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ABSTRACT

We study the complexity of voting control problems in multi-peaked elections. In particular, we focus on the constructive/destructive control by adding/deleting votes under Condorcet, Maximin and Copeland$^\alpha$ voting systems. We show that the $\mathcal{NP}$-hardness of these problems (except for the destructive control by adding/deleting votes under Condorcet, which is polynomial-time solvable in the general case) hold even in κ-peaked elections with κ being a very small constant. Furthermore, from the parameterized complexity point of view, our reductions actually show that these problems are $\mathcal{W}[1]$-hard in κ-peaked elections with $\kappa = 3, 4$, with respect to the number of added/deleted votes.

Categories and Subject Descriptors

F.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity; G.2.1 [**Combinatorics**]: Combinatorial algorithms; J.4 [**Computer Applications**]: Social Choice and Behavioral Sciences

General Terms

Algorithms

Keywords

single-peaked generalization, multi-peaked, parameterized complexity, election control, Condorcet, Maximin, Copeland

1. INTRODUCTION

Voting is a common method for preference aggregation and collective decision-making, and has applications in political elections, multiagent systems, web spam reduction, etc. Recently, the complexity of various voting problems in single-peaked elections has been attracting attention of many researchers from both theoretical computer science and social choice communities. It turned out that many voting problems being $\mathcal{NP}$-hard in general become polynomial-time solvable when restricted to single-peaked elections [1, 3].

In this paper, we consider a natural generalization of single-peaked elections, where more than one peak may occur in each vote. We mainly study control problems for Condorcet, Copeland$^\alpha$ and Maximin voting restricted to κ-peaked elections, aiming at exploring the complexity border for these control problems. Our results are summarized in Table 1.

2. PRELIMINARIES

Elections: An *election* is a tuple $\mathcal{E} = (\mathcal{C}, \Pi_\mathcal{V})$, where $\mathcal{C}$ is a set of candidates and $\Pi_\mathcal{V}$ is a multiset of votes casted by a set of voters $\mathcal{V}$. Each vote is defined as a linear order $\succ$ over $\mathcal{C}$. For two candidates c, c' and a vote $\succ$, we say c is ranked above c' in $\succ$ if $c \succ c'$. We use $N_\mathcal{E}(c, c')$ to denote the number of votes ranking c above c' in $\mathcal{E}$. We say c *beats* c' if $N_\mathcal{E}(c, c') > N_\mathcal{E}(c', c)$, and c ties c' if $N_\mathcal{E}(c, c') = N_\mathcal{E}(c', c)$. Moreover, the *position* of a candidate c in a vote $\succ$ is defined as $|\{c' \in \mathcal{C} \mid c' \succ c\}| + 1$. A *voting correspondence* φ is a function that maps an election $\mathcal{E} = (\mathcal{C}, \Pi_\mathcal{V})$ to a subset $\varphi(\mathcal{E})$ of $\mathcal{C}$. We call the elements in $\varphi(\mathcal{E})$ the *winners*.

For simplicity, we also use $(a_1, a_2, ..., a_n)$ to denote the linear order $a_1 \succ a_2 \succ, ..., \succ a_n$. For a vote $\succ$ and a subset $C \subseteq \mathcal{C}$, let $\succ (C)$ denote the *partial vote* of $\succ$ restricted to C. For example, for a vote $\succ$ defined as (a, b, c, d, e), we have that $\succ (\{b, d, e\}) = (b, d, e)$.

Single-peaked/κ-peaked elections: An election $(\mathcal{C}, \Pi_\mathcal{V})$ is *single-peaked* if there is a linear order $\mathcal{L}$ of $\mathcal{C}$ such that for every vote $\succ_v$ in $\Pi_\mathcal{V}$ and every three candidates $a, b, c \in \mathcal{C}$ with $a \ \mathcal{L} \ b \ \mathcal{L} \ c$ or $c \ \mathcal{L} \ b \ \mathcal{L} \ a$, $c \succ_v b$ implies $b \succ_v a$, where $a \ \mathcal{L} \ b$ means a is ordered before b in $\mathcal{L}$. The candidate ordered in the first position of $\succ_v$ is the *peak* of $\succ_v$ with respect to $\mathcal{L}$.

For an order $\mathcal{L} = (c_1, c_2, \ldots, c_m)$ of $\mathcal{C}$ and a vote $\succ_v$, we say $\succ_v$ is κ*-peaked* with respect to $\mathcal{L}$, if there is a κ-partition $L_1 = (c_1, c_2, ..., c_x)$, $L_2 = (c_{x+1}, c_{x+2}, ..., c_{x+y}), \ldots, L_\kappa = (c_z, c_{z+1}, ..., c_m)$ of $\mathcal{L}$ such that $\succ_v (\mathcal{C}(L_i))$ is single-peaked with respect to L_i for all $1 \leq i \leq \kappa$, where $\mathcal{C}(L_i)$ is the set of candidates appearing in L_i. An election is κ-peaked if there is an order $\mathcal{L}$ of $\mathcal{C}$ such that every vote in the election is κ-peaked with respect to $\mathcal{L}$.

Voting Correspondences: We study the following voting correspondences.

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	number of peaks κ								
	$\kappa = 1$	$\kappa = 3$				$\kappa \geq 4$			
	for all	CC		DC		CC		DC	
		AV	DV	AV	DV	AV	DV	AV	DV
Condorcet		$\mathcal{W}[1]$**-hard**		$\mathcal{P}$		$\mathcal{W}[1]$**-hard**		$\mathcal{P}$	
Maximin	$\mathcal{P}$	$\mathcal{W}[1]$**-hard**	?	$\mathcal{W}[1]$**-hard**	?	$\mathcal{W}[1]$**-hard**			
Copeland$^\alpha$	$(\alpha = 1)$	$\mathcal{W}[1]$**-hard**		$\mathcal{W}[1]$**-hard**		$\mathcal{W}[1]$**-hard**			

Table 1: A summary of the complexity of control problems under Condorcet, Maximin and Copeland$^\alpha$ in κ-peaked elections. Here, "$\mathcal{P}$" stands for polynomial-time solvable. Our results are in bold. Moreover, our results for Copeland$^\alpha$ apply to all $0 \leq \alpha \leq 1$. The $\mathcal{W}[1]$-hardness results of the control by adding/deleting votes are with respect to the number of added/deleted votes. The polynomial-time solvability results in single-peaked elections (1-peaked elections) are from [1]. The polynomial-time solvability of the destructive control by adding/deleting votes for Condorcet is from [4]. The entries filled with "?" means the corresponding problems are open.

Condorcet: A *Condorcet winner* is a candidate which beats every other candidate. A *weak Condorcet winner* is a candidate which is not beaten by any other candidate.

Copeland$^\alpha$ ($0 \leq \alpha \leq 1$): For a candidate c, let $B(c)$ be the set of candidates who are beaten by c and $T(c)$ the set of candidates who tie with c. The Copeland$^\alpha$ score of c is then defined as $|B(c)| + \alpha \cdot |T(c)|$. A Copeland$^\alpha$ winner is a candidate with the highest score.

Maximin: For a candidate c, the Maximin score of c is defined as $\min_{c' \in \mathcal{C} \setminus \{c\}} N_{\mathcal{E}}(c, c')$. A Maximin winner is a candidate with the highest Maximin score.

Problem Definitions: Problems studied here are characterized by three factors, CC|DC specifying constructive or destructive control, AV|DV specifying adding or deleting votes, φ specifying the voting correspondence. In the inputs of all these problems, we have a set $\mathcal{C}$ of candidates, a distinguished candidate p, and an integer $R \geq 0$. In the deleting votes case, there is only one multiset $\Pi_{\mathcal{V}_1}$ of (registered) votes in the input, while the adding votes case distinguishes two multisets of votes, $\Pi_{\mathcal{V}_1}$ the multiset of registered votes and $\Pi_{\mathcal{V}_2}$ the multiset of unregistered votes. The goal here is to make p win (CC) or lose (DC) the election by adding at most R unregistered votes (AV) or deleting at most R votes (DV). See Table 1 for a summary of our results.

3. RELATED WORK

Parameterized complexity of voting control problems have been extensively studied recently. In particular, Liu and Zhu [6] proved that both the constructive control and the destructive control by adding/deleting votes for Maximin are $\mathcal{W}[1]$-hard in the general case, with respect to the number of added/deleted votes. Moreover, Liu et al. [5] proved that the constructive control by adding/deleting votes for Condorcet is $\mathcal{W}[1]$-hard in the general case, with respect to the number of added/deleted votes. However, their reductions do not apply to 3,4-peaked elections. Liu and Zhu also studied parameterized complexity of other voting problems (see [7]). Recently, Yang and Guo [9] has also studied the complexity of control problems in κ-peaked elections. However, they considered only the r-approval voting systems. We complement their work by investigating the Condorcet, Maximin and Copeland$^\alpha$ voting. A special case of 2-peaked elections, called swoon-SP elections, were studied by Faliszewski et al. [2]. Further related work on complexity of strategic voting problems in generalized single-peaked elections can be found in [2, 8, 10].

4. CONCLUSION

We have studied the complexity of the control problems in κ-peaked elections which generalize single-peaked elections by allowing at most κ-peaks in each vote. In particular, we proved that the $\mathcal{NP}$-hardness of control by adding/deleting votes in the general case remains for Condorcet, Maximin and Copeland$^\alpha$ for every $0 \leq \alpha \leq 1$ in κ-peaked elections, even when κ is equal to 3 or 4. Our reductions imply that these problems are $\mathcal{W}[1]$-hard with respect to the number of added/deleted votes. See Table 1 for a summary of our results.

Several challenging and intriguing questions remain open. Among them is the complexity of control by adding/deleting votes for Condorcet, Maximin and Copeland$^\alpha$ in 2-peaked elections.

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