Controlling Elections by Replacing Candidates or Votes (Extended Abstract)

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ABSTRACT

We consider elections where the chair replaces either candidates or votes, with the goal of making a specific candidate win (constructive control) or lose (destructive control). We call this "replacement control" and study its computational complexity for several scoring rules (plurality, veto, Borda, k-approval), as well as for approval voting.

Categories and Subject Descriptors

I.2.11 [[**Distributed Artificial Intelligence**]: Multiagent systems

General Terms

Theory, Algorithms, Experimentation

Keywords

Social Choice, Voting, Control

1. INTRODUCTION

In multi-agent systems, we often want a centralised mechanism to make collective decisions. If preferences are total orders over the options, preference aggregation can be achieved with a voting rule. In the terminology of voting, the agents are the voters, the options are the candidates, and the collective decision is the winning candidate. Additionally an external agent (usually called the chair) may control which agents (voters) can vote and which options (candidates) can be considered. In this setting, several kinds of strategic actions can influence the result of the election. For instance, voters may submit insincere preferences, or the chair may introduce new candidates or choose the voting rule. We focus here on control by the chair [1, 5].

Control may be constructive when the chair's goal is for a certain candidate to win, or destructive when it is to prevent a candidate winning. Actions that the chair can take is adding or deleting candidates or votes [4]. Here we consider a specific form of combining the basic control actions, that we call *replacement control*, where the chair replaces some candidates (or votes). This is the combination of deletion and addition of candidates (or votes) in the same quantity.

We study the computational complexity of replacement control for plurality, veto, Borda, k-approval, and approval. Besides providing theoretical complexity results [2], where hardness informs us only about the worst case [9], we also perform an empirical evaluation using real-world data-sets (not included in this paper). For some of the considered voting rules, our empirical evaluation shows that rules are easy to control despite theoretical analysis.

Appears in: *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2015), Bordini, Elkind, Weiss, Yolum (eds.), May 4–8, 2015, Istanbul, Turkey.*

2. REPLACEMENT CONTROL

Replacement control can be seen as the combination of the addition and deletion of either votes or candidates in equal amount. That is, the chair can *replace* some candidates or some votes. We use RC (for Replacing Candidates) and RV (for Replacing Votes). These will be combined with either constructive or destructive control (CC and DC). Formally, we will study the following four problems.

Given two collections V_1, V_2 of votes over C, $V_1 \cap V_2 = \emptyset$, a distinguished candidate $p \in C$, and $k \in \mathbb{Z}_+$, we define:

- CCRV (Constructive Control via Replacing Votes) as the problem of finding subsets $A \subseteq V_2$ and $D \subseteq V_1$ such that $|A| = |D| \leq k$ and p is the winner of the election $E = (C, (V_1 \setminus D) \cup A)$;

- DCRV (Destructive Control via Replacing Votes) as the problem of finding subsets $A \subseteq V_2$ and $D \subseteq V_1$ such that $|A| = |D| \leq k$ and p is not the winner of the election $E = (C, (V_1 \setminus D) \cup A)$.

Also, given a collection V of votes over $C_1 \cup C_2$ (with C_1 and C_2 disjoint), a distinguished candidate $p \in C_1$, and $k \in \mathbb{Z}_+$, we define:

- CCRC (Constructive Control via Replacing Candidates) tas he poblem of finding subsets $A \subseteq C_2$ and $D \subseteq C_1$ such that $|A| = |D| \leq k$ and p is the winner of the election $E = ((C_1 \setminus D) \cup A, V)$;

- DCRC (Destructive Control via Replacing Candidates) as the problem correspond to finding subsets $A \subseteq C_2$ and $D \subseteq C_1$ such that $|A| = |D| \leq k$ and $p \in (C_1 \setminus D)$ is not the winner of the election $E = ((C_1 \setminus D) \cup A, V)$.

3. RELATIONSHIP WITH OTHER STRATEGIC ACTIONS

Replacement control is related to multi-mode control [3], where the chair uses two or more control actions at the same time. However, it is not possible in general to transfer complexity results between multi-mode control and replacement control. In fact, the decision problems connected to single control actions and to replacement control could belong to different complexity classes. For example, there exists a voting rule and a class of elections on which DCAC is in P, DCDC is NP-hard and DCRC is in P.

We consider also some axiomatic properties and their impact on the relationship between various types of control, showing the following results:

THEOREM 1. *Every unanimous voting rule resistant to CCAC is also resistant to CCRC.*

Given the notion of *Insensitive to Bottom-ranked Candidates (IBC)* defined in [6], we have:

THEOREM 2.

- *Every voting rule that is IBC and resistant to CCDC is also resistant to CCRC.*
- *Every voting rule that is IBC and resistant to DCAC or DCDC is also resistant to DCRC.*

4. POSITIONAL SCORING RULES

In general, positional scoring rules are vulnerable to DCRV. However, different scoring rules behave differently for the other problems.

THEOREM 3.

- *Positional scoring rules are vulnerable to DCRV.*
- *Plurality and veto are vulnerable to CCRV and DCRV, but resistant to CCRC and DCRC.*
- *k-approval is resistant to CCRC and DCRC, for all k. For $2 < k < m-2$, k-approval is resistant to CCRV and it is vulnerable to DCRV.*

Borda has been studied in [8], which provides some complexity results about control by adding/deleting voters and left open the control problems by adding/deleting candidates. We prove that Borda is resistant to constructive replacement control, while it is vulnerable to its destructive versions. We also close the open problems giving new results about the single control actions of adding or deleting candidates.

THEOREM 4.

- *Borda is resistant to CCRV and CCRC.*
- *Borda is resistant to CCAC.*
- *Borda is vulnerable to DCRC and DCRV.*
- *Borda is vulnerable to DCAC and DCDC.*

It is somewhat surprising that, although DCRC is NP-hard for plurality and veto, it is instead is polynomial for Borda. This is because the difference between two consecutive scores in the Borda's scoring vector are identical, unlike in plurality or veto.

5. APPROVAL VOTING

As expected [7], approval voting behaves very differently from positional scoring rules.

THEOREM 5. *Approval is resistant to CCRV, but it is vulnerable to CCRC, DCRC and DCRV.*

6. CONCLUSIONS

We have studied the computational complexity of replacement control, a control action with which the chair tries to influence an election by replacing candidates or votes, either constructively or destructively. The following table shows our complexity results for the various voting rules and types of replacement control.

Control	Plurality	Veto	Borda	Approval	k-approval
CCRV	V	V	R	R	R ($2 < k < m-2$)
DCRV	V	V	V	V	V
CCRC	R	R	R	V	R
DCRC	R	R	V	V	R

Table 1: Summary of results (V: vulnerable, R: resistant).

We also performed an experimental analysis (not included in this paper), using real-world data sets, which suggests that k-approval can be easy to control in practice despite our theoretical analysis classifying it as resistant to the control action.

REFERENCES

[1] J. J. Bartholdi, C. A. Tovey, and M. A. Trick. How hard is it to control an election. *Mathematical and Computer Modeling*, pages 27–40, 1992.

[2] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra. Using complexity to protect elections. *Commun. ACM*, 53(11):74–82, 2010.

[3] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra. Multimode control attacks on elections. *JAIR*, pages 305–351, 2011.

[4] P. Faliszewski, E. Hemaspaandra, L. A. Hemaspaandra, and J. Rothe. Llull and copeland voting computationally resist bribery and control. *JAIR*, pages 275–341, 2009.

[5] E. Hemaspaandra, L. A. Hemaspaandra, and J. Rothe. Anyone but him: The complexity of precluding an alternative. *Artificial Intelligence*, 171:255–285, 2005.

[6] J. Lang, N. Maudet, and M. Polukarov. New results on equilibria in strategic candidacy. In B. Vöcking, editor, *SAGT*, volume 8146 of *Lecture Notes in Computer Science*, pages 13–25. Springer, 2013.

[7] A. Lin. The complexity of manipulating k-approval elections. In J. Filipe and A. L. N. Fred, editors, *ICAART (2)*, pages 212–218. SciTePress, 2011.

[8] N. F. Russell. Complexity of Control of Borda Count Elections, 2007.

[9] T. Walsh. Is computational complexity a barrier to manipulation? *Annals of Mathematics and Artificial Intelligence*, 62(1-2):7–26, June 2011.